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ADRIAAN VAN MAANEN
1884–1946

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ADRIAAN VAN MAANEN

1884–1946

Adriaan van Maanen, for more than thirty-three years a member of the staff of the Mount Wilson Observatory, died from a heart ailment on January 26, 1946.

He was born in Sneek, Holland, on March 31, 1884, the son of Johan Willem Gerbrand and Catharine Adriana van Maanen. He was educated at the University of Utrecht, where he received his B.A. in 1906, his M.A. in 1909, and his Sc.D. in 1911. During 1909–11 he worked at the Groningen Observatory under the eminent Dutch astronomer, Kapteyn, who at that time was directing the attention of the astronomical world to the study of stellar motions through his discovery of the phenomena of star streaming. This contact with Kapteyn shaped van Maanen's life in two ways. It influenced him to devote his efforts to the field of astrometry and led him to follow his teacher on a visit to the United States to find larger instruments and better climatic conditions for such work. So in 1911, with the aid of a gift from an older cousin, he went to the Yerkes Observatory as a volunteer assistant and in July, 1912, came to the Mount Wilson Observatory, again as a volunteer assistant. On September 1, 1912, he was appointed a member of the Observatory staff.

In his early days on Mount Wilson, van Maanen assisted in the daily observations of the sun with the 5-foot spectroheliograph and took part in the general program of spectrographic observations with the 60-inch reflector. In 1913, however, he started his own program of observations for the determination of stellar parallaxes and in 1915 began observations for the determination of proper motions. In Pasadena he assisted Dr. Hale in measuring the plates taken to determine the general magnetic field of the sun, work which demanded a considerable portion of his office-time up to 1920.

His life was devoted primarily to four definite programs, three of them his own, the fourth Dr. Hale's: (1) parallaxes of stars and planetary nebulae; (2) proper motions of stars, clusters, and nebulae; (3) internal motions of clusters and spiral nebulae; and (4) the general magnetic field of the sun. His major papers, numbering about fifty, appeared as *Contributions from the Mount Wilson Observatory* in the *Astrophysical Journal*, and his bibliography shows over a hundred minor articles published in the United States and Europe, about half of them in the *Publications of the Astronomical Society of the Pacific*.

It should be pointed out that van Maanen's was the first serious attempt to use a large reflector for the measurement of the stellar parallax. He encountered many difficulties not met in the use of a refracting telescope and was successful in overcoming most of them. A few parallax plates with the 100-inch telescope, using a reducing sector, were unsatisfactory because of coma in the images too far from the center of the

plates; he showed, however, that the 100-inch telescope plates taken without the sector, permitting the use of comparison stars near the plate center, gave good results. Thereafter, in his parallax work he observed the brighter stars which needed reduction in light at the Cassegrain focus of the 60-inch telescope and used the Newtonian focus of the 100-inch only for the observation of faint stars requiring no reduction. The efficiency of this procedure was appreciated by other parallax observers, many of whom requested observations of stars so faint that exposure times with a refractor would be unduly long. After 1930, in a search for stars of low luminosity, van Maanen concentrated his attention on faint stars with large proper motion. This work, which could have been done efficiently only with a large telescope, has been of unique value in adding to our knowledge of star density and of the luminosity function in the neighborhood of the sun. By 1944, when the first symptoms of his illness developed, van Maanen had published parallaxes of objects in 475 fields. As soon as it appeared probable that he would be unable to continue his observational work, it became his ambition to finish the measurement of an even 500 fields. Although he was able to work but a few hours each day, he completed the measures of the final 25 fields only a few days before his death and left the results in such form that they can readily be prepared for publication.

The thesis presented by van Maanen for his doctorate in 1911 was entitled "The Proper Motions of 1418 Stars in and near the Clusters *h* and χ Persei." The probable errors of the proper motions, $\pm 0\overset{''}{.}0057$, were not large for that era; but, since the motions of most of the stars in the region amount to only a few thousandths of a second of arc, the observations were not accurate enough to indicate which belonged to the cluster and which did not. Almost the last paper he published (1944) was "The Proper Motion of the Cluster *h* Persei." He had reduced the probable errors of the motions nearly ninefold, to $\pm 0\overset{''}{.}00067$, and was able to assign 320 stars definitely to the cluster and to indicate the probability that 300 more are cluster members. This persistent search for greater accuracy was characteristic, and his measures of motions in the galactic clusters, such as Perseus, the Pleiades, and Messier 13 and 67, and in regions in the Hyades and Orion should prove of outstanding value in the study of the constitution and life-histories of these organizations. One program in the field of proper motions he was unable to complete, the very important determinations of the motions of a selected list of 125 variable stars, mainly Cepheids with both long and short periods. Nearly all the second-epoch plates have been secured, and the investigation will soon be completed by others.

Two of van Maanen's investigations—his measures of the apparent rotation of spiral nebulae and those of the general magnetic field of the sun—have been subject to wide discussion and criticism. The reasons are obvious: both sets of measures have far-reaching cosmical significance, and both involve quantities in general smaller than the probable errors of measurement.

When unimpeachable evidence of the great distances of the spiral nebulae was gathered (1925), it became clear that van Maanen's consistent measures after 1916 of the apparent rotations could not represent actual motion. Several pairs of plates measured by him were remeasured on the same comparator and reduced by the same methods by a number of other observers. Their measures indicated either no rotation or at most a small fraction of that originally indicated. Later, van Maanen himself remeasured some of the fields on plates separated by a greater interval of time and found much smaller internal motions, although the total displacements were much the same. When the difficulty of these measures, not on sharp stellar images but on nodules of fused stellar images and nebulosity, is appreciated, it seems safe and just to conclude that the large apparent motions measured by van Maanen were the results of systematic errors due to the character and distribution of the images.

With regard to the sun's magnetic field Dr. Hale wrote in 1914: "Our knowledge of the general magnetic field of the sun is based upon line displacements so minute as to fall within the ordinary limits of error of spectroscopic measurements." Measures of the

field were all made by van Maanen without knowledge of the part of the sun under observation and utilized the parallel-plate method, which was considered the most accurate available at that time for the purpose. This method is especially effective because the edges of the lines are the parts primarily affected by a magnetic field. The results were substantiated by measures made with the Koch registering microphotometer, which supposedly removed any personal equation. So, in spite of the fact that other measurers using the less desirable method of the wire-micrometer failed to detect the displacements, Hale and others felt that the existence of a general magnetic field during the period 1912–20 was definitely established. In a later series of measurements (1933–35), in which van Maanen had no part and for which new instrumental techniques had been developed, several experienced observers derived results, in Hale's words, "reasonably parallel to those of van Maanen," while others equally experienced got no positive results. Hale concluded that "possibly the general magnetic field of the sun varies in intensity in an unknown period." New methods developed by Babcock are now being applied to this study, and judgment as to the strength and possible variability of the sun's magnetic field should be reserved until the results of new measures have been carefully examined.

Van Maanen was an active member of numerous scientific organizations, among them the American Astronomical Society, the Astronomical Society of the Pacific, the Royal Astronomical Society, the Société Astronomique de France, the Astronomische Gesellschaft, the Amsterdam Academy of Sciences, the Utrecht Society, and Sigma Xi. He was especially interested in the International Astronomical Union, of which he was a member from its inception. He served as a member of four of its commissions: (24) Parallaxes and Proper Motions, (28) Nebulae and Star Clusters, (32) Selected Areas, and (33) Stellar Statistics; and he went abroad to attend its meetings at Cambridge (1925), Paris (1935), and Stockholm (1938).

He never married but, nevertheless, enjoyed an extensive social life. He was good company and was continually in demand to complete a table of bridge or fill out a dinner party. He was a member of the Valley Hunt Club of Pasadena and delighted in entertaining his friends there and at his home. Among all his activities, however, the one closest to his heart arose from his interest in the education of the promising young man of limited financial means. He was the prime organizer of the Students Fund, Inc., raised most of the money, and from 1928 to 1940 served as president. This organization has to date provided substantial aid in the education of 202 young men. He took great pride in the fact that this fund had never suffered a serious loss, the loans having been repaid after graduation, many times with added contributions, and he lived long enough to see some of the former recipients of loans become directors of the fund. In perpetuation of his memory this fund has recently been renamed the "van Maanen Fund."

Ralph E. Wilson

Mount Wilson Observatory

SIX-COLOR PHOTOMETRY OF STARS

IV. THE VARIATION OF α URSAE MINORIS AT DIFFERENT WAVE LENGTHS*

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Received November 21, 1945

ABSTRACT

The *amplitudes* of the light-variation of Polaris in the period of 3.96 days are found to range from 0.166 mag. at 3530 A to 0.036 mag. at 10,300 A in the simple sine-curves which characterize the changes of this star. The variation at each wave length is close to one-ninth of the corresponding variation of δ Cephei. The inferred *variation of spectral type* of Polaris is from cF6 to cF7. The *period* of the variation has become longer than predicted by any of the formulae derived in the past.

Ever since the light-variation of Polaris was established by E. Hertzsprung[2] in 1911, this star has stood out at one end of the sequence of Cepheid variables as having the smallest amplitude of change. Compared with a photographic range of 0.17 mag. for Polaris, no other star in H. Schneller's[3] catalogue for 1941 is clearly of the same type, with a variation of less than 0.30 mag. The continuous variables of small range, like β Cephei and 12 Lacertae, have early B-type spectra; and ordinarily they do not repeat their changes accurately even in successive cycles. Likewise, δ Scuti, though of spectrum F4, is irregular in its variation.[4] Polaris, however, seems to be a true variable of the δ Cephei or ζ Geminorum type, repeating its light and spectral changes in the period of 3.96 days continuously for years.

After the light-curves in six colors for δ Cephei[5] itself had been determined with a photocell on the 60-inch reflector at Mount Wilson, we planned to do the same for Polaris when opportunity offered. Accordingly, the star was measured on eight nights in the summer of 1944 to fix the amplitudes of the simple sine-curves which characterize the variation. As expected, there was a progressive decrease in the amplitudes with increasing wave length from 3530 A to 10,300 A. Because of the small total variation it would be very difficult to establish a slight retardation of phase like that of δ Cephei for the longer wave lengths. For Polaris the magnitudes in five of the colors seemed to change together in the same phase; but in the infrared the results pointed to a total amplitude of slightly more than 0.01 mag., exactly opposite in phase to the variation in the other wave lengths. Such a complicating circumstance for the pulsation theory of stellar variation would need to be thoroughly established; and a hundredth of a magnitude, even with a photocell, is scarcely enough evidence for far-reaching conclusions. As I simply did not believe this contradiction in phase, the measures were held over until the next year, when the cause of the trouble was found.

At the outset in 1945 it was found that the measures of 1944 had probably been affected by stray light leaking past the edge of the Newtonian flat of the 60-inch reflector. A 20-inch disk, placed on the Newtonian cage to fill up the central openings of the annular wire screens used by Seares for bright stars, had been removed sometime before 1944. The leak was found to be not large—less than half of 1 per cent of the light incident upon

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 718.

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[2] *A.N.*, **189**, 89, 1911.

[3] *Kl. Veröff. Berlin-Babelsberg*, No. 22, 1940.

[4] E. A. Fath, *Lick Obs. Bull.*, **18**, 77, 1937.

[5] J. Stebbins, *Mt. Wilson Contr.*, No. 704; *Ap. J.*, **101**, 47, 1945

the 60-inch mirror—but it amounted to about 0.07 mag. for the light transmitted by the 3-mag. screen used on Polaris; and, in addition, the leak may have changed by 0.01 mag., or more, in the shifting of the screen from night to night. In ordinary photometry such a circumstance would be fatal, but here with six colors the measures could be salvaged.

In Table 1 the observations of Polaris are referred to one comparison star, NPS 4 = HD 166926, visual magnitude 5.86, spectrum A3p. In 1944 each observation comprised a single setting on one star and then a setting on the other, without a check back on the first; but in 1945 the measures on the two stars were repeated immediately in the reverse order. The first column of the table gives the Julian date; the second, the phase from the elements in equation (1). For each color Δ mag. is used in the sense: Polaris through the 3-mag. screen *minus* NPS 4 in the clear. Polaris is always the brighter; hence all the tabular quantities are negative.

TABLE 1

OBSERVATIONS OF α URSAE MINORIS

JD 243	Phase	U 3530 A	V 4220 A	B 4880 A	G 5700 A	R 7190 A	I 10,300 A	Year
1293.698	$0^{\mathrm{P}}040$	$-0^{\mathrm{M}}268$	$-0^{\mathrm{M}}428$	$-0^{\mathrm{M}}645$	$-0^{\mathrm{M}}840$	$-1^{\mathrm{M}}108$	$-1^{\mathrm{M}}330$	1944
1294.682	0.288	.152	.330	.572	.800	1.075	1.335	
1295.678	0.539	.082	.310	.555	.775	1.060	1.350	
1320.667	0.834	.192	.385	.615	.818	1.090	1.325	
1321.653	0.083	.242	.418	.640	.840	1.090	1.325	
1322.675	0.340	.158	.335	.590	.752	1.082	1.345	
1323.676	0.592	.132	.308	0.568	.770	1.075	1.335	
1324.673	0.843	−0.188	−0.395	....	−0.802	−1.058	−1.345	
1679.792	0.303	−0.078	−0.212	−0.480	−0.690	−0.962	−1.222	1945
1679.847	0.317	.082	.275	.512	.785	1.028	1.258	
1680.728	0.539	.020	.225	.435	.692	0.990	1.210	
1680.737	0.541	.020	.215	.482	.698	0.970	1.225	
1681.703	0.784	.105	.345	.522	.805	1.068	1.275	
1681.715	0.787	.088	.292	.522	.728	1.005	1.240	
1682.700	0.035	.178	.345	.572	.795	1.020	1.248	
1682.711	0.038	−0.182	−0.368	−0.578	−0.802	−1.045	−1.272	

The current epoch of maximum was determined by combining the normal magnitudes of the two years and solving for the constants of the light-curve represented by the equation

$$\Delta \text{ mag.} = a + b \cos \theta + c \sin \theta ,$$

where θ is the angular phase. This solution was made for the first four colors, ultraviolet to green; and the resulting average time of maximum, combined with E. J. Meyer's[6] maximum at JD 2427689.13, gives

$$\text{Max.} = \text{JD } 2431495.988 + 3.96961\, E . \tag{1}$$

The lengthening of the period of Polaris is evident from the following determinations:

$3^{\mathrm{d}}96809$	J. H. Moore[7]	Spectroscopic	1899–1923
3.96894	N. Florja[8]	Predicted	1945
3.96907	E. J. Meyer[6]	Predicted	1945
3.96961	J. Stebbins	Observed	1936–1945

Meyer's elements have a cosine term which leads to a longest possible period of 3.96920 days; Florja gives a term in E^2 which has already run off from the observations. The

[6] *A.N.*, **256**, 425, 1935. [7] *Lick Obs. Bull.*, **11**, 166, 1924. [8] *A.N.*, **256**, 296, 1935.

variation of the 3.96-day period of Polaris has not been connected with the light-time in the spectroscopic orbit with the period of 29.6 years.[9] This variable is true to its type in showing a secular change which will require many years to evaluate.

With the period derived from our elements in equation (1), the observations in Table 1 were brought together in the normal magnitudes of Table 2, two observations to each normal. After approximate solutions the corrections for the light-leak of 1944 were made by raising or lowering all magnitudes at a given phase the same amount, to agree,

TABLE 2

NORMAL MAGNITUDES

Phase	U	V	B	G	R	I	Corr.	Year
$0^{\mathrm{P}}061$	$-0^{\mathrm{M}}262$	$-0^{\mathrm{M}}430$	$-0^{\mathrm{M}}649$	$-0^{\mathrm{M}}847$	$-1^{\mathrm{M}}106$	$-1^{\mathrm{M}}335$	$-0^{\mathrm{M}}007$	1944
0.314	.150	.327	.576	.771	1.073	1.335	+ .005	
0.566	.089	.291	.544	.754	1.050	1.324	+ .018	
0.839	−0.209	−0.409	−0.634	−0.829	−1.093	−1.354	−0.019	
0.037	−0.180	−0.356	−0.575	−0.798	−1.032	−1.260	....	1945
0.310	.080	.244	.496	.738	0.995	1.240	....	
0.540	.020	.220	.458	.695	0.980	1.218	....	
0.785	−0.096	−0.318	−0.522	−0.766	−1.036	−1.258	....	

TABLE 3

CONSTANTS OF LIGHT-CURVES

	U	V	B	G	R	I
1944 *a*	$-0^{\mathrm{M}}174$	$-0^{\mathrm{M}}361$	$-0^{\mathrm{M}}599$	$-0^{\mathrm{M}}798$	$-1^{\mathrm{M}}079$	$-1^{\mathrm{M}}334$
1945 *a*	−0.097	−0.287	−0.515	−0.751	−1.012	−1.245
Difference	−0.077	−0.074	−0.084	−0.047	−0.067	−0.089
1944 *b*	0.088	0.078	0.059	0.052	0.029	0.010
1945 *b*	0.078	0.075	0.059	0.052	0.030	0.022
α UMi *b*	0.083	0.076	0.059	0.052	0.030	0.018
δ Cep *b*/9	0.082	0.078	0.060	0.048	0.034	0.024
Difference	+0.001	−0.002	−0.001	+0.004	−0.004	−0.006

on the average, with the magnitudes of 1945. The procedure is similar to the correction for "night error" of a short-period variable. These corrections are in the next to last column of Table 2 and have already been applied to the normals of 1944.

Solutions for the light-curves in each of the six colors were next made from the normals of the two years separately, assuming the simple relation

$$\Delta \text{ mag.} = a + b \cos \theta ,$$

where a and b were determined from four equations for each color. The results are in Table 3. With our convention on signs both a and b come out negative, but the sign of b is omitted in the table.

[9] J. H. Moore, *Pub. A.S.P.*, **41**, 254, 1929.

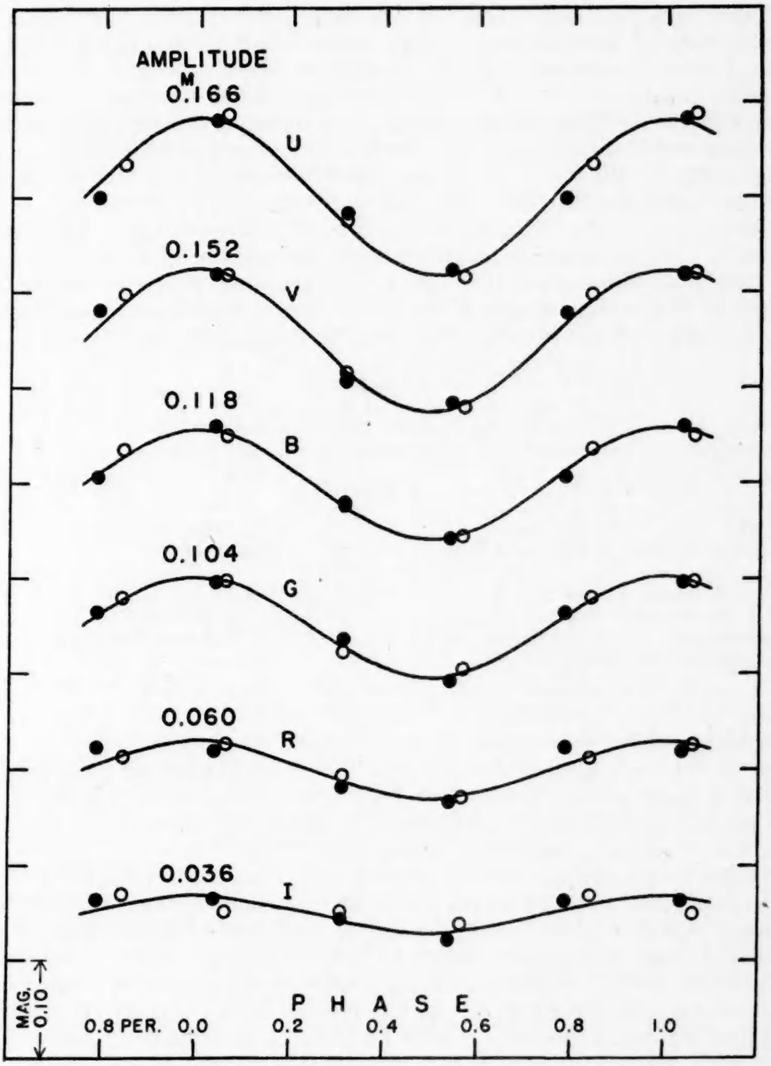

FIG. 1.—Light-curves of Polaris: open circles, 1944; solid circles, 1945

The effect of the light-leak of 1944 is shown in the systematic difference in a between the two years. The values of the semiamplitude b agree well enough, except perhaps in the infrared. The anomalous result for this color in 1944 was probably caused partly by the light-leak and partly by the small deflections of the comparison star, which were 16 or 17 mm, compared with about 50 mm for Polaris. In 1945 extra measures were made in the infrared; and the result was given double weight over that of 1944, shifting the mean of b for the two years by 0.002 mag. By a coincidence the variation of Polaris in each color turns out to be very nearly one-ninth of the corresponding variation of δ Cephei. The last three lines of the table give a comparison between the two stars, by b also denoting the semiamplitude of δ Cephei. The values from δ Cephei, with their smoother run and with the errors divided by 9, are probably better measures of the variation of Polaris than are the direct observations.

If we do not like the doctoring of the measures of 1944, we can reject them all and use only those of 1945, but the average difference in the last line of Table 3 would still remain ± 0.003 mag. However, in 1945 the sky was changing on two of the four nights, as shown by the progressive increase of the galvanometer deflections for an hour or more after dark. The photocell always gives a check on the sky. On the first and third nights

TABLE 4

AMPLITUDES OF THE VARIATION OF POLARIS

Observer	Amplitude	Reference
E. Hertzsprung, photographic	$0^{M}171$	*A.N.*, **189**, 95, 1911
J. Stebbins, mean of 3530 A and 4220 A	0.159	This paper
K. F. Bottlinger, photoelectric	0.146	*B.A.N.*, **4**, 287, 1928
E. J. Meyer, photoelectric	.136	*A.N.*, **256**, 424, 1935
Stebbins and Whitford, photoelectric	.150	*Mt. W. Contr.*, No. 586, p. 11, 1938
J. Stebbins, interpolated for 4500 A	0.138	This paper

the over-all atmospheric transmission varied at the rate of 0.05 mag. per hour, while on the other two the measured variation was 0.00 and 0.02 mag. per hour. Despite these difficulties it would require more effort than seems worth while to improve upon the present results, which, after all, are not so bad, considering that we had a variable light-leak the first year and a variable sky the second.

The curves for the data in Table 3 are in Figure 1. Each observed point on the graph is referred to the mean Δ mag. a for the corresponding color and year. For example, in the first normal of Table 2 at phase $0^{P}061$, the Δ mag. for U is -0.262, from which we subtract $a = -0.174$, the first quantity in Table 3, and get -0.088 for the first point to be plotted for 1944. The curves are averaged for the two years; and the probable errors of a normal magnitude, computed from the residuals from the mean curves, range from ± 0.004 for the blue and green to ± 0.008 for the ultraviolet and infrared, with an average of ± 0.006 mag.

The total amplitudes $2b$ of the curves in Figure 1 are the best to be derived from the present observations, although we probably would do still better to take the values from δ Cephei in Table 3. Comparisons with some other amplitudes are in Table 4. The photoelectric measures were all made with cells with maximum sensitivity presumably near 4500 A. The agreement is as good as can be expected without a more exact knowledge of the effective wave length for each observer. The amplitude of 0.078 mag. which I determined in 1912 with the selenium cell[10] is about 0.02 mag. smaller than would follow from

[10] J. Stebbins, *A.N.*, **192**, 192,1912.

the known sensitivity of that cell, whose effective wave length was about 6000 A. There is little point in a further discussion of the photographic and visual results on Polaris, some of which are obviously of inferior precision. The most recent summary seems to be that by Florja.[8]

The change of amplitude with wave length means, of course, that the over-all color of Polaris is variable, as shown in Table 5. As in our other work in six colors, the intensities are referred in magnitude to the mean of the blue, green, and red. Combining the data in Table 3 for the two years, we get the first two lines of Table 5, the colors of Polaris at maximum and minimum referred to the comparison star. When these Δ magnitudes are added to the respective values for NPS 4, the results in the fourth and fifth lines are the colors of Polaris on our standard system, referred to the mean of ten stars

TABLE 5

VARIATION IN COLOR: α URSAE MINORIS AND δ CEPHEI

	Phase	Spec.	U	V	B	G	R	I	V−I
Δ mag	Maximum	....	$+0^{\mathrm{M}}62$	$+0^{\mathrm{M}}44$	$+0^{\mathrm{M}}22$	$+0^{\mathrm{M}}01$	$-0^{\mathrm{M}}24$	$-0^{\mathrm{M}}47$	$+0^{\mathrm{M}}91$
	Minimum	....	+ .69	+ .50	+ .25	+ .02	− .27	− .53	+1.03
NPS 4	....	A3p	−0.49	−0.60	−0.29	−0.04	+0.33	+0.68	−1.28
α UMi	Maximum	(cF6)	+0.13	−0.16	−0.07	−0.03	+0.09	+0.21	−0.37
	Minimum	(cF7)	+0.20	−0.10	−0.04	−0.02	+0.06	+0.15	−0.25
δ Cep	Maximum	cF4	+0.02	−0.33	−0.14	−0.02	+0.15	+0.24	−0.57
	Minimum	cG2	+0.67	+0.25	+0.11	−0.01	−0.09	−0.18	+0.43
α UMi	Max.−Min.	....	−0.07	−0.06	−0.03	−0.01	+0.03	+0.06	−0.12
δ Cep	(Max.−Min.)/9	....	−0.07	−0.06	−0.03	0.00	+0.03	+0.05	−0.11

of average spectrum dG6. The variation in color of Polaris and of δ Cephei are compared in the last two lines; it would require another decimal place to show the discordances.

The inferred variation of the spectral type of Polaris, from cF6 to cF7, is found from the spectrum of δ Cephei by interpolation between the values of V − I in the last column of Table 5. The Mount Wilson classification of Polaris is cF7, but we have seen no reference to an observed small change of type in the period of 3.96 days.

The present study was undertaken not to determine complete light-curves of Polaris in different colors but simply to find the approximate amplitudes. The star turns out to be a reduced copy of δ Cephei, except that the latter has a nonsymmetrical curve while Polaris is more like ζ Geminorum, with its equal times of rise and fall. The data for tests of the pulsation theory are best obtained from stars of large variation, but it is of some interest to know that the stars of small variation like Polaris will not introduce complications into the theory.

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ORBITAL ELEMENTS OF THE ALGOL VARIABLE SS BOÖTIS*

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ABSTRACT

Orbital elements of the Algol variable SS Boötis derived from 25 spectrograms obtained at Mount Wilson from 1934 to 1945, inclusive, are compared with the elements obtained by Struve from 25 spectrograms obtained at the McDonald Observatory in a 3½-week interval in January and February, 1945. The only significant differences are in the systemic velocity, −48 km/sec (Mount Wilson) as compared to −43 km/sec (McDonald), and in the semiamplitude of velocity variation for the primary star, 69 km/sec as compared to 47 km/sec. The values of $a_1 \sin i$, $m_1 \sin^3 i$, and $m_2 \sin^3 i$ here found are considerably larger than Struve's.

Twenty-five spectrograms of the Algol variable SS Boötis were obtained at the McDonald Observatory by O. Struve in the interval from January 27 to February 20, 1945. A velocity-curve and orbital elements have been published.[1] Although absorption lines in both primary and secondary were measured, the published velocity-curve depends upon radial velocities from emission lines of H and K of *Ca* II belonging to the secondary star.

The results from another series of 25 spectrograms of SS Boötis accumulated at the Mount Wilson Observatory since 1934 are reported briefly in the present *Contribution.* The emission lines of H and K of *Ca* II in the secondary's spectrum were found independently by the writer[2] in May, 1944.

The journal of my observations is in Table 1. The velocities in the fifth column are from absorption lines of the primary or from blends of absorption lines of primary and secondary. The absorption-line velocities for the secondary are in the sixth column. Radial velocities for the secondary obtained from the two emission lines H and K are in the seventh column.

The phases in the fourth column are reckoned from primary minimum = 1915 July 28.920 + $7^{d}_{.}60605$ E, U.T.,[3] as in Struve's paper.

Elements have been derived from my velocities by means of standard velocity-curves. The secondary's elements depend upon the velocities from both absorption and emission lines.

Table 2 contains these elements together with Struve's for comparison. The Mount Wilson curves are shown with full lines in Figure 1.

The Mount Wilson systemic velocity is 5 km/sec less, algebraically, than Struve's. The semiamplitude for the secondary's velocity variation lies between the two values given by Struve, viz., 73 km/sec for absorption lines and 77 km/sec for the emission lines. The Mount Wilson semiamplitude of velocity variation for the primary is, however, 22 km/sec larger than Struve's, and hence the values for $a_1 \sin i$, $m_1 \sin^3 i$, and $m_2 \sin^3 i$ are markedly higher than his.

There is probably no significance to the difference between the Mount Wilson eccentricity of 0.05 and Struve's 0.00 (circular elements), since the probable error of this element is no doubt fully as large as the difference.

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 719.

[1] *Contr. McDonald Obs., U. Texas,* No. 110; *Ap. J.,* **102,** 118, 1945.

[2] *Pub. A.S.P.,* **57,** 217, 1945.

[3] "Katalog und Ephemeriden Veränderlicher Sterne für 1940," *Kleinere Veröff., U.-Sternwarte Berlin-Babelsberg,* No. 21, p. 191, 1939.

TABLE 1

RADIAL VELOCITIES FOR SS BOÖTIS

Plate	Disp. (A/Mm at $H\gamma$)	Date U.T.	Phase	Velocities (Km/Sec) Primary Abs.	Secondary Abs.	Secondary Em.
C 6456	73	1934 June 29.201	3ᵈ988	−55	...	−52
6643	73	1935 Apr. 11.368	1.125	−48	...	...
6650	73	12.388	2.145	−115	−2	...
6669	82	May 18.368	0.094	−66	...	...
6722	82	June 19.204	1.506	−127	+27	...
6734	82	21.192	3.494	−46	...	...
6765	73	July 13.255	2.739	−107	...	...
V 1018	39	Aug. 11.219	1.279	−105	+28	...
C 6906	73	1936 May 1.387	6.841	−55	...	...
6908	73	2.201	0.049	−47	...	...
γ23473	78	June 13.326	4.692	−43	...	−96
24170	78	1942 May 30.223	5.711	+26	−134	−125
24310	78	July 29.199	4.838	−55	...	−98
25922	36	1944 May 5.405	4.530	−11	−92	−88
25924	36	6.281	5.406	+18	−115	−112
25927	78	7.259	6.384	+22	−111	−109
25933	78	8.326	7.451	−44	...	−64
Ce 3426	37	19.302	3.215	−95	...	−14
3445	37	June 6.294	5.995	+16	−113	−118
3490	37	July 7.284	6.561	−9	...	−107
γ26028	36	9.263	0.934	−91	0	−10
Ce 3717	37	1945 Jan. 29.542	7.456	−51	...	−65
γ26670	78	June 1.383	0.994	−65	...	−5
Ce 3821	33	17.197	1.596	−106	+15	0
3829	33	18.288	2.687	−112	+7	−12

TABLE 2

ORBITAL ELEMENTS OF SS BOÖTIS

Elements	Primary: Sanford (Mt. Wilson)	Primary: Struve (McDonald)	Secondary: Sanford (Mt. Wilson)	Secondary: Struve (McDonald)
P	7.60605	7.606	...	...
ω	0°	...	180°	...
e	0.05	...	0.05	...
K	69 km/sec	47 km/sec	74 km/sec	{73 km/sec abs. 77 km/sec em.
γ	−48 km/sec	−43 km/sec	−48 km/sec	−43 km/sec
T	1915 Aug. 3.770 U.T.	...	...	...
$a \sin i$	7.2×10^6 km	4.9×10^6 km	7.7×10^6 km	7.6×10^6 km
$m \sin^3 i$	1.19☉	0.83☉	1.11☉	0.55☉

Struve found difficulty in obtaining satisfactory velocity measurements from the absorption lines of the two components. The writer also found difficulty, especially with low dispersion. Most of the spectrograms on which absorption lines from both components have been measured are those with the highest dispersion (col. 2 of Table 1). Struve's results and mine agree perhaps as well as could be expected, in view of the difficulties involved.

The period, 7.6060 days, represents satisfactorily all the spectroscopic observations from 1934 to 1945.

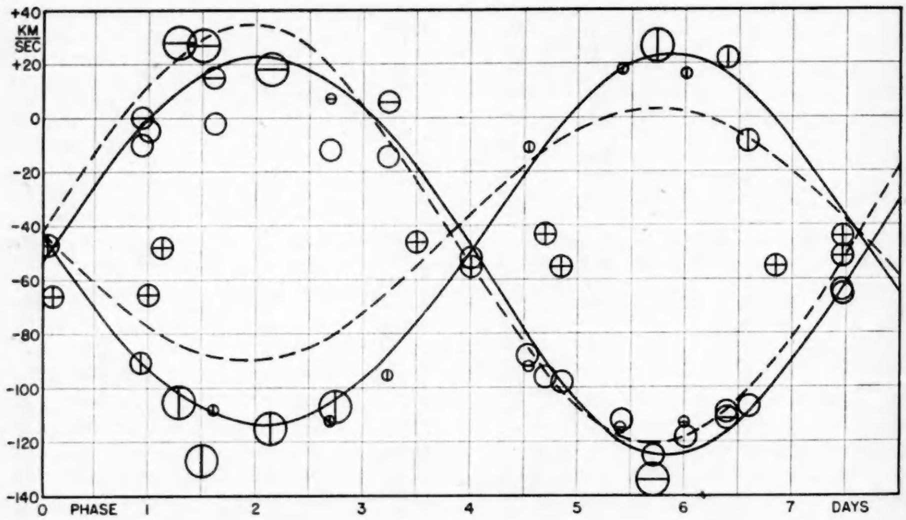

FIG. 1.—Circles with vertical lines represent the absorption-line velocities of the primary; circles with horizontal lines, the absorption-line velocities of the secondary; circles with both vertical and horizontal lines, velocities derived from blends of the absorption line of primary and secondary; and open circles, the velocities of the secondary from the emission lines H and K. The smallest circles represent the best-determined absorption-line velocities, and the largest the poorest. Velocity-curves from Struve's elements are shown with broken lines; the curves from the Mount Wilson elements with solid lines.

SPECTRA OF BD STARS WITHIN FIVE DEGREES OF THE NORTH POLE

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Received December 1, 1945

ABSTRACT

The spectral and luminosity classifications of nearly all the BD stars within 5° of the north pole have been determined with the 4°-objective prism of the Burrell telescope. Spectral classification criteria are established, as well as criteria for giant and dwarf stars of spectral class G2 and later. A general agreement exists between our classification and the HD system.

Our results show that the percentages of dwarfs for magnitude groups from 6 to 11 are in general agreement with the results obtained by other investigators. The selective absorption in the region is found to increase linearly to 0.30 mag. at 450 parsecs and to remain constant thereafter. The average intrinsic color is determined for each spectral class and for giant and dwarf stars later than G0.

The present investigation deals primarily with the spectral luminosity classifications of all the BD stars within 5° of the north celestial pole. Since stellar spectra[1] in this general region of the sky have been observed extensively, ample comparisons can be made with other classifications.

OBSERVATIONAL PROCEDURE

The spectra were secured with the 4° prism[2] mounted on the 24-inch Burrell telescope which has a field 5° in diameter. To widen the spectra of stars within $2^\circ\!.5$ of the pole, the polar axis of the instrument was set out of adjustment in altitude and the prism was adjusted so that the trailing due to the axis maladjustment produced spectra 0.25 mm wide in an exposure of 1 hour. To obtain spectra of the bright stars near the pole, a diaphragm was used in front of the prism and the exposure was kept the same. For other parts of the region the widening was produced by moving the telescope in declination, and for the bright stars short exposures were made and the telescope was moved accordingly. No guiding was found necessary.

For nearly all of the observations we have used Eastman 33 or II*a*-O plates, the latter yielding somewhat more satisfactory results. Each part of the area under investigation was covered by at least two plates, and the spectrum of each star on each plate was estimated independently by both observers.

CRITERIA FOR SPECTRAL CLASSIFICATION

Our criteria for classification were derived primarily by comparison with the known HD spectra of the stars at the pole as well as with bright stars in other regions. The adopted criteria are as follows:

B0 *H* lines weak. *He* lines λ 4026 and λ 4471 nearly as strong as the *H* lines.
B2 *H* lines well visible. *He* lines present.
B5 *H* lines prominent; *He* lines λ 4026 and λ 4471 just visible.
B8 *H* lines strong; *He* lines and K line invisible.
B9 *H* lines strong; K line just visible.
A0 *H* lines strong and broad; K line weak.
A2 *H* lines nearly as strong and broad as in A0; K line well visible, approximately one-third (H + $H\epsilon$).

[1] Seares and Joyner, *Ap. J.*, **98**, 244, 1943.

[2] J. J. Nassau, *Ap. J.*, **101**, 275, 1945.

A3	*H* lines only slightly weaker than in A0; K line strong, about one-half of (H + *H*ε).
A5	*H* lines weaker than A3. K approximately three-fourths of (H + *H*ε).
F0	*H* lines weaker than in A5 but still strong; K slightly weaker than (H + *H*ε). G band generally invisible.
F2	K nearly equal to (H + *H*ε); G band sometimes visible; *H* lines pronounced but weaker than in F0.
F5	K = (H + *H*ε); G band about one-half of *H*γ.
F8	G band equal to or slightly weaker than *H*γ. *H* lines well visible.
G0	G band stronger than *H*γ, *H* lines weak but visible.
G2	G band prominent, at least twice as strong as *H*γ, *H*δ still a well-formed line.
G5	G band very strong; perceptible break in continuum at the G band; *H*δ generally absent or very weak, λ 4227 of *Ca* I absent.
G8	Break at G band visible; λ 4227 sometimes visible; *H* lines absent.
K0	Break at G band well developed; G band very much stronger than λ 4227.
K2	G band slightly stronger than λ 4227.
K3	G band = λ 4227.
K5	Line λ 4227 > G band; *TiO* bands invisible.
M0	Band head at λ 4950 visible.
M2	Band head at λ 4950 strong; band at λ 4760 present.
M5	Band heads at λ 4950 and λ 4760 and double band at λ 4585 all present.
M8	All *TiO* bands very prominent.

The foregoing criteria represent only the prominent features which are visible on weak, as well as strong spectra and on plates taken under average conditions of seeing and sky fog. Plates taken under ideal conditions of exposure and sky show lines which, when used, are bound to increase the accuracy of the classifications. At this time we feel that it is best to describe the criteria used with average plates.

In order to establish criteria for giants and dwarfs, multiple-exposure plates have been taken of stars with known trigonometric parallax and of stars listed in the Mt. Wilson catalog of spectroscopic parallaxes.[3] The spectra measured by P. C. Keenan[4] at the north celestial pole and the north galactic pole have also been used. The *Atlas of Stellar Spectra* by Morgan, Keenan, and Kellman[5] has been used extensively, as well as some unpublished classifications kindly furnished by Dr. Morgan.

Giant and dwarf characteristics begin to show in our spectra at G2, although supergiants may sometimes be detected in earlier spectral types. From G2 to K3, inclusive, the *CN* bands at λ 3883 and λ 4215 are used as luminosity criteria. If these absorption bands are strong for a given spectral type, the star is a giant; if weak, it is a dwarf. The λ 3883 band is most useful for stars at G2, whereas from G8 to K3 the *CN* band at λ 4215 is more sensitive as a luminosity criterion. It is important to classify each star first, before assigning giant or dwarf characteristics.

For stars of class K5 and later the intensity of the continuum between λ 4227 and the G band is the best indicator of luminosity on our plates. If the continuum on the red side of λ 4227 is as strong or stronger than the continuum to the violet of λ 4227, the star is a giant. If the continuum on the red side of λ 4227 is weaker than on the violet side, the star is a dwarf.

DATA

Table 1 lists all the BD stars within 5° of the north pole. The photovisual magnitudes (second column) and the color index (third column) are taken from the paper by Seares, Ross, and Joyner.[6] The fourth column gives the spectral types as obtained from our

[3] *Ap. J.*, **81**, 187, 1935.

[4] *Ap. J.*, **91**, 113, 1940; **91**, 506, 1940.

[5] ("Astrophysical Mono."), University of Chicago Press, 1943.

[6] *Pub. Carnegie Inst. Wash.*, No. 532, 1941.

TABLE 1

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+85°1	9.36	0.44	F8	+85°51	10.31	0.35	F8
2	8.37	0.42	dG2	52	10.05	0.24	F2
3	8.52	1.65	g:K5	53	9.04	0.19	F0
4	10.96	0.93	G5	54	9.37	0.32	F8
5	10.35	0.40	G0	55	10.48	0.57	G2
6	9.63	1.81	gK3:	56	10.09	0.41	A2
7	9.49	1.12	gG8	57	8.70	0.35	G0
8	9.83	0.49	G0	58	10.86	0.38	F8
9	7.90	0.63	dG5	59	....	....	G0
10	8.68	1.65	gK3	60	....	....	gG5
11	8.39	−0.02	A3	61	....	....	gG5
12	9.22	0.89	gG5	62	9.19	0.20	F2
13	9.81	0.23	A2	63	6.53	0.44	F8
14	9.82	0.67	dG5	64	8.81	0.41	dG2
15	10.21	1.36	G8	65	8.40	0.20	F5
16	10.10	0.19	F2	66	10.01	0.96	gG5
17	10.18	0.48	F8	67	10.24	0.35	F2
18	9.45	1.16	gG8	68	....	....	A3
19	4.28	1.19	gK0	69	....	....	A0
20	8.58	1.16	gK0	70	10.58	1.00	gG8
21	9.63	1.15	gG8	71	....	....	(dG5)
22	....	....	gG8	72	....	....	dG5
23	10.59	0.46	G2	73	10.25	0.54	dG5
24	9.97	0.33	F5	74	6.50	0.24	A3
25	9.71	1.15	gG8	75	8.06	0.80	dG8
26	10.51	1.10	gG8	76	10.64	0.32	F5
27	10.46	0.70	dG5	77	9.35	0.02	A0
28	10.00	0.69	G2	78	6.71	−0.10	A0
29	9.90	1.80	K3	79	9.73	1.06	gG5
30	9.99	0.43	G0	80	6.17	1.52	gK3
31	10.67	0.54	(F5)	81	7.47	0.96	gG5
32	9.14	0.19	F2	82	9.01	1.00	d:K5
33	9.71	1.37	gK2	83	9.73	0.26	A5
34	10.11	0.94	G5	84	9.52	0.92	g:G5:
35	9.48	1.01	gG8	85	9.53	0.42	G0
36	9.25	0.38	G0	86	10.09	1.09	d:G8
37	10.56	0.39	G0	87	8.99	0.84	gG5
38	9.52	0.27	A2	88	10.85	0.50	G0
39	10.34	1.05	....	89	10.05	0.50	G0
40	10.41	1.23	....	90	10.77	0.69	g:G5
41	6.95	0.84	g:G8	91	9.70	0.37	F5
42	10.49	1.37	g:K2	92	9.57	1.13	g:G8
43	10.74	0.41	....	93	10.08	−0.04	A0
44	10.72	0.96	....	94	8.75	1.03	gG8
45	7.84	−0.05	B9	95	9.87	1.35	gK0
46	8.97	0.71	F8	96	10.22	0.91	dG8
47	10.45	0.37	F5	97	10.60	0.70	dG5
48	8.81	0.15	F2	98	8.13	1.62	gM2:
49	10.22	0.45	dG2	99	8.79	0.96	g:G8
50	8.83	−0.21	B9	100	8.76	2.05	g:K2

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+85°101.....	9.02	0.12	A5	+85°151.....	8.43	0.27	F8
102.....	10.30	1.30	gG8	152.....	10.89	0.19	A5
103.....	9.16	0.87	d:G8	153.....	10.72	0.57	G0
104.....	10.01	1.20	gG8	154.....	9.42	0.48	F8
105.....	7.96	0.34	F8	155.....	8.53	0.87	gG5
106.....	10.62	0.32	F8	156.....	10.24	1.69	gK3
107.....	9.76	0.33	F5	157.....	9.20	0.90	gG8
108.....	10.38	0.14	A3	158.....	8.63	1.06	gG8
109.....	10.59	0.62	G8	159.....	10.01	0.52	dG5
110.....	9.96	1.02	g:K0	160.....	8.36	0.37	F8
111.....	8.94	1.23	gK0	161.....	7.27	0.72	(dG5)
112.....	10.61	1.08	d:G8	162.....	10.42	0.63	dG5
113.....	10.66	0.40	F8:	163.....	10.72	0.44	G0
114.....	10.34	1.27	gK2	164.....	10.03	0.42	F8
115.....	9.54	1.13	gK0	165.....	9.11	0.46	G0
116.....	10.47	1.44	K3:	166.....	8.05	1.04	gK0
117.....	8.44	1.34	gK0	167.....	10.46	0.51	dG2
118.....	9.74	0.42	G0	168.....	10.43	0.24	F0
119.....	10.59	0.58	F8	169.....	10.67	0.32	F0
120.....	10.60	0.96	....	170.....	8.56	0.96	gG8
121.....	10.62	0.35	F2	171.....	10.43	1.37	(gK2)
122.....	10.26	1.37	gM5:	172.....	9.61	1.75	(gK5)
123.....	10.78	0.33	F8	173.....	8.87	1.11	gG8
124.....	8.58	1.23	gK2	174.....	10.54	1.12	(gG8)
125.....	9.35	0.36	F5	175.....	9.71	1.05	gG5
126.....	10.57	0.20	F5	176.....	10.01	0.44	F5
127.....	9.06	0.95	dK2	177.....	10.54	0.57	F8
128.....	7.32	0.33	F8	178.....	10.49	0.57	(dG5)
129.....	8.39	0.99	gK0	179.....	10.71	0.61	(dG0)
130.....	10.60	0.63	dG5	180.....	10.63	0.65	(F8)
131.....	9.58	0.29	F0	181.....	10.46	0.52	dG5
132.....	8.41	0.11	F0	182.....	9.79	1.79	(gK5)
133.....	10.45	1.03	g:K5:	183.....	7.22	0.88	gG8
134.....	10.93	0.25	F2	184.....	9.50	0.41	F2
135.....	8.79	0.37	F8	185.....	10.74	0.63	G2:
136.....	9.16	1.51	gK2	186.....	10.57	0.35	F2
137.....	10.71	1.01	....	187.....	10.43	0.33	F8
138.....	10.44	0.61	dG5	188.....	10.30	1.33	(gK2)
139.....	....	....	F8::	189.....	9.52	0.65	dG5
140.....	....	....	g:K0	190.....	9.43	0.44	G0
141.....	10.34	0.38	F8	191.....	8.63	0.20	F5
142.....	8.72	0.30	F5	192.....	9.66	1.20	gG8
143.....	10.77	0.44	F8	193.....	10.31	0.48	dG2
144.....	10.79	0.77	d:G5	194.....	10.65	0.57	dG2
145.....	10.68	0.46	F8	195.....	10.38	0.79	dG5
146.....	10.68	0.38	F2	196.....	8.68	0.03	A3
147.....	8.20	1.57	gK5	197.....	9.68	0.38	F8
148.....	10.17	1.61	g:M2	198.....	10.56	0.63	dG5:
149.....	10.57	0.46	G0	199.....	9.29	1.26	gK0
150.....	7.79	0.88	g:G8	200.....	10.58	0.70	dG5

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+85°201	10.31	0.64	d:K0	+85°251	....	....	g:G8
202	9.42	0.73	dG5	252	10.25	0.58	dG5
203	9.53	1.75	gK5	253	10.69	0.51	F8
204	10.89	1.12	G5:	254	10.22	0.49	G0
205	9.79	1.79	gK3	255	11.10	0.46	G0:
206	10.40	1.21	gG8	256	9.82	1.69	g:M2
207	10.05	1.56	gG5	257	9.43	0.29	F0
208	9.70	1.89	K5:	258	10.60	0.40	F5
209	9.09	0.63	dG0	259	10.84	0.58	G0
210	9.96	0.47	G0	260	10.80	0.93	K2:
211	9.55	0.48	G0	261	9.96	1.12	....
212	10.81	0.64	dG2	262	....	....	....
213	9.41	0.32	F5	263	6.88	0.91	gG5
214	9.24	0.56	dG8	264	9.04	0.62	dG5
215	10.95	−0.14	B8	265	9.10	0.95	g:G5
216	10.78	0.92	gG5	266	8.71	0.38	G0
217	9.77	0.55	dG2	267	10.50	0.20	....
218	10.93	0.89	(dG0)	268	10.45	0.55	G0
219	10.34	0.28	F8	269	6.91	0.09	A5
220	10.03	0.44	F5	270	10.22	1.37	(gK2)
221	....	....	dG2:	271	10.65	0.70	(dG0)
222	7.27	0.39	G0	272	10.37	0.46	G0
223	9.70	0.87	dK2	273	10.40	1.13	gG8
224	9.08	1.05	dG8	274	10.14	0.44	(dG0)
225	9.23	1.52	gK2	275	9.83	0.50	G0
226	10.94	0.28	F2p	276	10.59	0.93	dG8
227	10.67	0.37	F8	277	10.43	0.37	G0
228	10.48	0.25	A5	278	8.65	0.27	F8
229	10.87	0.98	G5:	279	9.78	1.24	(gG8)
230	10.56	0.78	d:K0	280	....	....	G5
231	10.07	1.21	gG8	281	9.95	1.87	K3
232	10.18	0.72	dG2	282	10.23	1.31	gG8
233	8.86	0.39	G0	283	10.09	1.16	dG5
234	7.74	1.15	gG8	284	....	....	g:G8
235	8.45	1.57	gK2	285	10.91	0.51	d:G5
236	10.50	0.92	gG5	286	9.49	0.92	dK0
237	10.39	1.25	gG8	287	10.60	0.47	F2p
238	10.62	0.30	F5	288	10.25	0.53	d:G2
239	9.14	0.19	A5	289	9.46	1.22	gG8
240	9.84	0.35	F5	290	10.66	1.11	G8
241	9.98	1.34	gG8	291	11.19	0.45	G0
242	8.61	1.28	gK0	292	10.58	0.74	d:G8
243	10.67	1.04	G5:	293	10.60	0.82	dG5
244	10.72	0.40	F8	294	7.64	0.14	F0
245	9.88	1.75	K2::	295	10.90	0.88	gG8
246	10.63	0.57	dG5:	296	10.86	0.94	gG5
247	9.18	0.37	F8	297	9.61	2.35	gM5
248	8.86	0.28	F5	298	10.65	1.35	(gM:)
249	7.50	0.87	gK0	299	10.84	0.44	F8
250	....	....	F2	300	....	....	(gK2)

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+85°301	10.88	0.64	dG5	+85°351	9.57	0.61	dG5
302	10.98	0.49	G0	352	9.83	0.30	F0
303	10.28	0.98	d:K0	353	9.04	1.40	gK2
304	9.05	0.51	F8:	354	7.88	1.44	gK3
305	10.95	0.45	F8	355	9.38	0.25	A5
306	10.87	0.69	G5:	356	10.49	1.25	g:G5
307	10.02	0.35	G0	357	9.10	0.57	dG2
308	10.82	0.75	d:G5	358	10.44	0.96	dG8
309	10.11	1.27	K0	359	8.43	0.10	A3
310	10.89	0.44	F2	360	11.09	0.45	A5
311	11.07	0.37	G0	361	....	....	A2
312	10.62	1.13	g:G8	362	10.51	1.15	gG8
313	10.64	1.22	gK0	363	10.66	0.10	A2p
314	10.63	0.39	F8	364	9.71	0.63	dG5
315	10.54	0.58	G0	365	10.62	1.09	g:G8
316	10.59	0.89	dG5	366	9.02	1.35	gK0
317	10.63	0.59	G2	367	9.01	−0.06	B9
318	10.43	1.22	gG5	368	....	....	G8
319	10.79	0.92	gG8	369	9.87	1.14	gG8
320	9.06	0.12	A0	370	9.45	0.38	F2
321	10.51	0.72	dG5	371	9.20	0.44	F8
322	10.78	0.72	(F8)	372	....	....	M5:
323	10.68	1.10	....	373	10.36	1.16	gG8
324	9.37	0.52	G0	374	10.16	0.34	A2
325	9.55	0.54	G0	375	10.57	0.44	G0
326	10.19	1.51	gG8	376	8.89	0.11	A5
327	10.88	0.98	gG8	377	10.83	0.63	G0:p
328	11.00	0.60	F8	378	9.87	0.91	dG8
329	7.93	1.06	gK0	379	10.76	0.98	gK0
330	8.64	0.79	d:G5	380	10.44	0.35	A3
331	9.51	1.49	gK0	381	10.16	1.56	gK0
332	9.53	2.06	R8	382	9.63	0.51	G0
333	10.36	1.10	dG5	383	5.30	−0.08	(A0)
334	9.73	1.19	gG8	384	6.65	1.07	gK0
335	9.21	1.64	gK0	385	10.85	0.82	d:G8
336	10.12	0.46	F8	386	9.36	1.30	gG8
337	8.82	0.09	A2	387	10.48	0.52	A0
338	10.01	1.08	gG5	388	....	....	G5
339	9.14	0.66	G0	389	9.38	0.39	G0
340	7.75	1.54	gK3	390	8.55	0.73	gG5
341	10.57	1.13	g:G5	391	10.51	1.17	G8
342	10.27	1.40	G8	392	9.44	0.39	A0
343	10.04	1.08	gG8	393	10.77	0.50	G0
344	10.44	0.19	A2	394	10.52	0.42	G0
345	9.42	1.25	gK0	395	9.99	1.44	gG8
346	10.69	0.59	F8	396	10.44	0.16	A3
347	8.58	0.74	dG5	397	10.76	0.62	A5
348	9.60	0.34	A5	398	8.59	1.03	gK0
349	9.75	0.44	F8	399	6.63	0.16	F0
350	10.34	0.42	G0	400	7.60	0.70	gG8

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+85°401.....	7.23	0.31	F8	+86° 36.....	10.38	1.02	gG5
402.....	10.10	0.35	F8	37.....	10.47	0.84	g:G8
403.....	7.16	−0.01	A3	38.....	9.26	0.02	A3
404.....	10.13	0.71	dG5	39.....	7.88	0.13	F2
405.....	10.74	0.45	F8	40.....	10.74	0.64	G0
406.....	8.90	−0.15	A2	41.....	9.81	1.09	gG8
407.....	9.93	0.21	F0	42.....	10.32	0.17	A2
408.....	10.57	0.13	A5	43.....	9.33	0.07	A3
409.....	6.82	−0.11	A2	44.....	9.17	0.30	F8
410.....	9.61	0.54	dG5	45.....	9.54	0.14	A5
411.....	10.09	0.18	F0	46.....	10.34	0.31	F8
412.....	8.48	0.32	G0	47.....	10.52	0.23	F5
				48.....	10.31	0.44	F0
				49.....	9.63	0.05	B8:
				50.....	....	....	G0
+86°1.....	10.16	0.18	F0	51.....	5.83	0.28	F5
2.....	9.14	1.67	gK3	52.....	9.66	0.31	A5
3.....	10.24	0.72	G2	53.....	10.81	0.82	g:G5
4.....	10.56	0.26	F2	54.....	8.95	0.06	A5
5.....	10.92	0.44	F8	55.....	9.56	0.36	F8
6.....	9.96	1.35	gK0	56.....	10.70	0.50	A2
7.....	9.05	0.22	F5	57.....	10.70	0.27	F2
8.....	10.45	0.21	F2	58.....	10.68	0.30	F5
9.....	8.78	−0.09	A5	59.....	9.80	1.46	gG8
10.....	10.60	1.43	(gK5:)	60.....	10.66	0.39	G0
11.....	9.99	0.39	F8	61.....	9.87	1.77	g:M2:
12.....	10.76	0.95	(gG8)	62.....	9.83	0.36	F8
13.....	10.94	0.72	dG5	63.....	10.21	0.39	G0
14.....	8.86	0.51	dG5	64.....	9.45	0.09	A5
15.....	10.13	0.88	dG5	65.....	8.53	0.42	G0
16.....	10.37	0.55	(F8:)	66.....	8.05	−0.18	B9
17.....	6.25	1.00	gK0	67.....	....	....	F8
18.....	9.34	1.02	gG8	68.....	10.51	0.28	G0
19.....	10.36	0.31	F8	69.....	9.97	0.36	G0
20.....	10.34	0.33	G0	70.....	9.85	1.13	gG8
21.....	8.50	0.40	dG5	71.....	10.74	0.18	F2
22.....	10.62	0.92	gG5	72.....	9.32	1.34	gK2
23.....	10.66	0.28	A3	73.....	9.81	0.68	dG2
24.....	10.44	1.45	gG8	74.....	10.70	0.51	G0
25.....	8.12	0.14	F5	75.....	9.61	0.32	F2
26.....	10.09	0.44	G0	76.....	10.91	0.28	F0
27.....	9.45	1.23	gG8	77.....	8.69	1.19	gG8
28.....	10.84	0.39	G0	78.....	10.43	0.34	dG2
29.....	9.85	0.23	F0	79.....	6.55	1.15	gK0
30.....	9.01	1.41	gK0	80.....	10.47	0.03	B8
31.....	9.95	0.02	B9	81.....	10.06	1.14	gG5
32.....	9.27	0.92	gG8	82.....	10.77	0.77	g:G5:
33.....	9.84	1.16	gG8	83.....	10.11	1.35	gG8
34.....	10.48	0.21	A5	84.....	10.49	0.92	gG8
35.....	10.78	0.52	dG2	85.....	10.82	0.40	G0

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+86°86	....	....	F8	+86°136	10.86	0.64	dG5
87	10.22	0.39	G0	137	10.97	0.52	F5
88	10.60	0.24	F0	138	10.72	1.04	gG5
89	10.54	1.21	K0:	139	10.61	0.59	d:G2
90	10.07	0.98	g:G8	140	....	....	....
91	9.53	0.00	A0	141	10.56	0.61	dG5
92	9.57	0.61	dG5	142	....	....	....
93	10.51	0.47	F8	143	8.37	−0.03	A2
94	9.80	1.08	gG8	144	10.54	0.41	G0
95	11.01	0.31	F8	145	10.34	1.06	gG8
96	8.96	0.27	F5	146	9.25	−0.05	A0
97	10.66	0.47	G0	147	10.79	1.09	g:G5:
98	9.99	1.03	gK2	148	10.69	0.48	dG2
99	10.87	0.75	d:G5	149	8.85	1.19	gG8
100	10.10	0.51	dG2	150	10.42	0.42	d:G2
101	10.35	1.08	gG8	151	9.96	0.68	dG5
102	8.84	−0.18	A0	152	....	....	A3–F2
103	7.85	0.43	(dG0)	153	10.11	1.32	gG8
104	10.18	0.50	dG2	154	8.02	0.36	G0
105	10.23	1.16	g:G8	155	....	....	B5:
106	....	....	F0	156	....	....	gG8:
107	9.41	0.31	F8	157	9.34	1.19	gG5
108	10.65	0.98	gG8	158	10.68	0.87	(gG8:)
109	9.81	0.54	dG2	159	8.40	−0.05	A3
110	8.36	1.55	gM5	160	9.60	0.65	d:G5
111	10.67	0.47	d:G2	161	7.32	0.12	A5
112	10.19	0.49	dG2	162	10.81	0.39	F8
113	7.33	0.81	gG5	163	9.18	0.44	G0
114	10.40	0.69	dG5	164	....	....	....
115	10.68	1.25	K0::	165	9.71	0.14	A5
116	9.44	0.98	gG8	166	10.37	0.46	F2:
117	9.62	1.12	gG8	167	9.87	0.95	g:G8:
118	10.74	0.96	dK2	168	10.27	0.40	F8
119	9.61	0.31	G0	169	9.39	0.26	F2
120	8.48	0.95	gG8	170	7.31	0.25	F0
121	10.75	0.46	G0	171	8.96	0.29	F2
122	9.54	1.03	gG8	172	8.04	0.78	gK0
123	9.82	1.16	gG8	173	9.15	1.31	gK0
124	10.33	0.11	A5	174	10.65	1.11	gG5
125	10.58	1.09	d:G5	175	10.60	1.4	(gK5:)
126	7.82	1.39	gK2	176	6.31	0.44	F8
127	9.37	1.02	gG8	177	8.81	0.20	F5
128	10.47	1.22	gK3:	178	9.67	1.40	gK0
129	10.73	0.40	F2	179	10.43	1.44	K0:
130	9.52	0.51	F8	180	8.91	−0.05	A0
131	10.04	0.38	F8	181	....	....	K3:
132	9.04	0.73	G0	182	7.24	0.11	A5
133	10.18	0.55	dG5	183	10.25	0.67	dG2
134	9.34	1.35	gG8	184	8.79	0.61	dG5
135	10.76	0.67	d:G2	185	10.97	0.40	G0

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+86°186.....	10.74	0.59	dG2	+86°236.....	10.72	0.32	F0
187.....	7.81	0.24	F8	237.....	10.32	1.16	gG5
188.....	10.18	0.14	A5	238.....	10.58	1.35	(gM)
189.....	11.02	0.63	d:G2	239.....	10.84	0.47	F8
190.....	....	....	F0	240.....	9.97	1.34	gG8
191.....	9.34	0.27	F2	241.....	10.73	0.59	d:G5
192.....	10.06	0.62	G2	242.....	8.46	0.97	gG8
193.....	7.95	0.19	F5	243.....	10.95	0.61	G0
194.....	10.64	0.75	dG5	244.....	....	....	A0
195.....	9.70	1.30	gK0:	245.....	10.96	0.46	F8:
196.....	10.43	1.21	gK0	246.....	9.76	0.54	G0
197.....	10.86	0.86	G8:	247.....	10.36	0.52	G0
198.....	10.71	0.58	d:G5:p	248.....	11.05	0.57	dG2
199.....	9.22	0.20	F0	249.....	10.29	1.12	g:G5
200.....	10.55	0.55	dG2	250.....	10.41	0.81	dK0
201.....	7.30	0.29	F5	251.....	10.43	1.18	G5
202.....	....	....	F5	252.....	9.93	1.16	gG8:
203.....	10.40	1.59	K3:	253.....	10.68	0.97	gG8
204.....	10.53	1.04	d:G5	254.....	9.54	0.60	dG5
205.....	....	....	G0:	255.....	10.78	0.61	F8
206.....	....	....	....	256.....	8.23	1.01	gG8
207.....	10.85	0.42	F5	257.....	10.02	0.61	dG5
208.....	10.92	0.58	d:G2	258.....	....	....	G8
209.....	10.86	1.07	g:G5	259.....	10.75	0.46	dG5
210.....	10.58	0.25	A3	260.....	9.90	0.48	G0
211.....	8.82	0.51	dG2	261.....	10.70	1.01	g:G8
212.....	9.78	0.39	F8	262.....	9.96	1.35	g:K0
213.....	10.65	0.18	F0	263.....	....	....	G0
214.....	10.39	0.27	F5	264.....	....	....	F5
215.....	9.60	0.30	F8	265.....	10.63	1.15	dG5
216.....	10.60	1.29	K0:	266.....	9.75	0.86	dG5
217.....	....	....	gG8	267.....	10.18	0.39	F2
218.....	10.63	1.16	gK0:	268.....	10.39	0.49	dG2
219.....	10.41	0.86	dG5	269.....	4.41	−0.02	(A0)
220.....	10.94	0.72	G0	270.....	10.74	0.93	d:G8
221.....	7.80	0.31	G0	271.....	10.04	0.50	dG8
222.....	9.11	0.46	G0	272.....	5.83	0.13	A2:
223.....	....	....	dG5	273.....	10.90	0.44	F8
224.....	10.50	0.53	dG2	274.....	10.96	0.46	G0
225.....	10.09	0.54	dG5p	275.....	7.66	1.06	gK0
226.....	10.86	1.05	G8:	276.....	10.91	0.49	G0
227.....	10.08	1.31	g:G8	277.....	9.15	0.22	A0
228.....	....	....	(d:K2:)	278.....	10.99	0.86	dG5
229.....	9.48	0.64	dG5	279.....	10.16	0.67	dG8
230.....	9.67	0.63	dG5:p	280.....	....	....	K3
231.....	9.72	1.15	dG8	281.....	9.09	1.12	gG5
232.....	10.44	0.31	F8	282.....	6.60	1.53	gM2
233.....	9.88	1.42	gG8	283.....	10.40	0.31	F2
234.....	9.59	0.35	F2	284.....	10.75	1.01	gG8
235.....	10.63	0.79	dG5	285.....	10.53	0.44	G0

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+86°286.....	9.31	0.40	F8	+86°336.....	10.92	1.06	G8
287.....	10.90	0.61	dG5	337.....	10.68	0.43	F5
288.....	10.44	1.12	gK0	338.....	10.51	1.15	gG8
289.....	9.29	1.40	gK0	339.....	10.72	0.24	F5
290.....	9.13	0.36	F2	340.....	10.40	0.17	F5
291.....	10.69	0.42	F0p	341.....	10.48	0.41	F8
292.....	10.87	0.49	G0	342.....	10.63	0.32	F8
293.....	9.96	0.54	G0	343.....	10.21	1.34	gK0
294.....	10.81	0.62	F8	344.....	5.60	0.14	A5
295.....	10.46	0.59	dG5	345.....	10.79	0.67	dG5
296.....	10.24	1.11	dG8	346.....	9.49	0.50	G0
297.....	9.21	0.44	F8	347.....	7.87	0.09	F0
298.....	10.23	0.56	dG2				
299.....	10.54	0.59	F8				
300.....	10.03	0.44	F8				
301.....	10.67	0.83	dG5	+87°1.....	9.17	0.03	A2
302.....	10.77	0.94	G5	2.....	9.92	0.89	dG5
303.....	8.74	1.05	gG8	3.....	10.75	0.61	dG5
304.....	10.91	0.34	A2	4.....	10.38	1.64	gK5
305.....	10.29	0.16	F0	5.....	9.45	0.30	F8
306.....	10.47	0.55	dG2	6.....	....	....	gG5
307.....	10.24	1.19	gG8	7.....	....	....	A3
308.....	10.59	1.13	gG8	8.....	9.10	1.06	gG8
309.....	10.62	1.04	g:G8	9.....	8.92	0.09	F2
310.....	10.64	0.96	gG8	10.....	10.86	0.61	dG2:
311.....	10.48	0.34	F5	11.....	10.22	0.21	F2
312.....	10.26	0.40	G0	12.....	7.89	1.10	gK0
313.....	10.86	0.65	F2p	13.....	9.72	0.40	G0
314.....	11.15	0.48	dG5	14.....	10.48	1.16	gG5
315.....	11.10	0.41	A2	15.....	8.18	0.03	A5
316.....	9.04	1.14	gK0	16.....	8.88	0.34	G0
317.....	9.56	0.56	G0	17.....	10.64	1.22	gG8:
318.....	8.56	0.21	F5	18.....	10.17	1.02	gG5
319.....	7.49	−0.07	A2	19.....	10.30	1.35	gG8
320.....	....	....	gG8	20.....	10.57	0.23	F5
321.....	....	....	A2	21.....	10.84	0.68	d:G5
322.....	10.77	0.59	F8	22.....	9.77	1.20	gG8
323.....	10.53	1.24	gK2	23.....	8.50	0.95	gK0
324.....	8.44	0.01	A2	24.....	11.09	0.51	d:G2
325.....	....	....	G0	25.....	10.92	0.55	dG2
326.....	8.77	1.96	g:M2	26.....	8.86	0.26	G0
327.....	9.74	0.49	G0	27.....	8.98	1.18	gG8
328.....	9.66	1.23	gG8	28.....	10.83	0.68	G8
329.....	10.90	1.04	G5	29.....	10.30	0.53	dG2
330.....	10.20	0.57	dG2	30.....	10.67	0.63	dG2
331.....	10.23	0.41	G0	31.....	9.76	0.41	G0
332.....	9.30	0.13	F2	32.....	10.03	1.84	(gK5:)
333.....	....	....	A0	33.....	8.61	0.15	F2
334.....	10.29	0.25	B9	34.....	10.47	1.20	gG2
335.....	8.29	−0.17	A0	35.....	9.47	0.42	G0

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+87°36	10.46	0.44	G0	+87°86	9.53	1.01	gG8
37	10.20	0.46	dG2	87	10.59	0.86	d:G2
38	10.07	0.32	F2	88	10.73	1.32	(gK5)
39	10.50	0.44	G0	89	9.79	0.32	F5
40	10.75	0.38	F8	90	10.40	0.26	F2
41	8.11	1.94	gK5	91	10.33	1.22	d:K5:
42	11.11	0.40	G0	92	10.60	0.38	G0
43	10.12	1.37	gG5	93	10.61	1.30	K2
44	10.60	0.42	A5	94	10.15	0.43	G0
45	9.25	0.14	A3	95	10.57	1.32	g:G5
46	8.31	0.88	gG8	96	....	....	....
47	9.42	1.69	gK0	97	10.96	0.44	G0
48	9.45	1.99	K3:p	98	....	....	dG5
49	10.88	0.47	F5	99	8.28	0.59	dG8
50	10.36	1.16	gG5	100	....	....	(F2)
51	5.06	1.61	g:M0	101	....	....	A0
52	....	....	d:K2:	102	9.59	0.33	F8
53	9.56	1.04	gG8	103	9.95	0.95	G5
54	10.17	0.43	G0	104	8.05	1.03	gK0
55	10.56	0.63	G0	105	9.97	1.26	G5
56	....	....	F8	106	10.79	0.58	dG5
57	10.77	0.86	dG5	107	6.31	0.33	F5
58	10.35	0.97	gG8p	108	9.70	0.40	G0
59	11.07	0.33	A3	109	10.07	0.29	F0
60	10.18	1.31	G8	110	9.96	0.91	gG8
61	10.86	0.50	d:G2	111	10.78	0.40	G0
62	11.07	0.56	G0	112	9.63	0.52	F2
63	9.90	0.48	G0	113	9.52	0.37	F8
64	10.82	0.72	d:G5	114	10.92	0.49	F0
65	....	....	gG5	115	8.64	0.34	G0
66	....	....	g:G5	116	10.38	0.74	G5
67	9.57	0.35	F0	117	9.40	0.34	F5
68	8.74	0.26	F8	118	8.84	1.02	dK3
69	9.06	0.83	gK0	119	....	....	(F8)
70	10.67	0.94	d:G5	120	....	....	G5
71	9.28	0.32	F8	121	8.71	0.97	gG8
72	9.81	0.41	F8	122	8.53	0.10	F2
73	9.86	1.11	gG8	123	10.76	0.39	F8
74	10.40	0.30	F8	124	9.57	0.15	A5
75	10.03	0.54	dG8	125	9.61	0.54	dG5
76	10.61	1.13	G5	126	10.75	0.54	dG2
77	10.65	0.43	G0	127	10.08	0.14	A2
78	8.31	0.12	F2	128	10.38	0.50	G0
79	8.32	0.99	gK0	129	8.78	1.44	gK0
80	8.66	1.09	gK0	130	9.14	1.14	gG8
81	9.49	0.47	G0	131	9.72	0.93	G5
82	9.28	0.29	F2	132	10.41	1.51	gK2:
83	7.76	0.94	gG8	133	9.30	0.86	g:G8
84	10.90	0.49	G0	134	10.47	0.47	G0
85	8.27	0.84	gG8	135	10.29	1.57	K0:

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+87°136	10.86	0.08	A0	+87°186	9.72	0.35	F8
137	9.83	1.03	gG8	187	8.08	1.11	gK0
138	10.21	0.35	G0	188	10.80	0.44	G0
139	10.45	1.27	d:G8	189	10.72	0.49	F5
140	10.72	0.63	G0	190	9.73	0.76	dG8
141	9.97	0.41	G0	191	10.01	1.21	gG5
142	10.41	0.52	dG5	192	9.99	0.44	dG2
143	6.89	1.39	gK0	193	9.41	0.27	G0
144	10.01	1.65	gK0	194	9.23	1.20	gK0
145	10.32	0.35	A5	195	9.49	0.42	dG2
146	....	....	A5	196	9.95	0.34	F5
147	8.16	0.31	G0	197	....	....	G0
148	9.62	0.52	dG2	198	....	....	g:K3
149	9.59	1.00	gG8	199	9.74	0.19	A5
150	10.39	1.18	gG5	200	11.00	0.47	G0
151	9.14	0.28	F8	201	8.42	0.22	F5
152	10.04	1.26	G5	202	10.46	1.11	gG8
153	10.77	0.97	G5:	203	10.31	1.20	gG5
154	10.57	0.58	dG2	204	10.58	1.30	g:G8
155	9.92	1.21	gG5	205	7.51	−0.05	A2
156	10.83	0.25	F5	206	9.51	0.31	F8
157	10.55	1.02	G5:	207	10.80	0.49	A5
158	10.01	0.55	dG5	208	11.14	(−0.16)	G0, F8
159	10.62	0.77	dG5	209	10.65	1.19	gK0
160	9.55	1.15	gK0	210	10.36	1.59	g:M2:
161	9.77	1.40	K0	211	10.42	1.08	g:G8
162	10.78	0.90	G8:	212	10.42	0.35	F5
163	10.56	0.28	F8	213	10.70	0.39	F8
164	9.87	0.51	G0	214	10.71	0.36	F5
165	10.15	0.40	G0	215	10.48	0.35	F8
166	8.97	1.12	gK0	216	10.76	0.51	F0
167	10.70	0.61	G2	217	9.07	−0.10	A2
168	9.56	0.74	G2	218	10.87	0.97	G5:
169	8.35	0.11	F0	219	10.99	0.37	F5
170	10.14	0.86	gG5	220	8.95	0.97	gK0
171	10.23	1.02	gG8	+88°1	10.50	0.44	dG2
172	10.51	1.27	g:G8	2	8.13	0.75	gG8
173	8.62	1.72	gK3	3	10.38	1.00	dK5p
174	9.74	1.10	gG8	4	6.48	0.01	A0
175	10.52	0.52	F5	5	8.93	0.58	gG8
176	10.82	0.38	F8	6	9.74	0.26	F5
177	10.30	0.42	G0	7	....	....	....
178	10.65	0.99	G8	8	....	....	G0
179	10.96	0.37	A5	9	8.14	0.14	F2
180	8.58	−0.07	A5	10	9.29	1.05	gG8
181	8.67	−0.08	A2	11	8.61	0.31	G0
182	10.81	0.39	F2	12	9.16	1.65	g:K0
183	9.65	1.01	dK5	13	8.86	0.07	F2
184	10.68	0.98	g:G8	14	9.53	1.32	gG8
185	9.44	0.18	F2	15	10.35	1.16	G8

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+88°16	9.67	0.45	G0	+88°66	10.81	0.45	dG2
17	9.72	1.85	K3	67	10.30	0.31	G0
18	10.50	0.85	dG5	68	10.47	0.43	gG2
19	9.62	1.64	gK0	69	9.76	1.01	gG8
20	9.39	1.18	gG8	70	10.72	1.02	g:G8
21	10.74	0.28	A2	71	6.31	0.20	F0
22	10.48	0.40	G0	72	9.67	0.24	F5
23	10.86	0.26	F8	73	10.24	0.33	F8
24	10.58	0.32	G0	74	9.63	1.34	gK0
25	10.61	0.66	dG2	75	9.14	0.32	F8:
26	10.48	0.56	G0	76	7.52	1.35	gK2
27	10.41	0.30	F8	77	8.66	0.13	F5
28	10.63	0.69	dG5	78	9.73	0.86	gG8
29	9.23	0.83	gG8	79	....	....	K0
30	10.52	0.34	G0	80	9.12	0.01	A2
31	9.49	1.66	gK0	81	10.88	0.47	F8
32	9.88	1.74	gK2	82	10.51	0.52	dG2
33	10.00	0.39	G0	83	10.41	0.33	G0
34	10.31	0.77	gG5	84	10.58	0.52	G0
35	9.35	0.33	F8	85	9.43	0.95	gK0
36	10.04	1.30	gG5	86	....	....	B9
37	10.41	1.20	gG8	87	10.88	0.47	A5
38	10.98	0.44	F2	88	10.74	0.57	G0
39	9.10	0.18	A2	89	10.44	1.24	gG8
40	10.29	1.06	dG5	90	9.08	0.77	g:G8
41	9.86	0.41	F8	91	10.33	0.14	F0
42	....	....	g:G5	92	9.93	0.23	F5
43	9.61	0.32	F2	93	10.42	0.48	G0
44	9.82	1.34	gG8	94	....	....	gM0
45	10.77	0.80	dG5	95	....	....	G2
46	9.93	1.46	gG8	96	9.78	0.32	F2
47	10.12	0.45	G0	97	10.69	0.44	F8
48	10.13	1.05	gG5	98	10.71	0.17	F2
49	10.60	0.32	G0	99	10.20	0.41	d:G2
50	10.88	0.97	G5	100	9.09	0.41	G0
51	11.05	0.48	dG5	101	9.32	0.29	F5
52	10.60	0.46	dG2	102	9.54	0.34	F8
53	10.44	0.43	G0	103	10.24	1.35	g:K0
54	10.04	0.42	G0	104	8.20	0.20	F8
55	9.97	0.32	F5	105	8.38	0.66	gG8
56	10.50	1.14	gG5	106	10.31	0.20	A3
57	10.47	0.54	G0	107	9.85	1.26	g:K0
58	10.43	0.55	dG5	108	10.26	0.34	G0
59	10.62	0.49	dG5	109	9.49	1.20	gG8
60	8.82	0.27	F8	110	9.09	0.15	F5
61	10.36	0.40	G0	111	....	....	F5
62	10.70	1.03	gG5	112	6.37	1.53	g:M2
63	10.38	0.36	G0	113	10.36	1.35	K3:
64	7.58	−0.22	A0	114	8.29	0.90	gK0
65	10.01	0.27	F5	115	9.22	−0.03	A3

TABLE 1—*Continued*

BD	m_{pv}	CI	Sp.	BD	m_{pv}	CI	Sp.
+88°116	10.19	0.29	F5	+89°6	10.31	0.35	A3
117	....	....	dG2	7	9.99	0.29	F2
118	9.56	0.44	G0	8	10.67	0.93	dK2p
119	10.64	0.19	F2	9	9.22	1.31	gG8
120	....	....	gG5	10	10.36	0.39	G0
121	....	....	....	11	10.70	0.41	G0
122	9.94	1.77	K5	12	9.82	0.50	G0
123	10.45	0.11	F2	13	7.09	0.09	A2
124	9.55	1.49	gG8	14	10.72	0.33	G0
125	10.62	0.78	dG5	15	9.63	0.69	gG5
126	10.94	0.51	G0	16	10.73	0.98	d:K3
127	10.59	0.66	d:G5	17	9.49	0.41	G0
128	10.37	0.36	F5	18	9.61	0.15	F2
129	10.64	0.26	F8	19	10.84	0.60	d:G2
130	9.28	0.23	F2	20	10.66	0.96	d:G5
131	8.98	−0.02	A3	21	....	....	F8
132	10.64	0.58	dG5	22	8.62	1.58	gK5
133	9.89	0.05	A5	23	....	....	g:G5
134	9.27	1.01	gG5	24	....	....	g:G5
135	....	....	g:K5:p	25	9.81	0.26	A3
136	10.44	0.23	F5	26	10.71	0.64	dG8
137	9.93	0.39	G0	27	10.53	0.40	G0
138	10.4:	1.4	g:M2	28	8.68	1.59	gK3
139	9.39	0.37	G0	29	10.40	0.15	A5
140	10.57	1.04	gG5	30	10.26	0.28	F8
141	10.10	0.38	G0	31	10.45	1.01	gG8
142	9.41	1.02	gG5	32	10.49	0.38	F2
143	10.46	0.98	dG5	33	10.42	0.18	F0
				34	10.02	0.60	dG5
+89°1	10.52	0.39	F2	35	9.85	1.15	gG8
2	9.15	1.70	gK2				
3	9.07	0.08	A3	36	10.39	0.34	F8
4	10.47	0.43	G0	37	....	....	G5
5	10.44	1.11	gG8	38	9.75	0.13	A0

NOTES TO TABLE 1

BD 85° 46	Very red for F8 star
100	Very red for g:K2 star
226	λ 4000 and λ 4040 prominent
246	λ 4227 strong
284	λ 4270 strong
287	Hγ weak, Hδ strong
363	Strong *He* λ 4027
377	λ 4150 present
387	Very red for A0 star
397	Very red for A5 star
86° 56	Very red for A2 star
152	λ 4077 strong; G band present
179	λ 4550 strong
198	λ 4227 strong
BD 86°225	λ 4077 and λ 4400 blend strong, Hγ weak
230	λ 4040, λ 4077, and Hδ strong
261	λ 4270 and λ 4400 blend strong
291	Strong line at λ 4470
313	Very red for F2 star; λ 4383 strong
87° 47	Very red for gK0
48	Many additional strong lines
58	Lines at λ 4470 and λ 4550 strong
139	Very red for d:G8
208	Two stars
88° 3	λλ 4040, 4077, 4470, and 4550 strong
135	Lines at λ 4470 and λ 4620
89° 8	λ 4550 strong

plates. For the sake of completeness we include in parentheses the spectra of a few stars which, for various reasons, we were unable to classify on our plates. These were secured, when available, from the Upsala spectral classifications as made by Petersson.[7]

COMPARISON WITH OTHER SYSTEMS

Table 1 includes 150 stars for which Henry Draper spectral types are available; these are plotted against our spectral classifications in Figure 1. Although the material is limited, it appears that a general agreement exists between the two systems. The large spread at G8 (Case) is due to the absence of that subclass in the *Henry Draper Catalogue*. The three stars shown by crosses in Figure 1 are: BD+85°412, 86°96, and 88°39. They

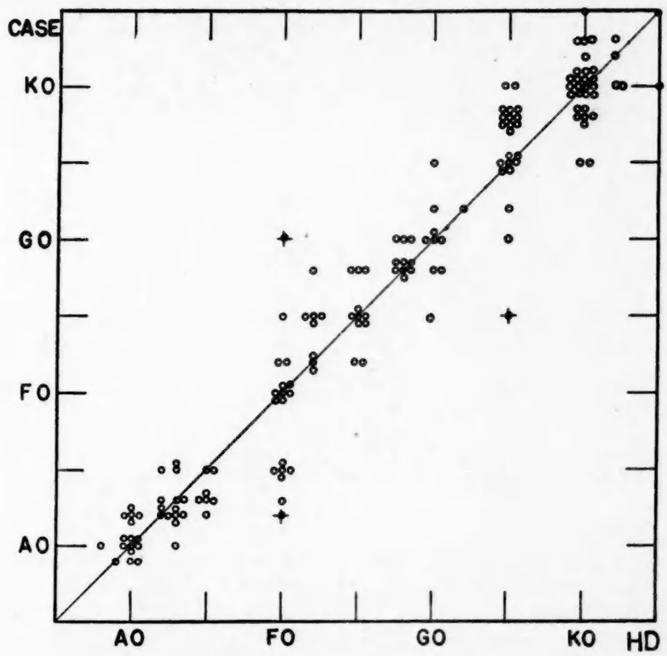

FIG. 1.—Plot of HD spectral type against spectral type determined at the Warner and Swasey Observatory of the Case School of Applied Science.

seem to have been wrongly classified in the *Henry Draper Catalogue*. Investigation showed no detectable magnitude error in our spectral classification.

The stars common to the Upsala[7] and Case lists which are classified as giant and dwarf indicate that there is agreement between the two lists for 231 of the 262 stars, or for 88 per cent of the stars in common. On the other hand, a comparison for the 63 stars common to the *Bergedorfer Spectral Durchmusterung*[8] and Table 1 show an agreement as to giant and dwarf classifications in only 33 per cent of the cases.

The percentages of dwarf stars near the pole are given in Table 2 from photovisual magnitude 6 to 11 and from G2 to M5. Our results agree closely, particularly for the K stars, with those obtained by Van Rhijn,[9] Bok,[10] and Van de Kamp and Vyssotsky[11]

[7] *Medd. Astr. Obs. Upsala*, **29**, 1, 1927.

[8] Schwassmann and Van Rhijn, Bergedorf, 1935.

[9] *Zs. f. Ap.*, **10**, 161, 1935.

[10] *Harvard Circ.*, No. 400, 1935.

[11] *Pub. McCormick Obs.*, Vol. **7**, 1937.

from secular parallaxes and from proper motions. For the G2 and G5 stars Table 2 indicates a somewhat larger percentage of dwarfs than found by the foregoing authors. The rapid change in percentage of dwarfs from G2 to G8 is noteworthy and indicates the necessity for accurate spectral classifications in this range when attempts are made to draw conclusions regarding percentages of dwarfs.

TABLE 2

PERCENTAGES OF DWARF STARS WITHIN 5° OF THE NORTH POLE

m_{pv}	6 to 7		7 to 8		8 to 9		9 to 10		10 to 11	
Sp. Class	Percentage Dwarfs	No. Stars	Percentage Dwarfs	No. Stars	Percentage Dwarfs	No. Stars	Percentage Dwarfs	No. Stars	Percentage Dwarfs	No. Stars
G2	....	0	....	0	100	3	100	8	95	42
G5	0	1	50	4	62	8	59	27	65	102
G8	0	1	0	5	10	20	9	64	18	68
K0	0	4	0	3	0	17	7	27	27	15
K2	....	0	0	2	0	4	10	10	33	9
K3	0	1	0	2	25	4	0	3	40	5
K5	....	0	....	0	0	4	40	5	29	7
M0–M5	0	2	....	0	0	3	0	3	0	4
G8–K5	0	6	0	12	6	49	11	109	22	104

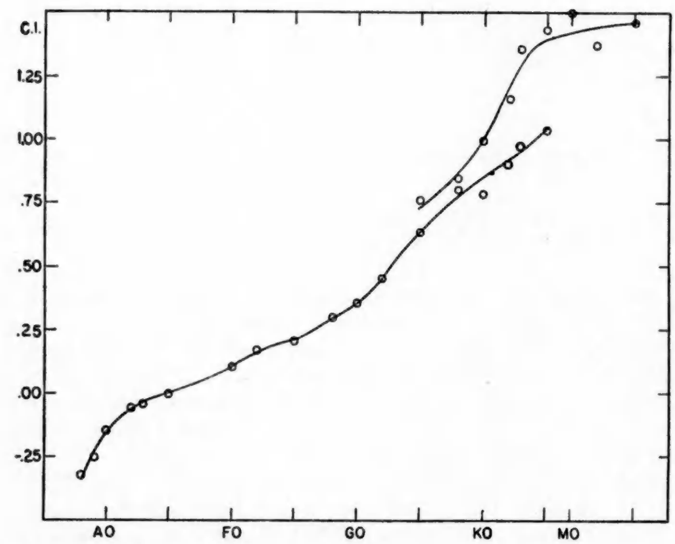

FIG. 2.—The intrinsic color-spectrum relation for stars near the north pole of rotation. The lower curve is for dwarf stars; the upper curve for giant stars.

COEFFICIENT OF SELECTIVE ABSORPTION AND THE COLOR-SPECTRUM RELATION

From a study of the spectra and corresponding color indices of stars near the pole, Seares and Joyner[1] have obtained the coefficient of selective absorption and established the intrinsic color-spectrum relation for stars within a region of 10° of the north celestial pole. All spectra used were reduced to the HD system, but the division of stars into giants and dwarfs was made primarily on the basis of the color indices.

In our case we are dealing with a uniform source of 1150 spectra, divided, as far as possible, into giants and dwarfs independent of their colors, with the stars confined within 5° of the north pole. With this material and with the colors obtained from Seares,

TABLE 3

INTRINSIC COLOR INDICES OF STARS

Sp.	CI (Case)	No. of Stars	CI (Sears and Joyner)	Diff.	Sp.	CI (Case)	No. of Stars	CI (Seares and Joyner)	Diff.
B8......	−0.35	3	−0.29	−0.06	dK0......	+0.85	6	+0.89	−0.04
B9......	− .24	6	− .23	− .01	dK2......	0.91	4	+1.01	.10
A0......	− .15	18	− .15	.00	dK3......	0.96	3	+1.06	.10
A2......	− .06	25	− .05	− .01	dK5......	1.03	4	....	....
A3......	− .04	21	− .02	− .02					
A5......	.00	35	.00	.00	gG5......	0.73	53	+0.78	.05
F0......	+ .11	31	+ .12	− .01	gG8......	0.87	138	+0.90	.03
F2......	+ .17	57	+ .16	+ .01	gK0......	0.99	61	+1.06	.07
F5......	+ .21	59	+ .26	− .05	gK2......	1.20	21	+1.25	.05
F8......	+ .30	105	+ .35	− .05	gK3......	1.30	12	+1.37	.07
G0......	+ .35	149	+ .42	− .07	gK5......	1.40	12	+1.45	.05
dG2......	+ .45	54	+ .50	− .05	gM0......	1.42	2	(1.47)	.05
dG5......	+ .63	91	+ .64	− .01	gM2......	1.44	8	(1.49)	−0.05
dG8......	+0.78	20	+0.79	−0.01	gM5......	+1.46	3	....	....

Ross, and Joyner[6] and with the values of absolute magnitude for the different spectral classes referred to in the Seares-Joyner[1] paper, the selective absorption at the pole was obtained. The results indicate that the selective absorption increases linearly to 0.30 mag. at 450 parsecs and remains constant thereafter. This result is in substantial agreement with the results of Seares and Joyner.[1] However, Stebbins, Huffer, and Whitford[12] obtained, from the photoelectric colors of 75 stars in a region within 10° of the pole, a value of the coefficient of selective absorption approximately two-thirds of the one given above. Using our derived coefficient of selective absorption, we obtain the average intrinsic color for each spectral type. These colors are shown in Figure 2 and Table 3. The fourth column of this table lists the corresponding values obtained by Seares and Joyner. The fifth column is the difference, Column 2 *minus* Column 3. A small systematic difference having an average value of −0.04 is present between the two determinations.

[12] *Ap. J.*, **94**, 215, 1941.

THE SPACE MOTIONS OF THE CLUSTER VARIABLES

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ABSTRACT

All available proper-motion and radial-velocity material was used in the study of the space motions of the cluster variables. The radial velocities of 67 stars gave a solar motion of 157 km/sec toward the apex $\alpha = 20^h50^m$, $\delta = +59°$. The tangential motions of 58 stars gave a solar motion of 142 km/sec toward the apex $\alpha = 21^h24^m$, $\delta = +38°$.

Four different solutions were completed for the velocity ellipsoid. These solutions were, however, not in agreement. The conclusion was reached that the velocity dispersion along the axis in the direction of the galactic center is 170 km/sec; and that along the axis in the direction of the galactic poles is 50 km/sec. But the velocity dispersion along the axis in the Cygnus direction is unknown. SW Bootis, RZ Cephei, and U Comae are possibly in retrograde motion about the galactic center.

I. INTRODUCTION

The problem of the origin of the cluster variables is closely related to the problem of their space motions. We would better understand the nature of these variables if we knew the place of their origin. It was with this problem in mind that the investigation was undertaken.

Baade[1] points out that there are really two Hertzsprung-Russell diagrams: the first is typical of the irregular galaxies and of the outer regions of the late-type spirals; the second is typical of globular clusters, elliptical nebulae, and the inner regions of spirals. This makes it probable that the galactic-cluster variables originated in the globular clusters, being thrown therefrom, or from near the galactic center, by perturbations resulting from the close approach of another star.

The cluster variables have at least one advantage as subjects for the study of space motions: the absolute magnitude is uniform and is known with considerable accuracy. Therefore, it is a simple matter to find the tangential velocity components of cluster variables from their median apparent magnitudes and proper-motion components. There is, however, the marked disadvantage that the cluster variables with known proper motions and radial velocities are pretty well confined to the second, third, and fourth quadrants of right ascension and to the northern hemisphere of the sky.

Previous studies of the motions of the cluster variables have been made by Strömberg,[2] by Bok and Boyd,[3] and by Wilson.[4] Of these investigations, that by Wilson was most complete.

It was largely in order to obtain the distribution of the peculiar motions of the cluster variables and to find what light, if any, was thereby thrown on the origin of these stars that the present investigation was undertaken.

II. THE MATERIAL

The 67 radial velocities used in this investigation were all drawn from A. H. Joy's catalogue of 1938.[5] This list is identical with that used by Wilson[4] in his investigation. The probable error of each determination of radial velocity is about 7 km/sec; this value was adopted.

[1] *Mt. W. Contr.*, No. 696; *Ap. J.*, **100**, 137, 1944.

[2] *Mt. W. Contr.*, No. 293; *Ap. J.*, **61**, 363, 1925.

[3] *Harvard Bull.*, No. 893, p. 1, 1933.

[4] *Mt. W. Contr.*, No. 604; *Ap. J.*, **89**, 218, 1939.

[5] *Pub. A.S.P.*, **50**, 302, 1938.

The proper motions were drawn from several different sources, the most important of these being Bok and Boyd's paper,[3] which furnishes about 40 determinations of proper motions and to which system all relative proper motions were reduced. Another important source, and one for which the writer wishes to express sincere appreciation, was a letter from Dr. A. van Maanen, which provided 25 unpublished determinations of the proper motions of cluster variables by Van Maanen and R. E. Wilson. W. J. Luyten's note[6] furnished 11 determinations of proper motions; J. C. Kapteyn and P. J. van Rhijn's paper[7] of 1922 furnished 14, including 5 meridian circle determinations; and R. E. Wilson's paper[8] of 1922 furnished a few.

The magnitudes used in the parallax determinations were, as far as possible, photographic median magnitudes obtained from Schneller's catalogue[9] for 1940.

Relative proper motions, mostly those published by Luyten and the unpublished proper motions by Van Maanen and Wilson, were corrected for parallactic motion and galactic rotation by means of the proper tables in *Groningen Publication* No. 45.[10] The adopted value of the proper motion for stars for which there were more than one determination was the arithmetic mean of all available determinations. The probable error for a single determination of one component of a proper motion was found to be, on the average, $\pm 0\overset{''}{.}008$; this value was adopted.

In determining the parallaxes of the cluster variables from their apparent magnitudes, the absolute median magnitude of 0.0 ± 0.2 was adopted, while a space absorption of 0.9 mag/kps for photographic light and an absorbing layer 300 parsecs thick on each side of the galactic plane were assumed.

III. THE SOLAR MOTION

Two solutions for the solar motions of the cluster variables were made; the first from the radial velocities and the second from the tangential velocities derived from the proper motions.

The solution from the radial velocities was made by Airy's method, which leads to the well-known equation of condition:

$$V = -X \cos\alpha \cos\delta - Y \sin\alpha \cos\delta - Z \sin\delta + K ,$$

in which V is the radial velocity, X is the solar-motion component in the direction of the vernal equinox, Y is the solar-motion component in the direction of 6 hours' right ascension and 0° declination; Z is the solar-motion component in the direction of the north pole, K is the speed of expansion or contraction of the stars as a group relative to the sun, α is the right ascension, and δ is the declination. The equations of condition for the tangential motions follow:

$$X' \sin\alpha - Y' \cos\delta = T_\alpha ,$$

$$X' \cos\alpha \sin\delta + Y' \sin\alpha \sin\delta - Z' \cos\delta = T_\delta ,$$

in which X', Y', Z' are the solar motion components and T_α and T_δ are the tangential components of the star's motion in right ascension and declination, respectively, relative to the sun.

Solving our two systems of normal equations for the solar motion gives the values in Table 1.

The motions and components are in kilometers per second, and the errors are probable errors. The results from the radial velocities agree with those found by Wilson from

[6] *Harvard Bull.*, No. 847, 1927.

[7] *B.A.N.*, Vol. 1, No. 8, 1922.

[8] *A.J.*, Vol. 35, No. 5, 1922.

[9] *Katalog und Ephemeriden Veränderlicher Sterne für 1940.*

[10] P. J. van Rhijn and Bart J. Bok, *Groningen Pub.*, No. 45 1931.

the same material[11] in spite of the fact that his solution was for galactic rotation as well as for solar motion. Both of the present derivations of the solar motion give a much larger motion than that obtained by Strömberg.[12] The reason for this is that the stars used in the present computation are, on the average, fainter and more distant than those used by Strömberg and have a greater motion relative to the sun. The K term is probably not real.

The determinations agree with each other as well as can be expected, considering the the fact that only a small number of widely scattered stars were represented. The agreement in the two values of the solar motion tends to prove that the value of the absolute magnitude adopted in computing the tangential velocities is correct.

TABLE 1

THE SOLAR MOTION OF THE CLUSTER VARIABLES

	Radial Velocities	Proper Motions
Number of stars	67	58
X	$+55\pm13$	$+88\pm21$
Y	-60 ± 16	-70 ± 19
Z	$+134\pm23$	$+87\pm19$
K	$+10\pm12$	....
Solar motion	157	142
Right ascension of apex	20^h50^m	21^h24^m
Declination of apex	$+59°$	$+38°$
Galactic longitude of apex	64°	53°
Galactic latitude of apex	$+9°$	$-10°$

IV. THE VELOCITY ELLIPSOID

Charlier's[13] method was used in computing the velocity ellipsoid. First, the second-order moments were found by means of least-squares solutions based upon the relations between these moments and the radial and tangential velocities. The equation of condition governing the relations between the radial velocities and the six second-order moments is as follows:

$$v^2 - e^2 = a^2 m_{200} + b^2 m_{020} + c^2 m_{002} + 2ab\,m_{110} + 2bc\,m_{011} + 2ac\,m_{101} \, ,$$

where v is the radial component of the star's peculiar motion; e is the mean error of the star's radial velocity; the m's are the six second-order moments of the velocity distribution; and $a = \cos\alpha \cos\delta$, $b = \sin\alpha\cos\delta$, and $c = \sin\delta$.

The three roots of the cubic from the six-moment radial velocity solution for the velocity ellipsoid were:

$$L_1 = -708 \; ; \qquad L_2 = +11{,}170 \; ; \qquad L_3 = +25{,}830 \, .$$

As the lengths of the axes of the ellipsoid are the square roots of the L's, the short axis is imaginary, making this solution a physical impossibility. The reason for this outcome is that the first three normal equations of this system are more or less indeterminate, with the result that the first three moments are not well separated.

Since the solution from the radial velocities alone led to a physical impossibility, it

[11] *Mt. W. Contr.*, No. 604; *Ap. J.*, **89**, 218, 1939.

[12] *Mt. W. Contr.*, No. 293; *Ap. J.*, **69**, 363, 1925.

[13] *The Motion and the Distribution of the Stars* ("Memoirs of the University of California," No. 7, 1926).

was decided to use the proper-motion material to derive the velocity ellipsoid. To find the moments from the tangential velocities we have the following relations:

$$m_{200} = m'_{200} - X^2\,; \qquad m_{020} = m'_{020} - Y^2\,; \qquad m_{002} = m'_{002} - Z^2\,,$$

$$m_{110} = m'_{110} - XY\,; \qquad m_{011} = m'_{011} - YZ\,; \qquad m_{101} = m'_{101} - XZ\,.$$

Also, there are three equations of condition connecting the tangential velocities with the m's:

$$T_\alpha^2 - E_T^2 = \sin^2\alpha\, m'_{200} + \cos^2\alpha\, m'_{020} - 2\sin\alpha\cos\alpha\, m'_{110}\,,$$

$$\begin{aligned} T_\delta^2 - E_T^2 = {} & \cos^2\alpha\sin^2\delta\, m'_{200} + \sin^2\alpha\sin^2\delta\, m'_{020} + \cos^2\delta\, m'_{002} + 2\cos\alpha\sin\alpha\sin^2\delta\, m'_{110} \\ & - 2\sin\alpha\sin\delta\cos\delta\, m'_{011} - 2\cos\alpha\sin\delta\cos\delta\, m'_{101}\,, \end{aligned}$$

$$\begin{aligned} T_\alpha T_\delta = {} & \sin\alpha\cos\alpha\sin\delta\, m'_{200} - \sin\alpha\cos\alpha\sin\delta\, m'_{020} + (\sin^2\alpha\sin\delta - \cos^2\alpha\sin\delta)\, m'_{200} \\ & + (\cos\alpha\cos\delta + \sin\alpha\cos^2\delta)\, m'_{011} - \sin\alpha\cos\delta\, m'_{101}\,. \end{aligned}$$

The symbol E_T represents the mean error of one component of the tangential velocity for individual stars. The average E_T is 63 km/sec, but this average has little significance, since the dispersion is great; it is the E_T's of the individual stars that have been used in forming the normal equations.

Two complete solutions for the velocity ellipsoid were undertaken: the first with the proper motions corrected for galactic rotation and the second with the proper motions not corrected for galactic rotation. The corrections for galactic rotation used were those of *Groningen Publication* No. 45. In each case, three sets of normal equations were formed, based on the three equations of condition involving the tangential motions. The corresponding equations of the different sets were then added together and the resulting set of equations transformed from m' to m. The solutions were then carried out. Because the orientation of the ellipsoid axes with the proper motions corrected for galactic rotation is more nearly normal, the set of normal equations connected with this solution was added to the corresponding equations for the radial velocity solution, and a third solution for the velocity ellipsoid was made.

Then, lastly, a solution for the radial velocities alone was made by assuming fixed axes as follows: X-axis, in the direction of the galactic center; Y-axis, in the galactic plane, 90° from the galactic center, in the direction of Cygnus; Z-axis, in the direction of the north galactic pole. The positions of the radial velocity stars were transformed to this system of galactic co-ordinates, the three normal equations formed, and the solution made for the axes of the velocity ellipsoid. The four resulting velocity ellipsoids are given in Table 2.

The writer does not have any explanation for most of the differences between the various velocity ellipsoids, beyond calling attention to the fact that the number of stars involved is small and that the stars are distributed in a rather peculiar fashion. The fact that the proper-motion solutions are more nearly spherical can be accounted for by the assumption that the adopted probable error of the proper motions is too small.

The dispersion along the axis through the galactic center is 170 km/sec, or thereabouts; this is fairly certain, since all four solutions agree on this point. The numerous radial-velocity stars in high northern galactic latitudes force the conclusion that the dispersion along the axis through the galactic poles is about 50 km/sec.

The dispersion along the axis in the galactic plane in the Cygnus direction is unknown. The best method to use in finding this dispersion would be to secure the radial velocities of about 20 more cluster variables in the Cygnus region. Until this is done, it will be impossible to make any valid interpretation of the space motions of the cluster variables.

The motions of those cluster variables for which both the proper motions and radial velocities are known were analyzed with respect to rectilinear components of motion in the galactic system of co-ordinates. Three stars were found that may possibly be in retrograde motion about the galactic center. These stars are given in Table 3. The velocity components are in km/sec.

TABLE 2

VELOCITY ELLIPSOIDS COMPUTED FOR THE CLUSTER VARIABLES

	Uncorrected Proper Motions	Corrected Proper Motions	Radial Velocity	Combined Data
	Axis toward Galactic Center			
Length	158 km/sec	185 km/sec	168 km/sec	168 km/sec
Right ascension	18^h33^m	21^h13^m	17^h28^m	18^h14^m
Declination	−35°	−57°	−30°	−43°
Galactic longitude	328°	304°	325°	319°
Galactic latitude	−15°	+43°	0°	−4°
	Axis toward Galactic Pole			
Length	233 km/sec	132 km/sec	47 km/sec	80 km/sec
Right ascension	15^h35^m	2^h56^m	12^h40^m	13^h37^m
Declination	+48°	+3°	+28°	+21°
Galactic longitude	42°	142°	0°	336°
Galactic latitude	+51°	−46°	+90°	+76°
	Third Axis			
Length	188 km/sec	239 km/sec	90 km/sec	216 km/sec
Right ascension	11^h24^m	20^h44^m	20^h56^m	20^h56^m
Declination	−22°	+32°	+46°	+41°
Galactic longitude	248°	43°	55°	51°
Galactic latitude	+37°	−7°	0°	−4°

TABLE 3

CLUSTER VARIABLES IN POSSIBLE RETROGRADE GALACTIC ROTATION

Star	X	Y	Z
SW Boo	−514	− 1	+121
RZ Cep	−510	+ 95	+323
U Com	−475	+173	+180

In closing, I wish to express my thanks to Professor R. J. Trumpler of the University of California, whose advice and guidance greatly facilitated this piece of research.

ON THE EQUATION OF STATE OF IONIZED HYDROGEN

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ABSTRACT

In order to ascertain the deviations from conditions of a perfect gas inside main-sequence stars, arising from electrical charges on the particles, an approximate form for the equation of state of totally ionized hydrogen has been derived, in which account has been taken of the charges of the electrons and protons. Expressions for the internal energy and for $\Gamma_2/(1-\Gamma_2)$, where Γ_2 is the adiabatic exponent, are obtained. The electrostatic correction is found to amount to 1 or 2 per cent of the total pressure for stars with masses smaller than that of the sun.

Since the early researches of Eddington on stellar interiors, it has been recognized[1] that a satisfactory first approximation to the pressure in the interior of a star is given by the perfect-gas law, despite the electrical charges of the particles composing the stellar material. The early calculations of the electrostatic corrections to the pressure were intended to indicate only the order of magnitude of the effect. Because of the increasing exactness of present-day stellar models,[2] it seems worth while to attempt a more precise estimate of the deviations from conditions of a perfect gas inside ordinary stars due to the charges of the ions and electrons.

In this paper we propose to determine the form and magnitude of the correction to the perfect-gas law for completely ionized hydrogen by taking account of the electrostatic interactions. One may expect that the results will be of some interest in evaluating conditions within actual stars. It is well known that hydrogen is responsible for about two-thirds of the pressure within a star; moreover, the corrections for heavier elements are even greater than those for hydrogen.[3] Therefore, the electrostatic corrections for hydrogen will, in some sense, represent a minimum for conditions in actual stars. The formulation of the problem as given below eliminates some of the difficulties of the earlier treatments and, as we shall see, affords a better insight into the nature and seriousness of the assumptions involved in deducing the magnitude of the effect.

1. *Formulation of the problem.*—Imagine n hydrogen atoms, completely ionized, in a spherical volume $V=(\frac{4}{3})\pi R^3$. The contents of the sphere is thus n protons and n electrons. It is well known[4] that the equation of state for such a group of particles with Coulomb fields is

$$PV=2nkT+\frac{1}{3}\left(\sum_{i=1}^{2n}\sum_{j=i+1}^{2n}\frac{\epsilon_i\epsilon_j}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}\right)_{\text{average}}, \tag{1}$$

where $\boldsymbol{r}_q$ is the position-vector of the qth particle, and ϵ_q is its charge in e.s.u. Our task is the evaluation of the long-time average of the double summation. Assigning to each

[1] Rosseland, *M.N.*, **84**, 720, 1924.

[2] E.g., Cowling, *M.N.*, **96**, 42, 1935; Henrich, *Ap. J.*, **96**, 106, 1942; Sen and Burman, *Ap. J.*, **100**, 347, 1944.

[3] Rosseland, *op. cit.*

[4] See, e.g., Jeans, *The Dynamical Theory of Gases*, p. 131, 4th ed.; Cambridge: Cambridge University Press, 1925.

proton a number from 1 to n, and to each electron a number from $n+1$ to $2n$, we may write

$$\left.\begin{aligned} \mathsf{U} &= \frac{1}{3}\left(\sum_{i=1}^{2n}\sum_{j=i+1}^{2n}\frac{\epsilon_i\epsilon_j}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}\right)_{\text{average}} \\ &= \tfrac{1}{3}\epsilon^2\left\{\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{1}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}+\sum_{i=n+1}^{2n}\sum_{j=i+1}^{2n}\frac{1}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}\right. \\ &\qquad\qquad\left.-\sum_{i=1}^{n}\sum_{j=n+1}^{2n}\frac{1}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}\right\}_{\text{average}}, \end{aligned}\right\} \tag{2}$$

where ϵ is the unit electric charge, 4.801×10^{-10} e.s.u.

The time-average of a typical term of equation (2) (say that belonging to the interaction of the qth and the pth particles) between the times t_0 and τ, is

$$\frac{1}{\tau-t_0}\int_{t_0}^{\tau}\frac{1}{r(t)}\,dt; \qquad r(t) = |\boldsymbol{r}_q(t) - \boldsymbol{r}_p(t)|.$$

If we call $\mathsf{P}_{qp}(r)\cdot 4\pi r^2 dr$ the probability of finding particle q within a spherical shell of radius r and thickness dr, centered at p, then, by the definition of a probability,

$$\lim_{\tau\to\infty}\frac{1}{\tau-t_0}\int_{t=t_0}^{t=\tau}\frac{1}{r(t)}\,dt = \int_{r=0}^{r=\infty}\frac{1}{r}\,\mathsf{P}_{qp}(r)\,4\pi r^2 dr.$$

Since the n electrons and the n protons are indistinguishable *inter se*, the probabilities P_{qp} are all of three types; we may call them

$\mathsf{P}_1(r)$	when	$q \leqslant n,\ p \leqslant n$	(proton-protron);
$\mathsf{P}_2(r)$	when	$q > n,\ p > n$	(electron-electron);
$\mathsf{P}_3(r)$	when	$q \leqslant n,\ p > n$	(proton-electron).

Then

$$\left.\begin{aligned} \mathsf{U} = \frac{4\pi}{3}\epsilon^2\Bigg[\frac{n(n-1)}{2}\int_0^\infty \mathsf{P}_1(r)\,r\,dr \\ +\frac{n(n-1)}{2}\int_0^\infty \mathsf{P}_2(r)\,r\,dr - n^2\int_0^\infty \mathsf{P}_3(r)\,r\,dr. \end{aligned}\right\} \tag{3}$$

For all regions in which classical mechanics applies, Boltzmann's principle gives the following form for the probability P of a particle of charge $\pm\,\epsilon$ being in a region where the potential is φ:

$$\mathsf{P} \propto e^{-(\pm\epsilon\varphi/kT)}.$$

Now the average potential about any particle in the assembly will be spherically symmetrical. Let us call the average potential at a distance r from a particle $\varphi_P(r)$ for the protons, and $\varphi_E(r)$ for the electrons. We formally identify this average, with the actual potential, which appears in the expression for the probability, so that

$$\mathsf{P}_1 = K_1 e^{-\epsilon\varphi P(r)/kT}; \qquad \mathsf{P}_2 = K_2 e^{+\epsilon\varphi E(r)/kT}; \qquad \mathsf{P}_3 = K_3 e^{+\epsilon\varphi P(r)/kT}. \tag{4}$$

The K's are normalizing factors.

Debye and Hückel[5] have derived a method for calculating an approximate form for $\varphi_P(r)$ and $\varphi_E(r)$. Applied to our problem, it gives

$$\varphi_P(r) = -\varphi_E(r) = \frac{\epsilon}{r} e^{-\kappa r}, \tag{5}$$

where

$$\kappa^2 = \frac{8\pi\epsilon^2 n}{kTV}. \tag{6}$$

Numerically,

$$\kappa = 5.013 \times 10^8 \left[\frac{(\rho/100)}{(T/10^7)}\right]^{\frac{1}{2}}.$$

Hence,

$$\left.\begin{aligned} \mathsf{P}_1 = \mathsf{P}_2 &= K \ \exp\left(-\frac{\epsilon^2}{rkT} e^{-\kappa r}\right), \\ \mathsf{P}_3 &= K_3 \exp\left(+\frac{\epsilon^2}{rkT} e^{-\kappa r}\right), \end{aligned}\right\} \tag{7}$$

for all regions in which classical mechanics is valid.

For values of r much less than δ, where

$$\frac{\epsilon^2}{\delta} = \tfrac{3}{2} kT, \tag{8}$$[6]

the whole particle-concept becomes inapplicable. It is consistent with this picture, then, to put $\mathsf{P}_1 = \mathsf{P}_2 = \mathsf{P}_3 = 0$, $r < \delta$, and to use equations (7) for them when $r \geqslant \delta$. For convenience, define

$$I_1 = \frac{\int_\delta^R \exp\left(-\frac{\epsilon^2}{rkT} e^{-\kappa r}\right) r\,d r}{\int_\delta^R \exp\left(-\frac{\epsilon^2}{rkT} e^{-\kappa r}\right) r^2 d r}, \tag{9}$$

and

$$I_3 = \frac{\int_\delta^R \exp\left(+\frac{\epsilon^2}{rkT} e^{-\kappa r}\right) r\,d r}{\int_\delta^R \exp\left(+\frac{\epsilon^2}{rkT} e^{-\kappa r}\right) r^2 d r}. \tag{9'}$$

Then

$$\mathsf{U} = \tfrac{1}{3}\,\epsilon^2 \left[n^2 (I_1 - I_3) - n I_1\right].$$

When use is made of the fact that the mean atomic weight μ is $\frac{1}{2}$ for ionized hydrogen, we find

$$\frac{\mathsf{U}}{V} = \frac{\epsilon}{3H^2}\,\rho^{\substack{2\\2}} \left[V (I_1 - I_3) - \frac{H}{\rho} I_1\right], \tag{10}$$

where H is the mass of the proton, and ρ is the density of the gas.

Combining equations (1), (2), and (10), we obtain

$$P = \frac{2k}{H}\,\rho T + \frac{\mathsf{U}}{V}. \tag{11}$$

For stellar conditions, the last term in brackets in equation (10) is seen to yield a correction which vanishes as $R \to \infty$. Even at R = 100 cm, the contribution is only 10^4 bar.

[5] *Phys. Zs.*, **24**, 185, 1923.

[6] Numerically, $\delta = (1.143 \times 10^{-3})/T$.

The ratio of this to the actual pressures is vanishingly small ($\sim 10^{-10}$). Since we are not interested in corrections of this size, it is sufficient to write

$$P = \frac{2k}{H}\,\rho T + \frac{\epsilon^2}{3H^2}\,\rho^2\,[V\,(I_1 - I_3)\,]\,. \tag{12}$$

Only the mathematical problem of evaluating $V(I_1 - I_3)$ for large R (of the order of meters, say) remains.[7]

2. *The explicit form of the equation of state.*—Introducing the new variable ξ, defined by

$$r = \delta\xi\,, \tag{13}$$

we find

$$\left.\begin{aligned} V\,(I_1 - I_3) = \tfrac{4}{3}\pi\,(\delta L)^3\cdot\frac{1}{\delta}\cdot\Bigg(&\frac{\int_1^L \exp\left(-\frac{3}{2\xi}\,e^{-a\xi}\right)\xi d\xi}{\int_1^L \exp\left(-\frac{3}{2\xi}\,e^{-a\xi}\right)\xi^2 d\xi} \\ &- \frac{\int_1^L \exp\left(\frac{3}{2\xi}\,e^{-a\xi}\right)\xi d\xi}{\int_1^L \exp\left(\frac{3}{2\xi}\,e^{-a\xi}\right)\xi^2 d\xi}\Bigg)\,, \end{aligned}\right\} \tag{14}$$

where we have written

$$a = \kappa\delta\,, \quad L = \frac{R}{\delta}\,. \tag{15}$$

Numerically,

$$a = 5.729\times 10^{-2}\left[\frac{(\rho/100)}{(T/10^7)^3}\right]^{\frac{1}{2}}.$$

Each of the four integrals is of the same type:

$$Q\,(q, n) = \int_1^L \exp\left(\frac{3q}{2\xi}\,e^{-a\xi}\right)\xi^n d\xi\,. \tag{16}$$

We re-write this as

$$\left.\begin{aligned} Q\,(q, n) = \int_1^\infty\left[\exp\left(\frac{3q}{2\xi}\,e^{-a\xi}\right) - 1\right]\xi^n d\xi + \int_1^L \xi^n d\xi \\ - \int_L^\infty\left[\exp\left(\frac{3q}{2\xi}\,e^{-a\xi}\right) - 1\right]\xi^n d\xi\,. \end{aligned}\right\} \tag{17}$$

Remembering that $L \sim 10^{11}$, we find to abundant accuracy that

$$Q\,(q, n) = C\,(q, n) + \frac{1}{n+1}\,L^{n+1}\,, \tag{18}$$

where

$$C\,(q, n) = \int_1^\infty\left[\exp\left(\frac{3q}{2\xi}\,e^{-a\xi}\right) - 1\right]\xi^n d\xi\,. \tag{19}$$

Substituting the expression (18) in equation (14) and noting that

$$\left.\begin{aligned} C\,(\pm 1, 1) &\approx \pm\frac{3}{2a}\,, \\ C\,(\pm 1, 2) &\approx \pm\frac{3}{2a^2}\,, \end{aligned}\right\} \tag{20}$$

[7] In order to make sure that the correction is really a "local" phenomenon, i.e., does not depend on conditions at great distances from the point under consideration, we do not immediately set $R = \infty$.

we see that the dominant term is

$$V\,(I_1 - I_3) = -8\pi\delta^2 \int^{\infty} \sinh\left(\frac{3}{2\xi}\,e^{-\alpha\xi}\right) \xi d\xi\,. \tag{21}$$

The next term is smaller by a factor of about 10^8. Let

$$D = \int_1^{\infty} \left[\sinh\left(\frac{3}{2\xi}\,e^{-\alpha\xi}\right) - \frac{3}{2\xi}\,e^{-\alpha\xi}\right] \xi d\xi\,. \tag{22}$$

Then

$$V\,(I_1 - I_3) = -8\pi\delta^2 \left[\frac{3}{2\alpha} + D\right]. \tag{23}$$

Now for conditions applicable in stellar interiors, α is of the order of 10^{-2}. Numerical integrations show that $D < 0.75$ for the whole range of relevant values of α. With an accuracy of better than 1 per cent,

$$V\,(I_1 - I_3) = -12\pi\,\frac{\delta^2}{\alpha} = -12\pi\,\frac{\delta}{\kappa}\,. \tag{24}$$

Using equations (12), (8), and (24), we finally obtain for the equation of state

$$P = \frac{2k}{H}\,\rho T - \frac{2\,(2\pi)^{1/2}\epsilon^3}{3H^{3/2}k^{1/2}}\,\rho^{3/2}\,T^{-1/2}\,. \tag{25}$$

3. *The adiabatic exponent.*—A quantity of considerable interest in the study of stellar structure is Γ_2,[8] governing adiabatic changes in the gas, according to the definition

$$\frac{dP}{P} + \frac{\Gamma_2}{1-\Gamma_2}\,\frac{dT}{T} = 0\,. \tag{26}$$

The second law of thermodynamics, stated for a gas with internal energy $U(T, V)$, shows that, for an adiabatic change,

$$0 = dQ = \frac{\partial U}{\partial T}\,dT + \left(\frac{\partial U}{\partial V} + P\right) dV\,. \tag{27}$$

But, since[9]

$$\frac{\partial U}{\partial V} = T\,\frac{\partial P}{\partial T} - P \tag{28}$$

and

$$dP = \frac{\partial P}{\partial V}\,dV + \frac{\partial P}{\partial T}\,dT\,, \tag{29}$$

we readily find that

$$\frac{\Gamma_2}{1-\Gamma_2} = \frac{\dfrac{\partial U}{\partial T}\,\dfrac{\partial P}{\partial V} - T\left(\dfrac{\partial P}{\partial T}\right)^2}{P\,\dfrac{\partial P}{\partial T}}\,. \tag{30}$$

We can find the internal energy of a gas with an equation of state of the form (25). Performing the indicated operations in equation (28), we find

$$U = \psi\,(T) - \frac{2\,(2\pi)^{1/2}\epsilon^3}{H^{3/2}k^{1/2}}\,m^{3/2}\,V^{-1/2}\,T^{-1/2}\,.$$

[8] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure*, p. 56, Chicago: University of Chicago Press, 1939.

[9] *Ibid.*, p. 36.

The function $\psi(T)$ is the expression for the internal energy of a perfect gas, as may be seen by considering the value of U as $V \to \infty$. Then

$$U = 3\frac{k}{H}mT - \frac{2(2\pi)^{1/2}\epsilon^3}{H^{3/2}k^{1/2}}m^{3/2}V^{-1/2}T^{-1/2}, \tag{31}$$

where m is the mass in volume V. Use of equations (25) and (31) allows us to evaluate $\Gamma_2/(1-\Gamma_2)$ explicitly. For convenience, set

$$\eta P = -\frac{2(2\pi)^{1/2}\epsilon^3}{3H^{3/2}k^{1/2}}\rho^{3/2}T^{-1/2} \quad \text{and} \quad (1-\eta)P = \frac{2k}{H}\rho T. \tag{32}$$

We then have

$$\frac{\Gamma_2}{1-\Gamma_2} = \frac{-\frac{5}{2}(1-\eta)^2 + \frac{1}{4}\eta(1-\eta) + 2\eta^2}{1-\frac{3}{2}\eta}. \tag{33}$$

For small values of η, this becomes approximately

$$\frac{\Gamma_2}{1-\Gamma_2} = -\frac{5}{2} + \frac{3}{2}\eta. \tag{34}$$

4. *Conclusions.*—Equation (25) gives the expression, within the framework of our assumptions, for the equation of state of ionized hydrogen. The pressure may be conveniently given as

$$P = \frac{k}{\frac{1}{2}H}\rho T\left\{1 - 0.0140\left[\frac{(\rho/100)}{(T/10^7)^3}\right]^{1/2}\right\}. \tag{35}$$

It is immediately obvious that, for conditions similar to those at the center of the sun, the electrostatic correction will be of the order of 1 per cent of the total pressure. A direct substitution of the values for ρ_c and T_c obtained from the standard model[10] for the sun shows that, for these values,

$$\frac{\Delta P_E}{P} = 0.43 \text{ per cent},$$

where ΔP_E refers to the electrostatic correction to the pressure. Since the electrostatic correction is proportional to $[\beta/(1-\beta)]^{1/2}$, where β is the ratio of the gas pressure to the total pressure, it is, therefore, for stars built on the standard model, very nearly inversely proportional to the mass.[11] For a star like o_2 Eri C, whose mass is 0.20 $\odot$,[12] the electrostatic correction corresponding to ρ_c and T_c for the standard model is 2.1 per cent.

However, equation (25) as it stands is not entirely adequate as a basis for more precise discussions. At best it is valid only for hydrogen. Moreover, there are several possible sources of error. The inaccuracy introduced by using an average potential around each particle may be appreciable; the expression (5) itself for the potential is inexact for small values of r. Finally, the electrostatic correction is directly proportional to δ (see eq. [24]), the value of the cutoff. In fact, it is likely that the somewhat arbitrary assignment of this value is the greatest weakness of the present theory. Calculations are now in progress, with a view to removal of these restrictions.

The writer is indebted to Professor Mark Kac, of the Cornell University Department of Mathematics, and to Professor S. Chandrasekhar, of the Yerkes Observatory, for valuable discussions.

[10] *Ibid.*, p. 232. [11] *Ibid.*, p. 229. [12] *Ibid.*, p. 489.

THE VARIATIONS OF ABSORPTION-LINE CONTOURS ACROSS THE SOLAR DISC

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ABSTRACT

In this paper a new method for calculating theoretical contours of absorption lines at various points on the solar disc is described, the general case in which the ratio of the line absorption coefficient to the continuous absorption coefficient is variable through the atmosphere being considered. The method requires the solution of the appropriate boundary-value problem of line formation in approximations higher than those hitherto considered. Various tables which are needed for the practical construction of line contours are provided.

Theoretical line contours are constructed on the "third approximation," which provides predictions for three points on the solar disc, representing the fractions 0.97, 0.75, and 0.36 of the solar radius from the center. From the comparison between theoretical and observed contours for certain selected lines, it is found that the general trends in the observed center-limb effects are accounted for.

1. *Introduction.*—The variations of line contours across the solar disc provide information concerning the structure of the solar atmosphere and the physical processes relevant to the formation of absorption lines. In most solutions to the theoretical problem predicting such variations, it has been customary to assume that the ratio η_ν of the line and continuous absorption coefficients is a constant through the atmosphere. While the approximate constancy of η_ν through model stellar atmospheres is met under certain special conditions, it is, nevertheless, an assumption which is hard to justify in general. It would, therefore, be useful to have a treatment of the problem which does not make such restrictive assumptions and which is, at the same time, sufficiently simple to enable applications to practical cases. It is the object of this paper to provide such a theory and show that it is feasible by constructing theoretical contours for some typical cases and comparing them with observations.

2. *A method for computing the theoretical contours of absorption lines.*—The equation of transfer appropriate for the problem of the formation of absorption lines, originating from a ground state, is, in a standard notation,[1]

$$\mu \frac{dI_\nu}{dt_\nu} = I_\nu - \frac{(1-\epsilon)\,\eta_\nu}{2\,(1+\eta_\nu)} \int_{-1}^{+1} I_\nu d\mu - \frac{1+\epsilon\eta_\nu}{1+\eta_\nu} B_\nu\,(t_\nu)\,. \tag{1}$$

By introducing the quantity

$$\lambda_\nu = \frac{1+\epsilon\eta_\nu}{1+\eta_\nu}\,, \tag{2}$$

equation (1) can be re-written in the form

$$\mu \frac{dI}{dt} = I - \tfrac{1}{2}\,(1-\lambda) \int_{-1}^{+1} I\,d\mu - \lambda B\,(t)\,, \tag{3}$$

where for convenience we have suppressed the suffix ν. For the problem of predicting the equivalent widths of absorption lines, we can, without loss of generality, set $\epsilon = 0$; for, as is well known, a nonvanishing ϵ is required only for accounting for the central intensities. Following a general method, based on Gauss's formula for numerical quadratures,

[1] See, e.g., B. Strömgren, *Ap. J.*, **86**, 1, 1937.

which has recently been developed by Chandrasekhar[2] for the solution of the various problems of radiative transfer in the theory of stellar atmospheres, we replace equation (3) by the system of $2n$ linear equations

$$\mu_i \frac{dI_i}{dt} = I_i - \tfrac{1}{2}(1-\lambda)\sum_{j=-n}^{+n} a_j I_j - \lambda B(t) \qquad (i = \pm 1, \ldots, \pm n) \quad (4)$$

in the nth approximation, where the μ_i's are the zeros of the Legendre polynomial of order $2n$ and the a_j's are the appropriate Gaussian weights.

In the "standard case" the ratio η of the line absorption coefficient to the coefficient of continuous absorption at the frequency ν of the absorption line is assumed to be independent of the optical depth t in the line. The quantity λ has then a constant value λ_0 (say), and the Planck intensity $B(t)$ is expressible as a linear function of t,

$$B(t) = a_{\nu_0} + bt\,, \quad (5)$$

where $b = \lambda_0 b_{\nu_0}$. Expressions for a_{ν_0} and b_{ν_0} are obtained by expanding B as a Taylor series from $t = 0$, using Milne's relation[3] for the temperature distribution, namely,

$$T^4 = T_0^4\left(1 + \tfrac{3}{2}\tau\right), \quad (5')$$

where T_0 is the surface temperature and τ is the optical depth in the continuous spectrum. The solution[4] to the system of equations (4) appropriate for this case is

$$I_i^{(0)}(t) = \sum_{\alpha=1}^{n} \frac{L_\alpha^{(0)} e^{-k_\alpha t}}{1+\mu_i k_\alpha} + (\mu_i b + a_{\nu_0} + bt) \qquad (i = \pm 1, \ldots, \pm n)\,, \quad (6)$$

where the k_α's are the n positive roots of the characteristic equation

$$1 = (1-\lambda_0)\sum_{j=1}^{n} \frac{a_j}{1-\mu_j^2 k^2} \quad (7)$$

and the $L_\alpha^{(0)}$'s ($\alpha = 1, \ldots, n$) are the n constants of integration to be determined by the boundary conditions

$$\sum_{\alpha=1}^{n} \frac{L_\alpha^{(0)}}{1-\mu_i k_\alpha} + a_{\nu_0} = \mu_i b \qquad (i = 1, \ldots, n)\,. \quad (8)$$

This solution has been obtained in its numerical form[5] in the first three approximations. The roots k_α ($\alpha = 1, \ldots, n$) for various values of λ_0 are tabulated in paper IV (Table 1), and the constants $L_\alpha = \lambda_0 L_\alpha^{(0)}/b$ ($\alpha = 1, 2, 3$) in the third approximation are given in paper VI (Table 1) for various values of λ_0 and

$$x = \frac{n}{u}\,\frac{e^u - 1}{e^u}\,, \quad (9)$$

where

$$u = \frac{h\nu_0}{kT_0} \quad \text{and} \quad n = \frac{\kappa_{\nu_0}}{\bar{\kappa}}\,, \quad (10)$$

[2] *Ap. J.*, **100**, 76, 1944. This paper will be referred to hereafter as "II."

[3] Cf. E. A. Milne, *Handb. d. Ap.*, **3**, 119, Berlin, 1929.

[4] Cf. II, p. 86.

[5] C. U. Cesco, S. Chandrasekhar, and J. Sahade, *Ap. J.*, **100**, 355, 1944, and *Ap. J.*, **101**, 320, 1945. These papers will be referred to hereafter as "IV" and "VI," respectively.

κ_{ν_0} being the continuous absorption coefficient at the center ν_0 of the line and $\bar{\kappa}$, the mean of the continuous absorption coefficient over the spectrum. The quantity n is assumed constant through the atmosphere.

We shall consider now the case in which η is not constant but variable through the atmosphere. However, we shall assume the change in η with optical depth to be such that we can write

$$\lambda(t) = \lambda_0 + \delta\lambda(t) , \tag{11}$$

where the quantity $\delta\lambda$ represents variations, small compared to λ, about a properly chosen constant value λ_0. Under the same assumption, the Planck intensity will be given by the expression

$$B(t) = a_{\nu_0} + bt + b_{\nu_0}\int_0^t \delta\lambda \, dt . \tag{12}$$

We have, accordingly, to solve the system of equations

$$\left.\begin{aligned} \mu_i \frac{dI_i}{dt} = I_i - \tfrac{1}{2}(1-\lambda_0)\Sigma a_j I_j - \lambda_0 (a_{\nu_0} + bt) + \tfrac{1}{2}\delta\lambda \Sigma a_j I_j - \delta\lambda (a_{\nu_0} + bt) \\ - b\int_0^t \delta\lambda\, dt \qquad (i = \pm 1, \ldots , \pm n) . \end{aligned}\right\} \tag{13}$$

In solving this system of equations we shall suppose that we can use for the I_j's, appearing in the second summation term on the right-hand side, the solution $I_j^{(0)}$ obtained in the "first approximation" on the assumption of constant $\lambda = \lambda_0$. We have then to consider the system of nonhomogeneous linear equations

$$\left.\begin{aligned} \mu_i \frac{dI_i}{dt} = I_i - \tfrac{1}{2}(1-\lambda_0)\Sigma a_j I_j - \lambda_0 (a_{\nu_0} + bt) + \tfrac{1}{2}\delta\lambda \Sigma a_j I_j^{(0)} - \delta\lambda (a_{\nu_0} + bt) \\ - b\int_0^t \delta\lambda\, dt \qquad (i = \pm 1, \ldots , \pm n) . \end{aligned}\right\} \tag{14}$$

It appears that the most convenient method for solving this system of equations is the method of the variation of the parameters. Thus, writing the general solution in the form (cf. II, eq. [18])

$$I_i(t) = \sum_{a=1}^{n}\left[\frac{L_a(t)\, e^{-k_a t}}{1+\mu_i k_a} + \frac{L_{-a}(t)\, e^{+k_a t}}{1-\mu_i k_a}\right] + (\mu_i b + a_{\nu_0} + bt) \quad (i = \pm 1, \ldots , \pm n), \tag{15}$$

where L_a and L_{-a} ($a = 1, \ldots , n$) are functions of t, we readily obtain the variational equation

$$\left.\begin{aligned} \mu_i \sum_{a=1}^{n}\left[\frac{e^{-k_a t}}{1+\mu_i k_a}\frac{dL_a}{dt} + \frac{e^{+k_a t}}{1-\mu_i k_a}\frac{dL_{-a}}{dt}\right] = \tfrac{1}{2}\delta\lambda \sum_{j=-n}^{n} a_j I_j^{(0)} - \delta\lambda (a_{\nu_0} + bt) \\ - b\int_0^t \delta\lambda\, dt \qquad (i = \pm 1, \ldots , \pm n) . \end{aligned}\right\} \tag{16}$$

Substituting for $I_j^{(0)}$ in the foregoing equation from equation (6), we have

$$\left.\begin{aligned} \mu_i \sum_{a=1}^{n}\left[\frac{e^{-k_a t}}{1+\mu_i k_a}\frac{dL_a}{dt} + \frac{e^{+k_a t}}{1-\mu_i k_a}\frac{dL_{-a}}{dt}\right] = \delta\lambda \left[\frac{1}{1-\lambda_0}\sum_{a=1}^{n} L_a^{(0)} e^{-k_a t}\right] \\ - b\int_0^t \delta\lambda\, dt \qquad (i = \pm 1, \ldots , \pm n) . \end{aligned}\right\} \tag{17}$$

Equation (17) represents a system of $2n$ linearly independent equations for the $2n$ functions L_a and L_{-a} ($a = 1, \ldots , n$).

The system (17) can be solved in the manner presented in a paper by Chandrasekhar[6] for the solution of a similar set of equations (cf. VII, eq. [46]). Multiplying equation (17) by $a_i\mu_i^{m-1}$ ($m = 1, \ldots, 2n$) and summing over all i's, we obtain

$$\left.\begin{aligned}\sum_{a=1}^{n}\left[D_{m,a}e^{-k_a t}\frac{dL_a}{dt}+(-1)^m D_{m,a}e^{+k_a t}\frac{dL_{-a}}{dt}\right]\\ =\frac{2}{m}\epsilon_{m,\,\text{odd}}\Delta \qquad (m=1,\ldots,2n),\end{aligned}\right\} \tag{18}$$

where we have written

$$\Delta=\delta\lambda\left[\frac{1}{1-\lambda_0}\sum_{a=1}^{n}L_a^{(0)}e^{-k_a t}\right]-b\int_0^t\delta\lambda\, dt \tag{19}$$

and

$$D_{m,a}=\sum_i\frac{a_i\mu_i^m}{1+\mu_i k_a}=(-1)^m\sum_i\frac{a_i\mu_i^m}{1-\mu_i k_a}. \tag{20}$$

The appearance of the quantity $\epsilon_{m,\,\text{odd}}$ in equation (18) results from the relation

$$\sum_i a_i\mu_i^{m-1}=\frac{2}{m}\epsilon_{m,\,\text{odd}} \qquad (m=1,\ldots,4n), \tag{21}$$

where

$$\left.\begin{aligned}\epsilon_{m,\,\text{odd}}&=1 \text{ if } m \text{ is odd}\\ &=0 \text{ otherwise}.\end{aligned}\right\} \tag{22}$$

The quantity $D_{m,a}$ can be evaluated directly by means of the recursion formula

$$D_{m,a}=\frac{1}{k_a}\left(\frac{2}{m}\epsilon_{m,\,\text{odd}}-D_{m-1,a}\right) \quad (m=1,\ldots,4n), \tag{23}$$

which for odd, respectively even, values of m takes the forms

$$D_{2j-1,a}=\frac{1}{k_a}\left(\frac{2}{2j-1}-D_{2j-2,a}\right) \tag{24}$$

and

$$D_{2j,a}=-\frac{1}{k_a}D_{2j-1,a}. \tag{25}$$

In this notation the equation for the characteristic roots k_a becomes (cf. eq. [7])

$$D_{0,a}=\frac{2}{1-\lambda_0}. \tag{26}$$

For odd, respectively even, values of m, equation (18) takes the forms

$$\sum_{a=1}^{n}D_{2j,a}\left[e^{-k_a t}\frac{dL_a}{dt}+e^{+k_a t}\frac{dL_{-a}}{dt}\right]=0 \qquad (j=1,\ldots,n) \tag{27}$$

and

$$\sum_{a=1}^{n}D_{2j-1,a}\left[e^{-k_a t}\frac{dL_a}{dt}-e^{+k_a t}\frac{dL_{-a}}{dt}\right]=\frac{2}{2j-1}\Delta \quad (j=1,\ldots,n). \tag{28}$$

[6] *Ap. J.*, **101**, 328, 1945. This paper will be referred to hereafter as "VII."

It is seen that equation (27) represents a system of homogeneous linear equations. Since the determinant of this system clearly does not vanish, we must have

$$e^{-k_a t}\frac{dL_a}{dt}+e^{+k_a t}\frac{dL_{-a}}{dt}=0 \qquad (a=1,\ldots,n). \tag{29}$$

Equation (28) now reduces to

$$\sum_{a=1}^{n} D_{2j-1,\,a}\, e^{-k_a t}\frac{dL_a}{dt}=\frac{1}{2j-1}\Delta \quad (j=1,\ldots,n). \tag{30}$$

Expanding $D_{2j-1,\,a}$ by means of the recursion formula (23), we have

$$\left.\begin{aligned} &2\sum_{a=1}^{n}\left\{\frac{-D_0}{2k_a^{2j-1}}+\frac{1}{k_a^{2j-1}}+\frac{1}{3k_a^{2j-3}}+\ldots+\frac{1}{(2j-1)\,k_a}\right\}\\ &\qquad\qquad\times X_a=\frac{1}{2j-1}\Delta \qquad (j=1,\ldots,n),\end{aligned}\right\} \tag{31}$$

where, for the sake of brevity, we have written

$$X_a=e^{-k_a t}\frac{dL_a}{dt}. \tag{32}$$

We can reduce the system of equations (31) to the simpler one[7]

$$\sum_{a=1}^{n}\frac{1}{k_a^{2j-1}}X_a=\frac{1}{2-D_0}U_j \qquad (j=1,\ldots,n), \tag{33}$$

where the U_j's are defined as follows:

$$\left.\begin{aligned} U_1&=\Delta\,,\\ U_2&=\tfrac{1}{3}U_1-\frac{2}{3(2-D_0)}U_1\,,\\ U_3&=\tfrac{1}{5}U_1-\frac{2}{3(2-D_0)}U_2-\frac{2}{5(2-D_0)}U_1\,,\\ &\ldots\ldots\ldots\ldots\ldots\ldots,\\ U_n&=\frac{1}{2n-1}U_1-\frac{2}{3(2-D_0)}U_{n-1}-\frac{2}{5(2-D_0)}U_{n-2}-\ldots\\ &\qquad-\frac{2}{(2n-1)(2-D_0)}U_1\,.\end{aligned}\right\} \tag{34}$$

The inverse[8] of the linear transformation which appears on the left-hand side in the system of equations (33) is known. The solution for X_a can, accordingly, be found. We have

$$\left.\begin{aligned} X_a=\left\{\begin{matrix} e^{-k_a t}\dfrac{dL_a}{dt}\\[2ex] -e^{+k_a t}\dfrac{dL_{-a}}{dt}\end{matrix}\right\}=\frac{1}{2-D_0}\frac{k_a^{2n-1}}{\prod_{j\neq a}(k_a^2-k_j^2)}\sum_{\lambda=0}^{n-1}S_{\lambda,\,a}U_{\lambda+1}&\\ (a=1,\ldots,n)\,,&\end{aligned}\right\} \tag{35}$$

[7] Cf. *ibid.*, p. 337.

[8] Cf. *ibid.*, pp. 339–341.

where $S_{\lambda,\alpha}$ are the n independent symmetric functions in the $(n-1)$ variables k_μ^2 ($\mu = 1, \ldots, r-1, r+1, \ldots, n$), defined as in equation (79) of paper VII.

It is convenient to re-write equation (35) in the form

$$e^{-k_\alpha t}\frac{dL_\alpha}{dt} = -e^{+k_\alpha t}\frac{dL_{-\alpha}}{dt} = (1-\lambda_0)\,Q_\alpha\Delta \quad (\alpha = 1, \ldots, n)\,, \tag{36}$$

where the Q_α's are defined by

$$Q_\alpha = -\frac{1}{2\lambda_0\Delta}\,\frac{k_\alpha^{2n-1}}{\prod_{j\neq\alpha}(k_\alpha^2 - k_j^2)}\sum_{\lambda=0}^{n-1} S_{\lambda,\alpha}U_{\lambda+1} \quad (\alpha = 1, \ldots, n)\,. \tag{37}$$

Substituting for Δ from equation (19) in equation (36), we obtain

$$\frac{dL_\alpha}{dt} = Q_\alpha\left[\delta\lambda\sum_{\beta=1}^{n} L_\beta^{(0)}e^{-k_\beta t} - (1-\lambda_0)\,b\int_0^t \delta\lambda\, dt\right]e^{+k_\alpha t} \tag{38}$$

and

$$\frac{dL_{-\alpha}}{dt} = -Q_\alpha\left[\delta\lambda\sum_{\beta=1}^{n} L_\beta^{(0)}e^{-k_\beta t} - (1-\lambda_0)\,b\int_0^t \delta\lambda\, dt\right]e^{-k_\alpha t}\,, \tag{39}$$

from which the solutions for L_α and $L_{-\alpha}$ readily follow. We have

$$\left.\begin{aligned} L_\alpha(t) = Q_\alpha\Big[&\int_0^t \delta\lambda\left(\sum_{\beta=1}^{n} L_\beta^{(0)}e^{-k_\beta t}\right)e^{+k_\alpha t}dt - \\ &(1-\lambda_0)\,b\int_0^t e^{+k_\alpha t}\left(\int_0^t \delta\lambda\, dt\right)dt\Big] + L_\alpha^{(0)} - \gamma_\alpha \quad (\alpha = 1, \ldots, n)\,, \end{aligned}\right\} \tag{40}$$

where γ_α ($\alpha = 1, \ldots, n$) are n constants of integration; and

$$\left.\begin{aligned} L_{-\alpha}(t) = Q_\alpha\Big[&\int_t^\infty \delta\lambda\left(\sum_{\beta=1}^{n} L_\beta^{(0)}e^{-k_\beta t}\right)e^{-k_\alpha t}dt - (1-\lambda_0)\,b\int_t^\infty e^{-k_\alpha t} \\ &\times\left(\int_0^t \delta\lambda\, dt\right)dt\Big] \quad (\alpha = 1, \ldots, n)\,. \end{aligned}\right\} \tag{41}$$

It will be noticed that in integrating equation (39) in the form (41) the constants of integration have been so chosen as to satisfy the boundary condition that none of the quantities increase exponentially as $\tau \to \infty$. At $t = 0$ the equations (40) and (41) become

$$L_\alpha(0) = L_\alpha^{(0)} - \gamma_\alpha \quad (\alpha = 1, \ldots, n) \tag{42}$$

and

$$\left.\begin{aligned} L_{-\alpha}(0) = Q_\alpha\Big[&\sum_{\beta=1}^{n}\left(\frac{L_\beta^{(0)}}{k_\alpha + k_\beta}\int_0^\infty \delta\lambda\, e^{-(k_\alpha+k_\beta)t}d\,(k_\alpha + k_\beta)\,t\right) \\ &-\frac{(1-\lambda_0)\,b}{k_\alpha^2}\int_0^\infty \delta\lambda\, e^{-k_\alpha t}d\,(k_\alpha t)\Big] \quad (\alpha = 1, \ldots, n)\,. \end{aligned}\right\} \tag{43}$$

The latter equation can be written in the form

$$L_{-a}(0) = Q_a \left[\sum_{\beta=1}^{n} \frac{L_\beta^{(0)}}{k_a + k_\beta} \overline{\delta\lambda}_{k_a+k_\beta} - \frac{(1-\lambda_0)\, b}{k_a^2} \overline{\delta\lambda}_{k_a} \right] \qquad (a = 1, \ldots, n)\,, \quad (44)$$

where we have introduced the quantities

$$\overline{\delta\lambda}_{k_a+k_\beta} = \int_0^\infty \delta\lambda\, e^{-(k_a+k_\beta)t} d\,(k_a + k_\beta)\, t \qquad (45)$$

and

$$\overline{\delta\lambda}_{k_a} = \int_0^\infty \delta\lambda\, e^{-k_a t} d\,(k_a t)\,. \qquad (46)$$

Equations (45) and (46) represent certain averages over the values of $\delta\lambda$ through the atmosphere, weighted according to the functions $\exp[-(k_a + k_\beta)t]$ and $\exp[-k_a t]$, respectively.

The constants of integration γ_a ($a = 1, \ldots, n$) which occur in the solution for $L_a(0)$ (cf. eq. [42]) are determined from the further boundary condition that there is no incident radiation on the surface $t = 0$:

$$I_{-i} = 0 \text{ at } t = 0 \text{ for } i = 1, \ldots, n\,. \qquad (47)$$

This condition requires that (cf. eq. [15])

$$\sum_{a=1}^{n} \left[\frac{L_a^{(0)} - \gamma_a}{1 - \mu_i k_a} + \frac{L_{-a}(0)}{1 + \mu_i k_a} \right] + a_{\nu_0} = \mu_i b \qquad (i = 1, \ldots, n)\,, \quad (48)$$

or, since $L_a^{(0)}$ satisfies the relation (cf. eq. [8])

$$\sum_{a=1}^{n} \frac{L_a^{(0)}}{1 - \mu_i k_a} + a_{\nu_0} = \mu_i b \qquad (i = 1, \ldots, n)\,, \quad (49)$$

the equations for γ_a are

$$\sum_{a=1}^{n} \frac{\gamma_a}{1 - \mu_i k_a} = \sum_{a=1}^{n} \frac{L_{-a}(0)}{1 + \mu_i k_a} \qquad (i = 1, \ldots, n)\,. \quad (50)$$

In terms of the matrices

$$\boldsymbol{G} = (G_{ij}) = \frac{1}{1 - \mu_i k_j} \quad \text{and} \quad \boldsymbol{H} = (H_{ij}) = \frac{1}{1 + \mu_i k_j} \qquad (51)$$

and the vectors

$$\boldsymbol{\gamma} = (\gamma_a) \quad \text{and} \quad \boldsymbol{L}_- = (L_{-a})\,, \qquad (52)$$

the formal solution to the foregoing system of equations is

$$\boldsymbol{\gamma} = \boldsymbol{G}^{-1}\boldsymbol{H}\boldsymbol{L}_-\,, \qquad (53)$$

where the inverse of the matrix $\boldsymbol{G}$ is known explicitly.[9]

This completes the solution to the variational equations (17) in the nth approximation.

The expression for the emergent intensity becomes

$$I_i(0) = \sum_{a=1}^{n} \left[\frac{L_a^{(0)} - \gamma_a}{1 + \mu_i k_a} + \frac{L_{-a}(0)}{1 - \mu_i k_a} \right] + \mu_i b + a_{\nu_0} \qquad (i = 1, \ldots, n)\,. \quad (54)$$

[9] Cf. VI, pp. 320–322.

The foregoing equation can be written alternatively in the form

$$I_i(0) = \sum_{a=1}^{n} \left[c_{i,a} \frac{L_a^{(0)}}{1-\lambda_0} + d_{i,a} \frac{L_{-a}(0)}{1-\lambda_0} \right] + \mu_i b + a_{\nu_0} \qquad (i = 1, \ldots, n) , \tag{55}$$

where

$$c_{i,a} = \frac{1-\lambda_0}{1+\mu_i k_a} \quad \text{and} \quad d_{i,a} = \frac{1-\lambda_0}{1-\mu_i k_a} \left[1 - \frac{1-\mu_i k_a}{1+\mu_i k_a} \frac{\gamma_a}{L_{-a}(0)} \right] . \tag{56}$$

The emergent intensity in the continuous spectrum is given by

$$I_i(0, \text{cont.}) = a_{\nu_0} + b_{\nu_0} \mu_i \qquad (i = 1, \ldots, n) , \tag{57}$$

obtained by putting $\lambda = \lambda_0 = 1$, in equation (55). Finally, the residual intensity in the line

$$r_i = \frac{I_i(0)}{I_i(0, \text{cont.})} \qquad (i = 1, \ldots, n) \tag{58}$$

can be written in the form

$$(\tfrac{8}{3} x + \mu_i) \, r_i = \sum_{a=1}^{n} [c_{i,a} \mathfrak{L}'_a + d_{i,a} \mathfrak{L}'_{-a}(0)] + \lambda_0 \mu_i + \tfrac{8}{3} x \qquad (i = 1, \ldots, n) , \tag{59}$$

where

$$\mathfrak{L}'_a = \frac{\lambda_0 L_a^{(0)}}{b(1-\lambda_0)} \tag{60}$$

and

$$\mathfrak{L}'_{-a}(0) = Q_a \left[\sum_{\beta=1}^{n} \frac{\mathfrak{L}'_\beta}{k_a + k_\beta} \overline{\delta\lambda}_{k_a+k_\beta} - \frac{\lambda_0}{k_a^2} \overline{\delta\lambda}_{k_a} \right] . \tag{61}$$

Equation (59) enables us, then, to compute in the nth approximation theoretical contours for n different points along a radius of the stellar disc, the emergent radiation being assumed to be divided into n streams symmetrically located along each radius.

It may be recalled that in the integrals over $\delta\lambda$ we can put

$$t_\nu = \frac{n_{\nu_0}}{\lambda_{0_\nu}} \tau . \tag{62}$$

We have

$$\overline{\delta\lambda}_{k_a+k_\beta} = \int_0^\infty \delta\lambda \, e^{-(k_a+k_\beta) n_{\nu_0} \tau / \lambda_0} d \left[(k_a + k_\beta) \frac{n_{\nu_0}}{\lambda_0} \tau \right] \tag{63}$$

and

$$\overline{\delta\lambda}_{k_a} = \int_0^\infty \delta\lambda \, e^{-k_a n_{\nu_0} \tau / \lambda_0} d \left[k_a \frac{n_{\nu_0}}{\lambda_0} \tau \right] . \tag{64}$$

The difference in the weighting functions for $\overline{\delta\lambda}_{k_a}$ and $\overline{\delta\lambda}_{k_a+k_\beta}$ is related to the difference between the two mechanisms operative in the formation of absorption lines. Absorption lines are produced by continuous absorption, depending upon the existence of a temperature gradient, and by scattering, causing a deviation from Kirchhoff's law. The integral $\overline{\delta\lambda}_{k_a}$, to which relatively deeper layers contribute, is associated with the first of these effects; the integral $\overline{\delta\lambda}_{k_a+k_\beta}$, to which relatively higher layers contribute, is associated with the second.

3. *The solutions in the first and second approximations: (a) The first approximation.*—The discussion of the approximation $n = 1$ is of particular interest, since the solution obtained can be compared directly with Strömgren's[10] work. For this case we have $\mu_1 = 1/\sqrt{3}$ and $k_1 = \sqrt{(3\lambda_0)}$. The final expression for the emergent intensity becomes

$$\left.\begin{aligned} I_1(0) = \frac{2a_{\nu_0}\sqrt{\lambda_0} + \frac{2}{\sqrt{3}}\,b}{1+\sqrt{\lambda_0}} &+ \frac{\frac{a_{\nu_0}}{\sqrt{\lambda_0}} - \frac{b}{\sqrt{3\lambda_0}}}{(1+\sqrt{\lambda_0})^2}\int_0^\infty \delta\lambda\, e^{-2\sqrt{3\lambda_0}\,t}\, d\,(2\sqrt{3\lambda_0}\,t) \\ &+ \frac{\frac{2b}{\sqrt{3\lambda_0}}}{1+\sqrt{\lambda_0}}\int_0^\infty \delta\lambda\, e^{-\sqrt{3\lambda_0}\,t}\, d\,(\sqrt{3\lambda_0}\,t)\,. \end{aligned}\right\} \qquad (65)$$

Performing the transformations carried out by Strömgren,[11] we can reduce equation (65) to the form

$$I_1(0) = \frac{2a_{\nu_0}\overline{\sqrt{\lambda}} + \frac{2}{\sqrt{3}}\,b_{\nu_0}\bar{\lambda}}{1+\overline{\sqrt{\lambda}}}\,, \qquad (66)$$

where

$$\bar{\lambda} = \int_0^\infty \lambda\, e^{-\sqrt{3\lambda_0}\,t}\, d\,(\sqrt{3\lambda_0}\,t) \qquad (67)$$

and

$$\overline{\sqrt{\lambda}} = \int_0^\infty \sqrt{\lambda}\, e^{-2\sqrt{3\lambda_0}\,t}\, d\,(2\sqrt{3\lambda_0}\,t)\,. \qquad (68)$$

Using the relation (cf. IV, eq. [7])

$$F(t=0) = \frac{2}{\sqrt{3}}\, I_1(t=0)\,, \qquad (69)$$

we have

$$F(t=0) = \frac{\frac{4}{3}\,a_{\nu_0}\sqrt{3}\,\overline{\sqrt{\lambda}} + \frac{4}{3}\,b_{\nu_0}\bar{\lambda}}{1+\overline{\sqrt{\lambda}}}\,. \qquad (70)$$

This is equivalent to Strömgren's result (cf. *op. cit.*, eq. [87]) except for the omission of the factor $2/\sqrt{3}$ before the term $\overline{\sqrt{\lambda}}$ in the denominator. This difference results from the fact that the Eddington approximation, $J = \frac{1}{2}F$ at the surface, is not made in the present work.

b) The second approximation.—In this approximation $\mu_1 = 0.34$ and $\mu_2 = 0.86$, and theoretical contours can accordingly be computed for these points. The final expressions for the emergent intensity are

$$\left.\begin{aligned} I_1(0) = \frac{L_1^{(0)}}{1+\mu_1 k_1} + \frac{L_2^{(0)}}{1+\mu_1 k_2} &+ \mu_1 b + a_{\nu_0} \\ &+ \frac{4\mu_1 k_1(k_1+k_2)(\mu_1+\mu_2)}{(1+\mu_1 k_1)^2(1+\mu_2 k_1)(1+\mu_1 k_2)(1-\mu_1 k_1)}\,L_{-1}(0) \\ &+ \frac{4\mu_1 k_2(k_1+k_2)(\mu_1+\mu_2)}{(1+\mu_1 k_2)^2(1+\mu_2 k_2)(1+\mu_1 k_1)(1-\mu_1 k_2)}\,L_{-2}(0) \end{aligned}\right\} \qquad (71)$$

[10] *Op. cit.* (see n. 1, above).

[11] *Ibid.*, pp. 14–16.

and

$$\left.\begin{aligned} I_2(0) &= \frac{L_1^{(0)}}{1+\mu_2 k_1} + \frac{L_2^{(0)}}{1+\mu_2 k_2} + \mu_2 b + a_{\nu_0} \\ &\quad + \frac{4\mu_2 k_1 (k_1+k_2)(\mu_1+\mu_2)}{(1+\mu_2 k_1)^2 (1+\mu_1 k_1)(1+\mu_2 k_2)(1-\mu_2 k_1)} L_{-1}(0) \\ &\quad + \frac{4\mu_2 k_2 (k_1+k_2)(\mu_1+\mu_2)}{(1+\mu_2 k_2)^2 (1+\mu_1 k_2)(1+\mu_2 k_1)(1-\mu_2 k_2)} L_{-2}(0)\,. \end{aligned}\right\} \quad (72)$$

The quantities $L_1^{(0)}$ and $L_2^{(0)}$ are given by (cf. VI, eq. [11])

$$L_1^{(0)} = \frac{(1-\mu_1 k_1)(1-\mu_2 k_1)}{k_1 - k_2}\, b \left[1 - k_2(\mu_1+\mu_2) + \frac{a_{\nu_0}}{b} k_2\right] \quad (73)$$

and

$$L_2^{(0)} = \frac{(1-\mu_2 k_2)(1-\mu_1 k_2)}{k_2 - k_1}\, b \left[1 - k_1(\mu_1+\mu_2) + \frac{a_{\nu_0}}{b} k_1\right]. \quad (74)$$

The solutions for $L_{-1}(0)$ and $L_{-2}(0)$ are

$$\left.\begin{aligned} L_{-1}(0) &= \frac{-k_1^3}{2\lambda_0 (k_1^2 - k_2^2)} \left(1 - \frac{k_2^2}{3\lambda_0}\right) \\ &\quad \times \left[\frac{L_1^{(0)}}{2k_1} \overline{\delta\lambda_{2k_1}} + \frac{L_2^{(0)}}{k_1+k_2} \overline{\delta\lambda_{k_1+k_2}} - (1-\lambda_0)\frac{b}{k_1^2} \overline{\delta\lambda_{k_1}}\right] \end{aligned}\right\} \quad (75)$$

and

$$\left.\begin{aligned} L_{-2}(0) &= \frac{-k_2^3}{2\lambda_0 (k_2^2 - k_1^2)} \left(1 - \frac{k_1^2}{3\lambda_0}\right) \\ &\quad \times \left[\frac{L_1^{(0)}}{k_1+k_2} \overline{\delta\lambda_{k_1+k_2}} + \frac{L_2^{(0)}}{2k_2} \overline{\delta\lambda_{2k_2}} - (1-\lambda_0)\frac{b}{k_2^2} \overline{\delta\lambda_{k_2}}\right]. \end{aligned}\right\} \quad (76)$$

4. *The solution in the third approximation in numerical form.*—In this approximation the points of the Gaussian division are $\mu_1 = 0.24$, $\mu_2 = 0.66$, and $\mu_3 = 0.93$. Accordingly, contours for these points, representing the fractional parts of the radius, 0.97, 0.75, and 0.36, respectively, from the center of the disc can be obtained. The expressions for the emergent intensities at these points become

$$\left.\begin{aligned} I_1(0) &= \frac{L_1^{(0)}}{1+\mu_1 k_1} + \frac{L_2^{(0)}}{1+\mu_1 k_2} + \frac{L_3^{(0)}}{1+\mu_1 k_3} + \mu_1 b + a_{\nu_0} \\ &+ \frac{4\mu_1 k_1 (k_1+k_2)(k_3+k_1)(\mu_1+\mu_2)(\mu_3+\mu_1)}{(1+\mu_1 k_1)^2 (1+\mu_2 k_1)(1+\mu_3 k_1)(1+\mu_1 k_2)(1+\mu_1 k_3)(1-\mu_1 k_1)} L_{-1}(0) \\ &+ \frac{4\mu_1 k_2 (k_1+k_2)(k_2+k_3)(\mu_1+\mu_2)(\mu_3+\mu_1)}{(1+\mu_1 k_2)^2 (1+\mu_2 k_2)(1+\mu_3 k_2)(1+\mu_1 k_1)(1+\mu_1 k_3)(1-\mu_1 k_2)} L_{-2}(0) \\ &+ \frac{4\mu_1 k_3 (k_3+k_1)(k_2+k_3)(\mu_1+\mu_2)(\mu_3+\mu_1)}{(1+\mu_1 k_3)^2 (1+\mu_2 k_3)(1+\mu_3 k_3)(1+\mu_1 k_1)(1+\mu_1 k_2)(1-\mu_1 k_3)} L_{-3}(0)\,, \end{aligned}\right\} \quad (77)$$

$$\left.\begin{aligned} I_2(0) &= \frac{L_1^{(0)}}{1+\mu_2 k_1}+\frac{L_2^{(0)}}{1+\mu_2 k_2}+\frac{L_3^{(0)}}{1+\mu_2 k_3}+\mu_2 b+a_{\nu_0} \\ &+\frac{4\mu_2 k_1 (k_1+k_2)\,(k_3+k_1)\,(\mu_1+\mu_2)\,(\mu_2+\mu_3)}{(1+\mu_1 k_1)\,(1+\mu_2 k_1)^2\,(1+\mu_3 k_1)\,(1+\mu_2 k_2)\,(1+\mu_2 k_3)\,(1-\mu_2 k_1)}L_{-1}(0) \\ &+\frac{4\mu_2 k_2 (k_1+k_2)\,(k_2+k_3)\,(\mu_1+\mu_2)\,(\mu_2+\mu_3)}{(1+\mu_1 k_2)\,(1+\mu_2 k_2)^2\,(1+\mu_3 k_2)\,(1+\mu_2 k_1)\,(1+\mu_2 k_3)\,(1-\mu_2 k_2)}L_{-2}(0) \\ &+\frac{4\mu_2 k_3 (k_3+k_1)\,(k_2+k_3)\,(\mu_1+\mu_2)\,(\mu_2+\mu_3)}{(1+\mu_1 k_3)\,(1+\mu_2 k_3)^2\,(1+\mu_3 k_3)\,(1+\mu_2 k_1)\,(1+\mu_2 k_2)\,(1-\mu_2 k_3)}L_{-3}(0)\,, \end{aligned}\right\} \quad (78)$$

and

$$\left.\begin{aligned} I_3(0) &= \frac{L_1^{(0)}}{1+\mu_3 k_1}+\frac{L_2^{(0)}}{1+\mu_3 k_2}+\frac{L_3^{(0)}}{1+\mu_3 k_3}+\mu_3 b+a_{\nu_0} \\ &+\frac{4\mu_3 k_1 (k_1+k_2)\,(k_3+k_1)\,(\mu_3+\mu_1)\,(\mu_2+\mu_3)}{(1+\mu_1 k_1)\,(1+\mu_2 k_1)\,(1+\mu_3 k_1)^2\,(1+\mu_3 k_2)\,(1+\mu_3 k_3)\,(1-\mu_3 k_1)}L_{-1}(0) \\ &+\frac{4\mu_3 k_2 (k_1+k_2)\,(k_2+k_3)\,(\mu_3+\mu_1)\,(\mu_2+\mu_3)}{(1+\mu_1 k_2)\,(1+\mu_2 k_2)\,(1+\mu_3 k_2)^2\,(1+\mu_3 k_1)\,(1+\mu_3 k_3)\,(1-\mu_3 k_2)}L_{-2}(0) \\ &+\frac{4\mu_3 k_3 (k_3+k_1)\,(k_2+k_3)\,(\mu_3+\mu_1)\,(\mu_2+\mu_3)}{(1+\mu_1 k_3)\,(1+\mu_2 k_3)\,(1+\mu_3 k_3)^2\,(1+\mu_3 k_1)\,(1+\mu_3 k_2)\,(1-\mu_3 k_3)}L_{-3}(0)\,. \end{aligned}\right\} \quad (79)$$

The equations for the constants $L_a^{(0)}$ are (cf. VI, eq. [9])

$$\left.\begin{aligned} L_1^{(0)} = \frac{(1-\mu_1 k_1)\,(1-\mu_2 k_1)\,(1-\mu_3 k_1)}{(k_1-k_2)\,(k_3-k_1)}\,b\Big[\,(k_2+k_3)-k_2 k_3(\mu_1+\mu_2+\mu_3) \\ +\frac{a_{\nu_0}}{b}k_2 k_3\Big]\,, \end{aligned}\right\} \quad (80)$$

$$\left.\begin{aligned} L_2^{(0)} = \frac{(1-\mu_1 k_2)\,(1-\mu_2 k_2)\,(1-\mu_3 k_2)}{(k_2-k_3)\,(k_1-k_2)}\,b\Big[\,(k_3+k_1)-k_3 k_1(\mu_1+\mu_2+\mu_3) \\ +\frac{a_{\nu_0}}{b}k_3 k_1\Big]\,, \end{aligned}\right\} \quad (81)$$

and

$$\left.\begin{aligned} L_3^{(0)} = \frac{(1-\mu_1 k_3)\,(1-\mu_2 k_3)\,(1-\mu_3 k_3)}{(k_3-k_1)\,(k_2-k_3)}\,b\Big[\,(k_1+k_2)-k_1 k_2(\mu_1+\mu_2+\mu_3) \\ +\frac{a_{\nu_0}}{b}k_1 k_2\Big]\,. \end{aligned}\right\} \quad (82)$$

The quantities $L_{-a}(0)$ are given by

$$L_{-1}(0) = Q_1\Big[\frac{L_1^{(0)}}{2k_1}\,\overline{\delta\lambda}_{2k_1}+\frac{L_2^{(0)}}{k_1+k_2}\overline{\delta\lambda}_{k_1+k_2}+\frac{L_3^{(0)}}{k_1+k_3}\,\overline{\delta\lambda}_{k_1+k_3}-(1-\lambda_0)\,\frac{b}{k_1^2}\,\overline{\delta\lambda}_{k_1}\Big]\,, \quad (83)$$

$$L_{-2}(0) = Q_2\Big[\frac{L_1^{(0)}}{k_1+k_2}\overline{\delta\lambda}_{k_1+k_2}+\frac{L_2^{(0)}}{2k_2}\,\overline{\delta\lambda}_{2k_2}+\frac{L_3^{(0)}}{k_2+k_3}\,\overline{\lambda\delta}_{k_2+k_3}-(1-\lambda_0)\,\frac{b}{k_2^2}\,\overline{\delta\lambda}_{k_2}\Big]\,, \quad (84)$$

and

$$L_{-3}(0) = Q_3\Big[\frac{L_1^{(0)}}{k_1+k_3}\,\overline{\delta\lambda}_{k_1+k_3}+\frac{L_2^{(0)}}{k_2+k_3}\,\overline{\delta\lambda}_{k_2+k_3}+\frac{L_3^{(0)}}{2k_3}\,\overline{\delta\lambda}_{2k_3}-(1-\lambda_0)\,\frac{b}{k_3^2}\,\overline{\delta\lambda}_{k_3}\Big]\,, \quad (85)$$

where

$$Q_1 = \frac{k_1^5}{2\lambda_0 (k_1^2 - k_2^2)\,(k_3^2 - k_1^2)}\left[1 - \frac{k_2^2 + k_3^2}{3\lambda_0} + \left(\frac{1}{5\lambda_0} + \frac{1-\lambda_0}{9\lambda_0^2}\right)k_2^2 k_3^2\right], \tag{86}$$

$$Q_2 = \frac{k_2^5}{2\lambda_0 (k_1^2 - k_2^2)\,(k_2^2 - k_3^3)}\left[1 - \frac{k_1^2 + k_3^2}{3\lambda_0} + \left(\frac{1}{5\lambda_0} + \frac{1-\lambda_0}{9\lambda_0^2}\right)k_1^2 k_3^2\right], \tag{87}$$

and

$$Q_3 = \frac{k_3^5}{2\lambda_0 (k_3^2 - k_1^2)\,(k_2^2 - k_3^2)}\left[1 - \frac{k_1^2 + k_2^2}{3\lambda_0} + \left(\frac{1}{5\lambda_0} + \frac{1-\lambda_0}{9\lambda_0^2}\right)k_1^2 k_2^2\right]. \tag{88}$$

The expressions for the residual intensities have the form

$$\left.\begin{aligned}\left(\tfrac{8}{3}x + \mu_i\right) r_i = {} & c_{i,1}\mathfrak{L}'_1 + c_{i,2}\mathfrak{L}'_2 + c_{i,3}\mathfrak{L}'_3 + d_{i,1}\mathfrak{L}'_{-1}(0)\\ & + d_{i,2}\mathfrak{L}'_{-2}(0) + d_{i,3}\mathfrak{L}'_{-3}(0) + \lambda_0\mu_i + \tfrac{8}{3}x \quad (i = 1, 2, 3)\end{aligned}\right\} \tag{89}$$

where $\mathfrak{L}'_a = \mathfrak{L}_a/(1 - \lambda_0)$. It is the quantities $\mathfrak{L}_a$ which are tabulated for different values of λ_0 and x in paper VI (Table 1). The coefficients $c_{i,a}$ and $d_{i,a}$ are equivalent to the corresponding coefficients of $L_a^{(0)}$ and $L_{-a}(0)$, respectively, in the expressions (77), (78),

TABLE 1

VALUES OF $c_{1,a}$ AND $d_{1,a}$

λ_0	$c_{1,1}$	$c_{1,2}$	$c_{1,3}$	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$
0	0.5668	1	0.7738	3.877	0	0.8268
0.10	.5037	0.7997	.6944	4.022	0.6235	.9639
0.20	.4421	0.6840	.6150	4.071	.7078	.9320
0.30	.3818	0.5844	.5356	4.105	.7049	.8652
0.40	.3231	0.4930	.4566	4.135	.6564	.7782
0.50	.2658	0.4064	.3782	4.160	.5800	.6760
0.60	.2100	0.3228	.3006	4.184	.4848	.5608
0.70	.1555	0.2409	.2241	4.208	.3759	.4344
0.80	.1024	0.1600	.1485	4.230	.2576	.2980
0.90	0.0506	0.0798	0.0738	4.253	0.1318	0.1529
1.00	0	0	0	4.274	0	0

and (79) for the emergent intensities, multiplied by the factor $(1 - \lambda_0)$. Values of these coefficients $c_{i,a}$ and $d_{i,a}$ for various values of λ_0 are tabulated in Tables 1, 2, and 3. The quantities Q_a have also been computed for different values of λ_0, the results being given in Table 4. This completes the formal solution to our problem. We now turn to applications of this theory.

5. *The variation of η through the solar atmosphere.*—For each selected point on the contour of the absorption line, the basis of the calculation is the function $\eta(\tau)$, which can be evaluated numerically in the manner of Strömgren.[12] For transitions arising from a ground state, the line absorption coefficient which results as a consequence of radiation damping, collision broadening, and Doppler broadening is given by

$$\sigma = k_0 N_1 H(a, v)\,, \tag{90}$$

[12] *Festschrift für E. Strömgren*, Kopenhagen: Einar Munksgaard, 1940.

TABLE 2

VALUES OF $c_{2,\alpha}$ AND $d_{2,\alpha}$

λ_0	$c_{2,1}$	$c_{2,2}$	$c_{2,3}$	$d_{2,1}$	$d_{2,2}$	$d_{2,3}$
0	0.3207	1	0.5524	−1.2454	0	4.694
0.10	.2830	0.6680	.4944	0.9162	1.0530	4.764
0.20	.2466	0.5443	.4362	0.7400	1.3168	4.654
0.30	.2115	0.4521	.3783	0.5977	1.4287	4.575
0.40	.1778	0.3746	.3208	0.4768	1.4292	4.556
0.50	.1453	0.3053	.2642	0.3720	1.3335	4.604
0.60	.1140	0.2405	.2088	0.2798	1.1596	4.715
0.70	.0839	0.1785	.1547	0.1980	0.9249	4.876
0.80	.0549	0.1181	.1020	0.1250	0.6456	5.074
0.90	0.0270	0.0588	0.0505	−0.0593	0.3344	5.298
1.00	0	0	0	0	0	5.544

TABLE 3

VALUES OF $c_{3,\alpha}$ AND $d_{3,\alpha}$

λ_0	$c_{3,1}$	$c_{3,2}$	$c_{3,3}$	$d_{3,1}$	$d_{3,2}$	$d_{3,3}$
0	0.2508	1	0.4667	−0.8498	0	−7.5841
0.10	.2210	0.6041	.4172	.5699	1.473	5.9517
0.20	.1922	0.4812	.3677	.4530	2.203	4.6880
0.30	.1645	0.3948	.3183	.3635	2.982	3.6022
0.40	.1380	0.3247	.2694	.2892	3.887	2.6970
0.50	.1126	0.2632	.2214	.2254	4.938	1.9690
0.60	.0882	0.2067	.1746	.1696	6.121	1.3936
0.70	.0648	0.1531	.1291	.1201	7.414	0.9372
0.80	.0423	0.1012	.0849	.0759	8.783	0.5680
0.90	0.0208	0.0502	0.0420	−0.0361	10.214	−0.2617
1.00	0	0	0	0	11.674	0

TABLE 4

VALUES OF Q_α

λ_0	Q_1	Q_2	Q_3
0.10	−0.9519	−2.4004	−0.1773
0.20	0.9750	1.4449	.2254
0.30	0.9914	0.9679	.2769
0.40	1.0014	0.6617	.3206
0.50	1.0061	0.4539	.3468
0.60	1.0065	0.3142	.3525
0.70	1.0034	0.2219	.3420
0.80	0.9977	0.1608	.3221
0.90	0.9898	0.1199	.2979
1.00	−0.9805	−0.0919	−0.2728

where k_0, the fictitious atomic absorption coefficient at the center of the line for vanishing damping, depends on the Einstein coefficient A_{k1}, corresponding to the transition in question, and where $H(a, v)$ is a certain definite integral involving a and v as two parameters.[13] The number of absorbing atoms N_1 per gram at a given optical depth τ can be determined with the aid of Saha's equation in terms of the temperature T and the electron pressure p_e prevailing at τ and certain relative abundances, including the ratio of hydrogen atoms to metal atoms (A in Strömgren's notation) and the ratio of the particular absorbing atoms to the metal atoms. The integral defining $H(a, v)$ has been tabulated by Hjerting[13] for various values of v, representing the deviation from the center of the line in units of the Doppler width, and

$$a=\frac{\Delta\nu_N+\Delta\nu_c}{\Delta\nu_D}. \tag{91}$$

In equation (91) $\Delta\nu_N$ is the coefficient of radiation damping, $\Delta\nu_c$, the coefficient of collision broadening, and $\Delta\nu_D$, the Doppler width. The transition probability A_{k1} determines $\Delta\nu_N$. In evaluating $\Delta\nu_c$ in the solar atmosphere, we shall follow Strömgren and consider only the effect of collisions with neutral hydrogen. Moreover, we shall put the partial pressure of the neutral hydrogen atoms equal to the total pressure P. Under these conditions the number of broadening collisions is proportional to $T^{-0.7}P$ and is related, in addition, to $\overline{R_k^2}$, the mean square radius corresponding to the upper stationary state of the broadened spectral line.

Thus, the evaluation of

$$\eta=\frac{\sigma}{\kappa}=\frac{\sigma}{n\bar{\kappa}} \tag{92}$$

requires the adoption of a model solar atmosphere, giving the temperature, the total pressure, and the electron pressure as functions of τ, computed with a certain mean absorption coefficient for various assumed values of the hydrogen-metal ratio and the relative abundance of the absorbing atoms. The present investigation will be limited to those lines for which reasonably accurate transition probabilities and the physical data for the calculation of the collision broadening are known.

From the numerical values of $\lambda = 1/1 + \eta$ so derived, appropriate values of λ_0, about which the relative variations of λ in the relevant range of τ are small, are chosen. It should be mentioned here that the final results are not particularly dependent on the choice of the precise value of λ_0. For two values of λ_0 which do not differ very greatly lead to the same results. This agreement is, in fact, guaranteed by the circumstance that our theory is correct to quantities of the first order. The proper averages of $\delta\lambda$ (cf. eqs. [63] and [64]) can then be evaluated by numerical quadratures. In carrying out these averages it was found convenient and sufficiently accurate to use a three-point integration formula,[14] according to which we set the integral

$$\overline{\delta\lambda}_{f(k)}=\int_0^\infty \delta\lambda\, e^{-f(k)n\tau/\lambda_0}d\left[f(k)\frac{n}{\lambda_0}\tau\right]=\sum_{i=1}^{3}q_i\delta\lambda(x_i)\,, \tag{93}$$

where $x_i = [f(k)n\tau/\lambda_0]$ $(i = 1, 2, 3)$ are the roots of the Laguerre polynomial $L_3(x)$:

$$x_1=0.41577\,,\qquad x_2=2.2943\,,\qquad x_3=6.2899\,. \tag{94}$$

[13] F. Hjerting, *Ap. J.*, **88**, 508, 1938.

[14] A. Reiz, *Arkiv för matematik, astronomi och fysik*, Vol. **29**, Part 4, 1944.

The corresponding "weights" q_i are

$$q_1 = 0.71109\ , \qquad q_2 = 0.27852\ , \qquad q_3 = 0.010389\ . \tag{95}$$

Thus, $\overline{\delta\lambda}_{f(k)}$ is determined as the weighted mean of the values of $\delta\lambda$ at

$$\tau_i = \frac{1}{f(k)} \frac{\lambda_0}{n} x_i \qquad (i = 1, 2, 3)\ . \tag{96}$$

6. *The comparison of theoretical contours with observed contours.*—In the present investigation a model solar atmosphere in radiative equilibrium, computed by Rudkjøbing[15] for a boundary temperature $\theta_0 = 5040/T_0 = 1.041$ and $\log g = 4.44$ and for the hydrogen-metal ratio $\log A = 3.8$, obtained by Strömgren,[12] was used. The mean absorption coefficient $\bar{\kappa}$ corresponds to the Rosseland mean over all frequencies of the sum of the continuous absorption of neutral hydrogen and the negative hydrogen ion, set equal to its maximum value 2.6×10^{-17} cm^2, as derived by Massey and Bates,[16] over the entire spectrum.

Houtgast's[17] observations of strong Fraunhofer lines at seven and eight points along a solar radius provide the most complete basis for the comparison of the theory with observations. Of the lines observed by Houtgast, those selected for application of the present theory of stellar absorption lines include the D lines of sodium, the H and K lines of ionized calcium, and the calcium line λ 4227. Strömgren's[12] relative abundances of sodium and calcium in the sun were adopted; the logarithm of the number of sodium atoms per gram matter is 17.7; of calcium atoms, 18.0, the manner of determination of the latter being more approximate. Values of n, to which the calculation of the residual intensities is sensitive, were obtained from Münch's[18] discussion of the observed emergent intensity distribution at the center of the solar disc and the emergent flux distribution of the continuous spectrum of the sun. For the H and K lines of *Ca* II, n is 0.674 and 0.694, respectively; for the *Ca* I line λ 4227, $n = 0.636$; and for the D_1 and D_2 lines of *Na* I, $n = 0.756$ and 0.755, respectively. For the sodium D lines the transition probability was adopted according to Ladenburg and Thiele,[19] $A_{kl} = 6.8 \times 10^7$, and the mean square radius, needed for the computation of the collision broadening, was derived from wave functions obtained by Fock and Petrashen,[20] $\overline{R_k^2} = 41a_0^2$, a_0 being the atomic unit of length. For the calcium lines, the transition probabilities obtained by Hartree and Hartree[21] were used; further, from their wave functions the mean square radii were determined. We have for the H and K lines, $A_{kl} = 1.66 \times 10^8$ and $\overline{R_k^2} = 23a_0^2$; for λ 4227, $A_{kl} = 1.40 \times 10^8$ and $\overline{R_k^2} = 69a_0^2$.

In the computed variation of η through the atmosphere, it is interesting to note that, for the *Ca* II H and K lines, η remains practically constant in the regions relevant to the formation of the lines, decreasing with τ at greater depths. In the case of the λ 4227 line of *Ca* I, η first increases and then decreases through the atmosphere. In the wings of the sodium D lines, the variation of η with τ is similar to that for λ 4227, but the relative changes are less. At the centers of the D lines, η decreases with τ.

Contours have been computed at three points on the solar disc, at $\vartheta = 21°$, $48°$, and $76°$, according to the third approximation. The theoretical results are compared with the observational material presented by Houtgast.[17] Tables 5–9 give the observed and computed values of the residual intensities at selected points on the contours as functions

[15] *Zs. f. Ap.*, **21**, 254, 1942.

[16] *Ap. J.*, **91**, 202, 1940.

[17] "The Variations in the Profiles of Strong Fraunhofer Lines along a Radius of the Solar Disc" (dissertation), Utrecht, 1942.

[18] *Ap. J.*, **102**, 385, 1945.

[19] *Zs. f. Phys.*, **72**, 697, 1931.

[20] *Phys. Zs. Sowjetunion*, **6**, 368, 1934.

[21] *Proc. R. Soc.*, A, **164**, 167, 1938.

of ϑ. The observed contours at the given angles were obtained by linear interpolation in Houtgast's tables of residual intensities for undisturbed points. The observations for the sodium D lines are the most free from error, the calcium lines, especially λ 4227, being more strongly disturbed by blends. In the present work no proper theory of the central intensities is included. It is to be noted that the center-limb variations are most pronounced in the inner wings of the lines. The predicted and the observed profiles of the K line of *Ca* II, the *Ca* I line λ 4227, and the D_1 line of *Na* I at $\vartheta = 48°$ are shown in Figures 1, 2, and 3, respectively. The agreement between theory and observations is satisfactory.

The quantities c, given by the equation

$$1 - r = \frac{c\,r}{\Delta\lambda^2}, \tag{97}$$

TABLE 5

RESIDUAL INTENSITIES FOR *Ca* II K

$\Delta\lambda$	$\vartheta=21°$		$\vartheta=48°$		$\vartheta=76°$	
	Computed	Observed	Computed	Observed	Computed	Observed
0	0	0.072	0	0.078	0	0.129
1.5	0.088	.150	0.105	.179	0.126	.270
3	.204	.241	.226	.285	.300	.397
5	.374	.395	.402	.421	.514	.531
7	.518	.558	.542	.562	.666	.647
10	.688	.725	.700	.724	.802	.767
15	.831	.849	.832	.848	.899	.868
20	0.895	0.924	0.896	0.920	0.940	0.936
c	47.7	38.8	46.8	38.0	25.9	32.4
Eq. widths	19,050	19,050	18,680	18,360	14,360	15,850

TABLE 6

RESIDUAL INTENSITIES FOR *Ca* II H

$\Delta\lambda$	$\vartheta=21°$		$\vartheta=48°$		$\vartheta=76°$	
	Computed	Observed	Computed	Observed	Computed	Observed
0	0	0.073	0	0.080	0	0.128
1.5	0.134	.193	0.150	.215	0.196	.320
3	.285	.325	.314	.353	.411	.469
5	.500	.500	.528	.523	.647	.616
7	.673	.655	.679	.660	.779	.731
10	.803	.791	.807	.799	.879	.842
15	.899	.891	.900	.894	.944	.928
20	0.941	0.932	0.941	0.936	0.967	0.958
c	24.3	26.0	23.9	24.8	13.5	18.1
Eq. widths	14,280	15,110	13,990	14,685	10,970	10,930

applicable to the wings, were found graphically for the computed contours. The equivalent widths were measured with a planimeter. The contributions to the equivalent widths from the far wings were obtained by the expression

$$\sqrt{c}\left(\frac{\pi}{2} - \tan^{-1}\frac{\Delta\lambda}{\sqrt{c}}\right). \tag{98}$$

The constants c and the equivalent widths of the lines, expressed in mA, are also given in Tables 5–9. The observed values were read from smooth curves drawn through Houtgast's observations across the solar disc for each line. In the ratios of the computed c's for the *Ca* II H and K lines and for the *Na* I D lines, the doublet ratio 1:2 is reflected. In

TABLE 7

RESIDUAL INTENSITIES FOR *Ca* I λ 4227

$\Delta\lambda$	$\vartheta=21°$		$\vartheta=48°$		$\vartheta=76°$	
	Computed	Observed	Computed	Observed	Computed	Observed
0	0.002	0.052	0.002	0.054	0.002	0.071
0.10	.066	.131	.080	.140	.11	.188
0.30	.280	.329	.303	.329	.398	.414
0.45	.466	.484	.479	.472	.598	.523
0.60	.609	.617	.624	.584	.725	.616
0.80	.740	.729	.735	.689	.817	.696
1.20	.867	.850	.858	.822	.902	.803
1.60	.923	.912	.916	.895	.942	.884
2.40	0.964	0.969	0.960	0.955	0.971	0.969
c	0.23	0.22	0.25	0.27	0.15	0.25
Eq. widths	1391	1425	1392	1519	1118	1449

TABLE 8

RESIDUAL INTENSITIES FOR *Na* I D_2

$\Delta\lambda$	$\vartheta=21°$		$\vartheta=48°$		$\vartheta=76°$	
	Computed	Observed	Computed	Observed	Computed	Observed
0	0.004	0.070	0.006	0.060	0.006	0.085
0.068	.031	.111	.033	.111	.039	.111
0.12	.177	.218	.196	.229	.253	.235
0.20	.316	.370	.344	.392	.430	.450
0.40	.610	.639	.630	.640	.723	.698
0.60	.781	.793	.782	.784	.852	.814
0.80	.861	.878	.864	.866	.909	.880
1.20	.932	.934	.940	.935	.956	.947
1.60	.959	.949	.957	.959	.975	.970
2.00	0.971	0.968	0.973	0.970	0.983	0.979
c	0.10	0.092	0.10	0.097	0.060	0.087
Eq. widths	1003	912	984	926	773	878

the case of the observed ratios for the H and K lines the deviation from the doublet ratio is attributed to the disturbance of the H line by $H\epsilon$.

The ratios between the c's and between the equivalent widths of a given line at various points on the solar disc are of greatest interest for comparison. These ratios, theoretical and observed, are recorded in Table 10. For the sodium D lines and λ 4227 of calcium the observations indicate an increase in the wings up to $\vartheta = 60°$ and 65°, respectively. The small observed increase for the sodium D lines is not predicted. In the case of λ 4227 the

TABLE 9

RESIDUAL INTENSITIES FOR *Na* I D_1

$\Delta\lambda$	$\vartheta=21°$		$\vartheta=48°$		$\vartheta=76°$	
	Computed	Observed	Computed	Observed	Computed	Observed
0	0.008	0.105	0.008	0.100	0.008	0.105
0.068	.041	.165	.043	.160	.050	.155
0.12	.248	.325	.272	.326	.329	.340
0.20	.445	.500	.474	.511	.576	.558
0.40	.732	.771	.753	.755	.830	.795
0.60	.881	.884	.879	.880	.918	.883
0.80	.929	.925	.926	.923	.952	.918
1.20	.965	.965	.967	.965	.978	.969
1.60	.980	.987	.980	.984	.986	.989
2.00	0.988	0.995	0.989	0.995	0.992	0.990
c	0.051	0.047	0.050	0.050	0.032	0.043
Eq. widths	724	645	693	664	565	610

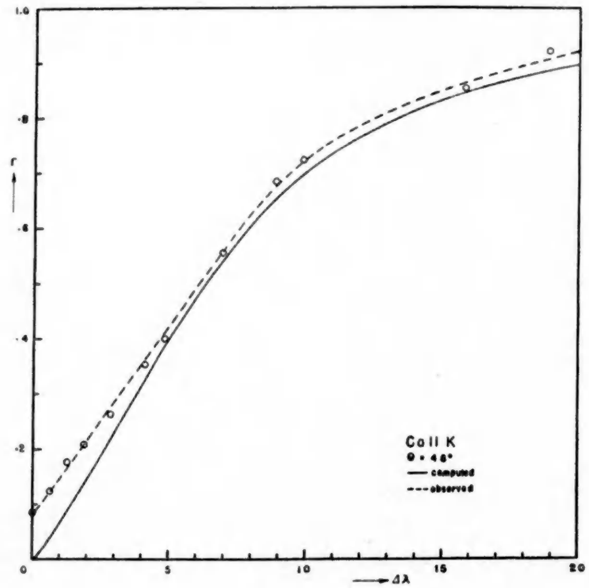

FIG. 1

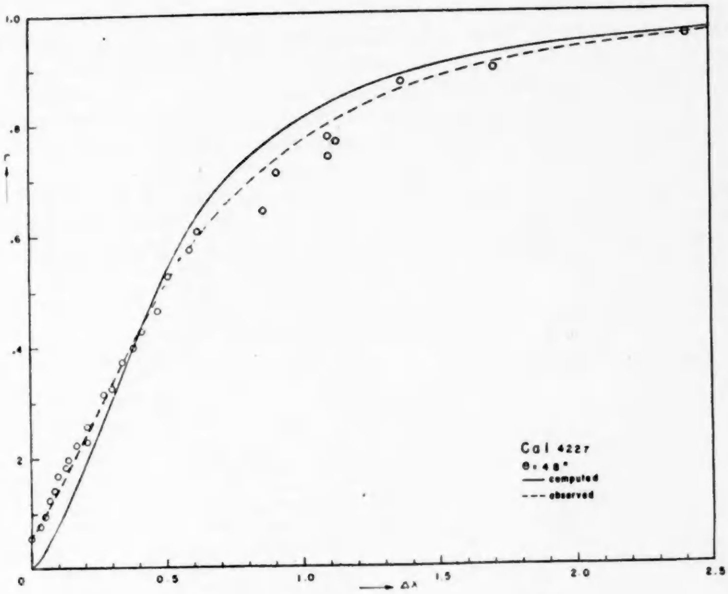
Ca I 4227
θ = 48°
computed
observed
r
Δλ
0
0.5
1.0
1.5
2.0
2.5
.2
.4
.6
.8
1.0

FIG. 2

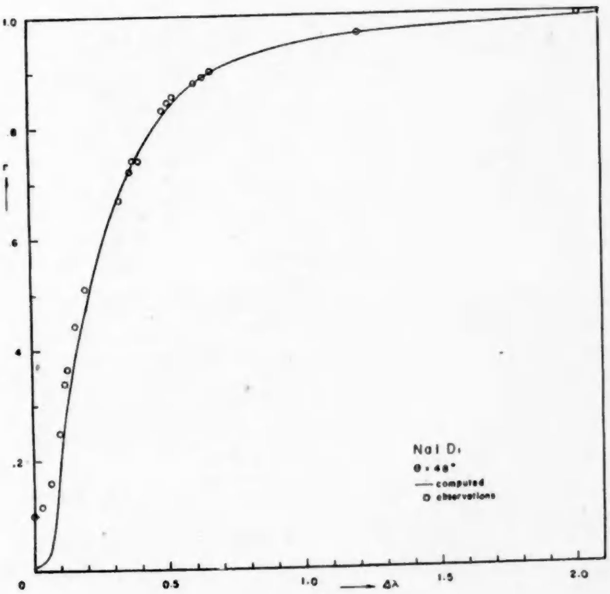
Na I D₁
θ = 48°
computed
observations
r
Δλ
0
0.5
1.0
1.5
2.0
.2
.4
.6
.8
1.0

FIG. 3

predicted increase in the wings is less than the observed. This phenomenon is not so pronounced in the observations made by Royds and Narayan.[22] In general, the theory predicts a greater decrease near the limb in the equivalent widths than is observed. Shane's[23] observations of the sodium D lines suggest, however, that the wings are appreciably weaker near the limb than Houtgast's data indicates. Also, for *Ca* I, λ 4227, Royds and Narayan[22] find a decrease in the equivalent width at $\vartheta = 76°$; the ratios of the equivalent widths obtained by them are 1.00, 1.02, and 0.904, the values of ϑ being 21°, 48°, and 76°, respectively.

In judging the differences which exist between the predicted and observed variations in the contours, it must be remembered that on the theoretical side the present work can be improved by the use of a more accurate model atmosphere for the sun. The Rosseland

TABLE 10

RATIOS OF THE c'S AND THE EQUIVALENT WIDTHS

LINE		$c(\vartheta)/c(21°)$		EQ. WIDTHS (ϑ) / EQ. WIDTHS (21°)	
		Computed	Observed	Computed	Observed
Ca II K	$\vartheta=21°$	1	1	1	1
	$\vartheta=48$	0.981	0.979	0.981	0.964
	$\vartheta=76$	0.543	0.835	0.754	0.832
Ca II H	$\vartheta=21°$	1	1	1	1
	$\vartheta=48$	0.984	0.954	0.980	0.972
	$\vartheta=76$	0.556	0.696	0.768	0.723
Ca I λ 4227	$\vartheta=21°$	1	1	1	1
	$\vartheta=48$	1.09	1.23	1.00	1.07
	$\vartheta=76$	0.652	1.14	0.804	1.02
Na I D_2	$\vartheta=21°$	1	1	1	1
	$\vartheta=48$	1.00	1.05	0.981	1.02
	$\vartheta=76$	0.600	0.946	0.771	0.963
Na I D_1	$\vartheta=21°$	1	1	1	1
	$\vartheta=48$	0.980	1.06	0.957	1.03
	$\vartheta=76$	0.627	0.915	0.780	0.946

mean absorption coefficient used by Rudkjøbing should be replaced by the proper mean absorption coefficient as defined by Chandrasekhar.[24] Further, this should be determined with the new absorption curve for H^-,[25] the contribution to the opacity from the free-free transitions of free electrons being included. On the basis of the model atmosphere obtained, the relative abundances of the various elements composing the solar atmosphere should be rederived. Such an investigation is proposed for a later time.

It is a pleasure to acknowledge my indebtedness to Dr. S. Chandrasekhar for his constant guidance and friendly encouragement.

[22] *Kodaikanal Obs. Bull.*, No. 109, p. 375, 1936.

[23] *Lick Obs. Bull.*, No. 507, 1941.

[24] *Ap. J.*, **101**, 328, 1945.

[25] S. Chandrasekhar, *Ap. J.*, **102**, 395, 1945.

ON THE RADIATIVE EQUILIBRIUM OF A STELLAR ATMOSPHERE. IX

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ABSTRACT

In this paper the problem of diffuse reflection by a semi-infinite plane-parallel atmosphere is considered along the lines of the earlier papers of this series. Explicit solutions are obtained for the cases when the scattering of radiation by the atmosphere takes place in accordance with the phase functions $\lambda(1 + x \cos \Theta)$, $(0 < \lambda \leqslant 1,\ -1 \leqslant x \leqslant 1)$, and $(1 + \cos^2 \Theta)$. It is shown how simple, closed expressions can be found for the angular distribution of the reflected radiation in a general nth approximation. Only certain simple algebraic equations need be solved for their "characteristic roots" to bring the solutions to their numerical forms.

Tables of certain constants and functions required for the practical use of the solutions are provided.

1. *Introduction.*—The phenomenon of diffuse reflection by a semi-infinite plane-parallel atmosphere is of particular interest for astrophysics. It occurs in the study of planetary illumination and of the reflection effect in eclipsing binaries. And it is basic for the interpretation of reflection nebulae. While these various aspects of the phenomenon have been the subject of numerous investigations, it is fair to say that, except in the context of the reflection effect in binaries,[1] the fundamental problem in the theory of radiative transfer has not received an adequately satisfactory treatment. However, interest in the general problem has been revived by a series of recent papers by V. A. Ambarzumian,[2] who has tried to eliminate the explicit solution of the equation of transfer by concentrating on the angular distribution of the reflected radiation alone. In this manner he has been able to reduce the problem of characterizing the reflected radiation to the solution of a number of relatively simple integral equations, which he then seeks to solve numerically by an iteration method. In this paper we shall show how, for the particular cases considered by Ambarzumian, the method which has been developed in the earlier papers of this series[3] can be successfully applied to yield explicit solutions for the angular distribution of the reflected radiation. To reduce these solutions to their numerical forms, it is necessary only to solve certain algebraic equations[4] for "characteristic roots." The method presented in this paper has, accordingly, an advantage over Ambarzumian's in that, in addition to reducing the necessary numerical work very considerably, it also yields simple, closed expressions for the solution in a general nth approximation.

As we have already indicated, the basic problem is that of the radiative equilibrium of a semi-infinite plane-parallel atmosphere exposed to a parallel beam of radiation of flux πF per unit area, normal to itself, and incident at an angle β, normal to the boundary of the atmosphere (see Fig. 1 in paper VIII). Moreover, in considering the general problem of diffuse reflection, it is necessary that we do not restrict ourselves to the case of isotropic scattering but allow for the anisotropy of the scattered radiation in accordance with a "phase function" $p(\cos \Theta)$. The meaning of this phase function is that

$$p(\cos\Theta)\,\frac{d\omega}{4\pi} \tag{1}$$

[1] Cf. S. Chandrasekhar, *Ap. J.*, **101**, 348, 1945; also, C. U. Cesco and J. Sahade (in press).

[2] *J. Physics Acad. Sci. U.S.S.R.*, **8**, 64, 1944, and references given in this paper.

[3] See particularly *Ap. J.*, **100**, 76, 117, 1944, and **101**, 328, 348, 1945. These papers will be referred to as "II," "III," "VII," and "VIII," respectively.

[4] The degree of these equations depends on the order of the approximation in which the solutions are sought.

governs the probability that a pencil of radiation will be scattered in a direction inclined at an angle Θ to the incident direction and confined to an element of solid angle $d\omega$. On these assumptions, the equation of transfer, in a standard notation, is

$$\left.\begin{aligned}\cos\vartheta\,\frac{dI(\tau,\vartheta,\varphi)}{d\tau} = I(\tau,\vartheta,\varphi) - \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} I(\tau,\vartheta',\varphi')\,p\,(\cos\vartheta\cos\vartheta' \\ +\sin\vartheta\sin\vartheta'\cos[\varphi-\varphi'])\sin\vartheta' d\vartheta' d\varphi' - \tfrac{1}{4}Fe^{-\tau\sec\beta}p\,(-\cos\vartheta\cos\beta \\ +\sin\vartheta\sin\beta\cos\varphi)\,,\end{aligned}\right\}\quad(2)$$

where it will be noted that we have assumed (as it entails no loss of generality) that the radiation πF is incident along the direction $\vartheta = \pi - \beta$ and $\varphi = 0$.

In this paper we shall restrict ourselves to the consideration of the following two phase functions:

$$p(\cos\Theta) = \lambda(1 + x\cos\Theta) \qquad (0 < \lambda \leqslant 1, 0 \leqslant |x| \leqslant 1)\,, \tag{3}$$

where λ and x are two constants and

$$p(\cos\Theta) = \tfrac{3}{4}(1 + \cos^2\Theta)\,. \tag{4}$$

The phase function (4) corresponds, of course, to Rayleigh's law of scattering. But the phase function (3), in addition to introducing an asymmetry in the backward and the forward scattering, allows also for the conversion on scattering of the radiant into other forms of energy in terms of the "albedo," λ. The study of diffuse reflection with the phase function (3) is particularly suitable for the analysis of planetary illumination.[5] This is, moreover, also the case for which Ambarzumian has obtained some numerical results. We shall, accordingly, study this case in some detail. In a later paper we shall outline the method for solving the equation of transfer (2) with a general phase function and relate our method to Ambarzumian's.

I. DIFFUSE REFLECTION IN ACCORDANCE WITH THE PHASE FUNCTION $\lambda(1 + x\cos\Theta)$

2. *The reduction of the equation of transfer.*—For a phase function of the form (3), equation (2) becomes

$$\left.\begin{aligned}\cos\vartheta\,\frac{dI(\tau,\vartheta,\varphi)}{d\tau} = I(\tau,\vartheta,\varphi) - \frac{\lambda}{4\pi}\int_0^{\pi}\int_0^{2\pi} I(\tau,\vartheta',\varphi')\,[1 + x(\cos\vartheta\cos\vartheta' \\ +\sin\vartheta\sin\vartheta'\cos[\varphi-\varphi'])]\sin\vartheta' d\vartheta' d\varphi' - \frac{\lambda}{4}Fe^{-\tau\sec\beta}[1 + x(-\cos\vartheta\cos\beta \\ +\sin\vartheta\sin\beta\cos\varphi)]\,.\end{aligned}\right\}\quad(5)$$

The form of equation (5) immediately suggests that we seek a solution in the form

$$I(\tau,\vartheta,\varphi) = I^{(0)}(\tau,\vartheta) + I^{(1)}(\tau,\vartheta)\cos\varphi\,. \tag{6}$$

Substituting this form for $I(\tau,\vartheta,\varphi)$ in equation (5), we find that the equation breaks up into two equations for $I^{(0)}$ and $I^{(1)}$, respectively. We have

$$\left.\begin{aligned}\mu\frac{dI^{(0)}}{d\tau} = I^{(0)} - \tfrac{1}{2}\lambda\int_{-1}^{+1} I^{(0)}(\tau,\mu')\,d\mu' - \tfrac{1}{2}x\lambda\mu\int_{-1}^{+1} I^{(0)}(\tau,\mu')\,\mu' d\mu' \\ - \tfrac{1}{4}\lambda Fe^{-\tau\sec\beta}(1 - x\mu\cos\beta)\end{aligned}\right\}\quad(7)$$

[5] See a forthcoming paper in which the solutions obtained in this paper are used to interpret the known data on planetary illumination.

and

$$\left.\begin{aligned}\mu \frac{dI^{(1)}}{d\tau} = I^{(1)} - \tfrac{1}{4}x\lambda \sqrt{(1-\mu^2)} \int_{-1}^{+1} I^{(1)}(\tau,\mu')\sqrt{(1-\mu'^2)}\,d\mu' \\ - \tfrac{1}{4}x\lambda F e^{-\tau \sec\beta} \sin\beta \sqrt{(1-\mu^2)}\,,\end{aligned}\right\} \quad (8)$$

where we have written μ for $\cos\vartheta$. We shall now show how the two foregoing equations for $I^{(0)}$ and $I^{(1)}$ can be solved.

3. *The solution of equation (7) in the nth approximation.*—As in paper II, we replace the integrals which occur on the right-hand side of equation (7.) by sums according to Gauss's formula of numerical quadratures and obtain an equivalent system of linear equations. In the nth approximation this is

$$\left.\begin{aligned}\mu_i \frac{dI_i^{(0)}}{d\tau} = I_i^{(0)} - \tfrac{1}{2}\lambda \Sigma a_j I_j^{(0)} - \tfrac{1}{2}x\lambda\mu_i \Sigma a_j \mu_j I_j^{(0)} \\ - \tfrac{1}{4}\lambda F e^{-\tau\sec\beta}(1 - x\mu_i\cos\beta) \qquad (i = \pm 1, \ldots, \pm n)\,,\end{aligned}\right\} \quad (9)$$

where the various symbols have the same meanings as in paper II.

In solving the system of equations represented by equation (9) we first seek the general solution of the associated homogeneous system

$$\mu_i \frac{dI_i}{d\tau} = I_i - \tfrac{1}{2}\lambda\Sigma a_j I_j - \tfrac{1}{2}x\lambda\mu_i\Sigma a_j\mu_j I_j \qquad (i = \pm 1, \ldots, \pm n) \quad (10)$$

and then add to it a particular integral of the nonhomogeneous system.

To obtain the different linearly independent solutions of the system (10), we proceed as follows: Setting

$$I_i = g_i e^{-k\tau} \qquad (i = \pm 1, \ldots, \pm n) \quad (11)$$

in equation (10), where the g_i's and k are constants, unspecified for the present, we obtain

$$(1 + \mu_i k)\, g_i = \tfrac{1}{2}\lambda\Sigma a_j g_j + \tfrac{1}{2}x\lambda\mu_i\Sigma a_j\mu_j g_j\,. \quad (12)$$

Equation (12) implies that g_i must be expressible in the form

$$g_i = \frac{A + B\mu_i}{1 + \mu_i k} \qquad (i = \pm 1, \ldots, \pm n)\,, \quad (13)$$

where A and B are two constants independent of i. Substituting equation (13) back into equation (12), we find

$$A + B\mu_i = \tfrac{1}{2}\lambda\Sigma \frac{a_j(A + B\mu_j)}{1 + \mu_j k} + \tfrac{1}{2}x\lambda\mu_i\Sigma \frac{a_j\mu_j(A + B\mu_j)}{1 + \mu_j k}\,. \quad (14)$$

Since this equation must be valid for all i's ,we must require that

$$A = \tfrac{1}{2}\lambda(A D_0 + B D_1) \quad (15)$$

and

$$B = \tfrac{1}{2}x\lambda(A D_1 + B D_2) \quad (16)$$

where we have introduced the quantity

$$D_m = \Sigma \frac{a_j \mu_j^{\,m}}{1 + \mu_j k}\,. \quad (17)$$

These D_m's satisfy the recursion formula (VIII, eq. [52])

$$D_m = \frac{1}{k}\left(\frac{2}{m}\epsilon_{m,\,\mathrm{odd}} - D_{m-1}\right); \tag{18}$$

in particular,

$$D_1 = \frac{1}{k}(2 - D_0) \qquad \text{and} \qquad D_2 = -\frac{1}{k}D_1 = -\frac{1}{k^2}(2 - D_0). \tag{19}$$

Returning to equations (15) and (16), we can re-write them in the forms

$$(2 - \lambda D_0) A - \lambda D_1 B = 0 \tag{20}$$

and

$$x\lambda D_1 A + (x\lambda D_2 - 2) B = 0. \tag{21}$$

In order that A and B do not vanish identically, we must require that

$$(2 - \lambda D_0)(x\lambda D_2 - 2) + x\lambda^2 D_1^2 = 0. \tag{22}$$

Using the recurrence relation (18), the foregoing equation can be reduced to give

$$2 = \lambda\left[D_0 - \frac{x(1-\lambda)}{k}D_1\right], \tag{23}$$

or equivalently (cf. eq. [19])

$$2 = \lambda\left[D_0 + x(1-\lambda)D_2\right]. \tag{24}$$

In other words, k must be a root of the equation

$$2 = \lambda\Sigma\,\frac{a_j\left[1 + x(1-\lambda)\mu_j^2\right]}{1 + \mu_j k}; \tag{25}$$

or, since $a_j = a_{-j}$, and $\mu_j = -\mu_{-j}$,

$$1 = \lambda\sum_{j=1}^{n}\frac{a_j\left[1 + x(1-\lambda)\mu_j^2\right]}{1 - \mu_j^2 k^2}. \tag{26}$$

This is the characteristic equation for k. Equation (26) is of order n in k^2 and for $\lambda \neq 1$[6] admits of $2n$ distinct nonvanishing roots, which must occur in pairs as

$$\pm k_a \qquad (a = 1, \ldots, n). \tag{27}$$

From equation (20) (or [21]) we now conclude that

$$B = \frac{2 - \lambda D_0}{\lambda D_1} A, \tag{28}$$

or, according to equation (23), that

$$B = -\frac{x(1-\lambda)}{k} A. \tag{29}$$

Hence (cf. eq. [13])

$$g_i = \text{constant}\ \frac{1 - x(1-\lambda)\mu_i/k}{1 + \mu_i k} \qquad (i = \pm 1, \ldots, \pm n). \tag{30}$$

Thus the homogeneous system of equations (10) admits the $2n$ linearly independent integrals

$$I_i = \text{constant}\ \frac{1 \mp x(1-\lambda)\mu_i/k_a}{1 \pm \mu_i k_a}\, e^{\mp k_a \tau} \qquad \begin{pmatrix} i = \pm 1, \ldots, \pm n \\ a = 1, \ldots, n \end{pmatrix}. \tag{31}$$

[6] We treat the case $\lambda = 1$ separately (cf. n. 8, p. 177).

The general solution can therefore be written in the form

$$\left.\begin{aligned} I_i = \tfrac{1}{4}\lambda F \Bigg\{ \sum_{a=1}^{n} \frac{M_a\left[1 - x(1-\lambda)\mu_i/k_a\right]}{1+\mu_i k_a}\, e^{-k_a\tau} \\ + \sum_{a=1}^{n} \frac{M_{-a}\left[1 + x(1-\lambda)\mu_i/k_a\right]}{1-\mu_i k_a}\, e^{+k_a\tau}\Bigg\} \qquad (i = \pm 1, \ldots, \pm n)\,, \end{aligned}\right\} \tag{32}$$

where $M_{\pm a}$ $(a = 1, \ldots, n)$ are $2n$ constants of integration.

To complete the solution of the nonhomogeneous system (9), we need a particular integral. This can be found in the following manner:

Setting

$$I_i^{(0)} = \tfrac{1}{4}\lambda F h_i e^{-\tau \sec\beta} \qquad (i = \pm 1, \ldots, \pm n) \tag{33}$$

in equation (9) (the h_i's are certain constants unspecified for the present), we verify that we must have

$$(1 + \mu_i \sec\beta)\, h_i = \tfrac{1}{2}\lambda \Sigma a_j h_j + \tfrac{1}{2} x\lambda \mu_i \Sigma a_j \mu_j h_j + 1 - x\mu_i \cos\beta\,. \tag{34}$$

Equation (34) implies that the constants h_i must be expressible in the form

$$h_i = \frac{\gamma + \mu_i \delta}{1 + \mu_i \sec\beta} \qquad (i = \pm 1, \ldots, \pm n)\,, \tag{35}$$

where the constants γ and δ have to be determined in accordance with the relation

$$\gamma + \mu_i \delta = \left[\tfrac{1}{2}\lambda(\gamma E_0 + \delta E_1) + 1\right] + \mu_i \left[\tfrac{1}{2} x\lambda(\gamma E_1 + \delta E_2) - x\cos\beta\right], \tag{36}$$

where we have used E_m to denote

$$E_m = \Sigma \frac{a_j \mu_j^m}{1 + \mu_j \sec\beta}\,. \tag{37}$$

From equation (36) we conclude that the equations which determine γ and δ are

$$(2 - \lambda E_0)\,\gamma - \lambda E_1 \delta - 2 = 0 \tag{38}$$

and

$$x\lambda E_1 \gamma + (x\lambda E_2 - 2)\,\delta - 2x\cos\beta = 0\,. \tag{39}$$

Solving these equations, we find

$$\gamma = \cfrac{1}{1 - \lambda \displaystyle\sum_{j=1}^{n} \frac{a_j\left[1 + x(1-\lambda)\mu_j^2\right]}{1 - \mu_j^2 \sec^2\beta}} \tag{40}$$

and

$$\delta = -\gamma x (1-\lambda)\cos\beta\,. \tag{41}$$

In reducing the solutions for γ and δ to the foregoing forms, use has been made of the recursion formula (cf. eq. [18])

$$E_m = \cos\beta\left(\frac{2}{m}\epsilon_{m,\,\mathrm{odd}} - E_{m-1}\right), \tag{42}$$

which the E_m's satisfy.

The expression (40) for γ has a simple representation in terms of the roots $k_1^2, \ldots, k_n^2$ of the characteristic equation (26);[7] for, considering the function

$$T(z) = 1 - \lambda \sum_{j=1}^{n} \frac{a_j [1 + x(1-\lambda)\mu_j^2]}{1 - \mu_j^2 z}, \tag{43}$$

we observe that it vanishes for

$$z = k_a^2 \qquad (a = 1, \ldots, n). \tag{44}$$

Accordingly,

$$\prod_{j=1}^{n} (1 - \mu_j^2 z) T(z), \tag{45}$$

which is a polynomial of degree n in z, cannot differ from

$$\prod_{a=1}^{n} (z - k_a^2) \tag{46}$$

except by a constant factor. The constant of proportionality can be determined by comparing the coefficients of the highest powers. In this manner we find that

$$T(z) = (-1)^n \mu_1^2 \ldots \mu_n^2 \frac{\prod_{a=1}^{n} (z - k_a^2)}{\prod_{j=1}^{n} (1 - \mu_j^2 z)}. \tag{47}$$

Hence,

$$\gamma = \frac{1}{T(\sec^2 \beta)} = \frac{(-1)^n}{\mu_1^2 \ldots \mu_n^2} \frac{\prod_{j=1}^{n} (1 - \mu_j^2 \sec^2 \beta)}{\prod_{a=1}^{n} (\sec^2 \beta - k_a^2)}, \tag{48}$$

or, somewhat differently,

$$\gamma = \frac{(-1)^n}{\mu_1^2 \ldots \mu_n^2} \frac{\prod_{j=1}^{n} (\cos^2 \beta - \mu_j^2)}{\prod_{a=1}^{n} (1 - k_a^2 \cos^2 \beta)}. \tag{49}$$

In terms of the functions (cf. II, eqs. [58] and [59])

$$P(\mu) = \prod_{j=1}^{n} (\mu - \mu_j) \tag{50}$$

and

$$R(\mu) = \prod_{a=1}^{n} (1 - k_a \mu), \tag{51}$$

[7] The analysis which follows is similar to that in paper VIII, following eq. (40).

we can express γ alternatively in the form

$$\gamma = \frac{1}{\mu_1^2 \ldots . \mu_n^2} \frac{P(\cos\beta)\,P(-\cos\beta)}{R(\cos\beta)\,R(-\cos\beta)} . \tag{52}$$

When we return to equations (33) and (35), it is seen that the nonhomogeneous system (9) admits the particular integral (cf. eq. [41])

$$I_i^{(0)} = \tfrac{1}{4}\lambda F e^{-\tau\sec\beta}\gamma \frac{1 - x(1-\lambda)\,\mu_i \cos\beta}{1 + \mu_i \sec\beta} \qquad (i = \pm 1, \ldots . , \pm n) . \tag{53}$$

Adding to this particular integral the general solution of the homogeneous system which is bounded for $\tau \to \infty$, we have

$$\left.\begin{aligned} I_i^{(0)} = \tfrac{1}{4}\lambda F \Big[\sum_{a=1}^{n} & \frac{M_a [1 - x(1-\lambda)\,\mu_i / k_a]\, e^{-k_a\tau}}{1 + \mu_i k_a} \\ & + \frac{\gamma [1 - x(1-\lambda)\,\mu_i \cos\beta]\, e^{-\tau\sec\beta}}{1 + \mu_i \sec\beta} \Big] \qquad (i = \pm 1, \ldots . , \pm n) , \end{aligned}\right\} \tag{54}$$

where the constants M_a ($a = 1, \ldots . , n$) have to be determined from the boundary conditions at $\tau = 0$.

At $\tau = 0$ we have no incident radiation derived from the material. Accordingly, we should require that

$$I_{-i}^{(0)} = 0 \quad \text{at} \quad \tau = 0 \quad \text{and for} \quad i = 1, \ldots . , n . \tag{55}$$

Hence the equations which determine M_a are

$$\left.\begin{aligned} \sum_{a=1}^{n} \frac{M_a [1 + x(1-\lambda)\,\mu_i / k_a]}{1 - \mu_i k_a} + \frac{\gamma [1 + x(1-\lambda)\,\mu_i \cos\beta]}{1 - \mu_i \sec\beta} &= 0 \\ (i = 1, \ldots . , n) &. \end{aligned}\right\} \tag{56}$$

If we now let $G(\mu)$ denote

$$G(\mu) = \sum_{a=1}^{n} \frac{M_a [1 + x(1-\lambda)\,\mu / k_a]}{1 - \mu k_a} + \frac{\gamma [1 + x(1-\lambda)\,\mu \cos\beta]}{1 - \mu \sec\beta} , \tag{57}$$

then

$$G(\mu_i) = 0 \qquad (i = 1, \ldots . , n) . \tag{58}$$

The angular distribution of the part of the reflected radiation corresponding to $I^{(0)}$ (cf. eq. [6]) can be found from the source function

$$\mathfrak{J}^{(0)} = \tfrac{1}{2}\lambda \Sigma a_j I_j^{(0)} + \tfrac{1}{2} x \lambda \mu \Sigma a_j \mu_j I_j^{(0)} + \tfrac{1}{4} F e^{-\tau\sec\beta} (1 - x\mu\cos\beta) , \tag{59}$$

according to the formula

$$I^{(0)}(0,\mu) = \int_0^\infty \mathfrak{J}^{(0)}(\tau)\, e^{-\tau/\mu} \frac{d\tau}{\mu} . \tag{60}$$

The quantities on the right-hand side of equation (59) can readily be evaluated in terms of the solution (54). We find

$$I^{(0)}(0,\mu) = \tfrac{1}{4}\lambda F G(-\mu) . \tag{61}$$

This is in agreement with solution (54) for $\tau = 0$ and at the points of the Gaussian division, $\mu = u_i$.

We shall now show how an explicit formula for $G(\mu)$ can be found without having to solve for the constants M_a.

Consider the function

$$(1-\mu \sec \beta) R(\mu) G(\mu) = (1-\mu \sec \beta) \prod_{a=1}^{n} (1-k_a \mu) G(\mu) . \tag{62}$$

This is a polynomial of degree $n+1$ in μ which vanishes for $\mu = \mu_i$, $i = 1, \ldots, n$. Consequently, there must exist a proportionality of the form

$$(1-\mu \sec \beta) R(\mu) G(\mu) \propto P(\mu)(\mu + c) , \tag{63}$$

where c is some constant. The constant of proportionality can be found from a comparison of the coefficients of the highest powers of μ on either side. On the left hand the coefficient of μ^{n+1} is

$$(-1)^n k_1 \ldots k_n x (1-\lambda) \left[\sum_{a=1}^{n} \frac{M_a}{k_a^2} \sec \beta + \gamma \cos \beta \right], \tag{64}$$

while on the right-hand side it is unity. Hence,

$$\left. \begin{aligned} G(\mu) = (-1)^n k_1 \ldots k_n x (1-\lambda) & \left[\sum_{a=1}^{n} \frac{M_a}{k_a^2} \sec \beta + \gamma \cos \beta \right] \\ & \times \frac{P(\mu)}{R(\mu)} \frac{\mu + c}{1 - \mu \sec \beta} . \end{aligned} \right\} \tag{65}$$

Now, according to equation (57),

$$\lim_{\mu \to \cos \beta} (1-\mu \sec \beta) G(\mu) = \gamma [1 + x(1-\lambda) \cos^2 \beta] . \tag{66}$$

Substituting for γ and $G(\mu)$ from equations (52) and (65) in equation (66), we obtain

$$\left. \begin{aligned} (-1)^n k_1 \ldots k_n x (1-\lambda) \left[\sum_{a=1}^{n} \frac{M_a}{k_a^2} \sec \beta + \gamma \cos \beta \right] & \frac{P(\cos \beta)}{R(\cos \beta)} (\cos \beta + c) \\ = \frac{1 + x(1-\lambda)\cos^2 \beta}{\mu_1^2 \ldots \mu_n^2} & \frac{P(\cos \beta) P(-\cos \beta)}{R(\cos \beta) R(-\cos \beta)} , \end{aligned} \right\} \tag{67}$$

or

$$\left. \begin{aligned} (-1)^n k_1 \ldots k_n x (1-\lambda) & \left[\sum_{a=1}^{n} \frac{M_a}{k_a^2} \sec \beta + \gamma \cos \beta \right] \\ & = \frac{1 + x(1-\lambda)\cos^2 \beta}{\mu_1^2 \ldots \mu_n^2} \frac{P(-\cos \beta)}{R(-\cos \beta)} \frac{1}{\cos \beta + c} . \end{aligned} \right\} \tag{68}$$

In virtue of this relation, equation (65) becomes

$$G(\mu) = \frac{1 + x(1-\lambda)\cos^2 \beta}{\mu_1^2 \ldots \mu_n^2} \frac{P(-\cos \beta) P(\mu)}{R(-\cos \beta) R(\mu)} \frac{\mu + c}{(\cos \beta + c)(1 - \mu \sec \beta)} . \tag{69}$$

Equation (69) specifies $G(\mu)$ completely except for the constant c, which remains to be determined.

From equations (57) and (69) it follows that

$$G(0) = \sum_{a=1}^{n} M_a + \gamma = (-1)^n \frac{1 + x(1-\lambda)\cos^2\beta}{\mu_1 \ldots \mu_n} \frac{P(-\cos\beta)}{R(-\cos\beta)} \frac{c}{\cos\beta + c}. \quad (70)$$

On the other hand, since (cf. eq. [57])

$$[1 + x(1-\lambda) k_a^{-2}] M_a = \lim_{\mu \to k_a^{-1}} (1 - k_a \mu) G(\mu), \quad (71)$$

we have

$$M_a = \frac{1 + x(1-\lambda)\cos^2\beta}{\mu_1^2 \ldots \mu_n^2} \frac{P(-\cos\beta)}{R(-\cos\beta)} \frac{c + k_a^{-1}}{\cos\beta + c} m_a, \quad (72)$$

where

$$m_a = \frac{P(k_a^{-1})}{[1 + x(1-\lambda) k_a^{-2}](1 - k_a^{-1}\sec\beta) R_a(k_a^{-1})}. \quad (73)$$

In equation (73) we have introduced the function $R_a(\mu)$, which is obtained from $R(\mu)$ by omitting the factor $(1 - k_a\mu)$ in its product representation. Thus (cf. eq. [51])

$$R_a(\mu) = \prod_{\beta \neq a} (1 - k_\beta \mu). \quad (74)$$

Substituting now for M_a and γ according to equations (72) and (52) in equation (70), we obtain, after some minor reductions, the following equation, which, as we shall see, determines c:

$$\left. \begin{aligned} c\Big[(-1)^n \mu_1 \ldots \mu_n - \sum_{a=1}^{n} m_a - \frac{1}{1 + x(1-\lambda)\cos^2\beta} \frac{P(\cos\beta)}{R(\cos\beta)}\Big] \\ = \sum_{a=1}^{n} \frac{m_a}{k_a} + \frac{\cos\beta}{1 + x(1-\lambda)\cos^2\beta} \frac{P(\cos\beta)}{R(\cos\beta)}. \end{aligned} \right\} \quad (75)$$

In order that we may use equation (75) to obtain an explicit formula for c, we have to sum the two series

$$\sum_{a=1}^{n} m_a \quad \text{and} \quad \sum_{a=1}^{n} \frac{m_a}{k_a}. \quad (76)$$

Considering first Σm_a, we have to evaluate

$$\sum_{a=1}^{n} \frac{P(k_a^{-1})}{[1 + x(1-\lambda) k_a^{-2}](1 - k_a^{-1}\sec\beta) R_a(k_a^{-1})}. \quad (77)$$

We re-write this in the form

$$\left. \begin{aligned} \sum_{a=1}^{n} m_a &= -\cos\beta \sum_{a=1}^{n} \frac{k_a P(k_a^{-1})}{[1 + x(1-\lambda) k_a^{-2}](1 - k_a\cos\beta) R_a(k_a^{-1})} \\ &= -\frac{\cos\beta}{R(\cos\beta)} \sum_{a=1}^{n} \frac{k_a P(k_a^{-1}) R_a(\cos\beta)}{[1 + x(1-\lambda) k_a^{-2}] R_a(k_a^{-1})} \\ &= -\frac{\cos\beta}{R(\cos\beta)} f(\cos\beta) \quad . \end{aligned} \right\} \quad (78)$$

where

$$f(z) = \sum_{\alpha=1}^{n} \frac{k_\alpha P(k_\alpha^{-1}) R_\alpha(z)}{[1 + x(1-\lambda) k_\alpha^{-2}] R_\alpha(k_\alpha^{-1})}. \tag{79}$$

As defined in the foregoing equation, $f(z)$ is a polynomial of degree $n - 1$ in z which assumes for $z = k_\alpha^{-1}$ the values

$$f(k_\alpha^{-1}) = \frac{k_\alpha}{1 + x(1-\lambda) k_\alpha^{-2}} P(k_\alpha^{-1}) \quad (\alpha = 1, \ldots, n). \tag{80}$$

Hence,

$$z[1 + x(1-\lambda) z^2] f(z) - P(z) \tag{81}$$

is a polynomial of degree $n + 2$ in z which vanishes for $z = k_\alpha^{-1}$ ($\alpha = 1, \ldots, n$). There must, accordingly, be a relation of the form

$$z[1 + x(1-\lambda) z^2] f(z) = P(z) + R(z)(\xi z^2 + \eta z + \zeta), \tag{82}$$

where ξ, η, and ζ are certain constants. These constants can be found in the following manner:

First putting $z = 0$ in equation (82), we conclude that

$$\zeta = (-1)^{n+1} \mu_1 \ldots \mu_n. \tag{83}$$

Next, comparing the coefficients of z^{n+2} on either side of equation (82), we have

$$\left.\begin{aligned} -(-1)^n k_1 \ldots k_n x(1-\lambda) \sum_{\alpha=1}^{n} \frac{P(k_\alpha^{-1})}{[1 + x(1-\lambda) k_\alpha^{-2}] R_\alpha(k_\alpha^{-1})} \\ = (-1)^n k_1 \ldots k_n \xi, \end{aligned}\right\} \tag{84}$$

or

$$\xi = -x(1-\lambda) \sum_{\alpha=1}^{n} \frac{P(k_\alpha^{-1})}{[1 + x(1-\lambda) \kappa_\alpha^{-2}] R_\alpha(k_\alpha^{-1})}. \tag{85}$$

And, finally, equating the coefficients of z^{n+1} on both sides of equation (82), we have

$$\left.\begin{aligned} (-1)^{n-2} x(1-\lambda) k_1 \ldots k_n \Bigg[\left(\sum_{\alpha=1}^{n} \frac{1}{k_\alpha}\right) \sum_{\alpha=1}^{n} \frac{P(k_\alpha^{-1})}{[1 + x(1-\lambda) k_\alpha^{-2}] R_\alpha(k_\alpha^{-1})} \\ - \sum_{\alpha=1}^{n} \frac{P(k_\alpha^{-1})}{k_\alpha [1 + x(1-\lambda) k_\alpha^{-2}] R_\alpha(k_\alpha^{-1})}\Bigg] = (-1)^{n-1} k_1 \ldots k_n \left(\sum_{\alpha=1}^{n} \frac{1}{k_\alpha}\right) \xi \\ + (-1)^n k_1 \ldots k_n \eta. \end{aligned}\right\} \tag{86}$$

Substituting for ξ from equation (85) in the foregoing equation, we find

$$\eta = -x(1-\lambda) \sum_{\alpha=1}^{n} \frac{P(k_\alpha^{-1})}{k_\alpha [1 + x(1-\lambda) k_\alpha^{-2}] R_\alpha(k_\alpha^{-1})}. \tag{87}$$

Returning to equation (78), we now have, according to equations (82) and (83),

$$\left.\begin{aligned} \sum_{\alpha=1}^{n} m_\alpha = -\frac{1}{1 + x(1-\lambda) \cos^2 \beta} \Bigg[\frac{P(\cos \beta)}{R(\cos \beta)} \\ + \xi \cos^2 \beta + \eta \cos \beta + (-1)^{n+1} \mu_1 \ldots \mu_n\Bigg]. \end{aligned}\right\} \tag{88}$$

With this expression for Σm_a, the terms in the square brackets on the left-hand side of equation (75) can be reduced to

$$- x(1-\lambda)\frac{\cos\beta}{1+x(1-\lambda)\cos^2\beta}(\rho\cos\beta+\sigma)\,, \tag{89}$$

where

$$\rho=\sum_{a=1}^{n}\frac{P(k_a^{-1})}{[1+x(1-\lambda)k_a^{-2}]R_a(k_a^{-1})}+(-1)^{n+1}\mu_1\ldots\mu_n \tag{90}$$

and

$$\sigma=\sum_{a=1}^{n}\frac{P(k_a^{-1})}{k_a[1+x(1-\lambda)k_a^{-2}]R_a(k_a^{-1})} \tag{91}$$

are two constants, depending only on the characteristic roots k_a.

Considering next the summation $\Sigma m_a/k_a$ which occurs on the right-hand side of equation (75), we have to evaluate (cf. eq. [73])

$$\sum_{a=1}^{n}\frac{m_a}{k_a}=\sum_{a=1}^{n}\frac{P(k_a^{-1})}{k_a[1+x(1-\lambda)k_a^{-2}](1-k_a^{-1}\sec\beta)R_a(k_a^{-1})}\,. \tag{92}$$

We re-write this in the form (cf. eq. [78])

$$\sum_{a=1}^{n}\frac{m_a}{k_a}=-\frac{\cos\beta}{R(\cos\beta)}\,g(\cos\beta)\,, \tag{93}$$

where

$$g(z)=\sum_{a=1}^{n}\frac{P(k_a^{-1})R_a(z)}{[1+x(1-\lambda)k_a^{-2}]R_a(k_a^{-1})} \tag{94}$$

is a polynomial of degree $(n-1)$ in z. It is seen that

$$g(k_a^{-1})=\frac{P(k_a^{-1})}{1+x(1-\lambda)k_a^{-2}}\,. \tag{95}$$

Hence,

$$[1+x(1-\lambda)z^2]\,g(z)-P(z) \tag{96}$$

is a polynomial of degree $n+1$ in z which vanishes for $z=k_a^{-1}$ $(a=1,\ldots,n)$. We conclude that

$$[1+x(1-\lambda)z^2]\,g(z)=P(z)+R(z)(az+b)\,, \tag{97}$$

where a and b are constants. To determine them, we first set $z=0$ in equation (97) and find (cf. eq. [90])

$$b=\sum_{a=1}^{n}\frac{P(k_a^{-1})}{[1+x(1-\lambda)k_a^{-2}]R_a(k_a^{-1})}+(-1)^{n+1}\mu_1\ldots\mu_n=\rho\,. \tag{98}$$

Next, comparing the coefficients of the highest powers of z on either side of equation (97), we find (cf. eq. [91])

$$a=-x(1-\lambda)\sum_{a=1}^{n}\frac{P(k_a^{-1})}{k_a[1+x(1-\lambda)k_a^{-2}]R_a(k_a^{-1})}=-x(1-\lambda)\,\sigma\,. \tag{99}$$

We have thus shown that

$$[1+x(1-\lambda)z^2]\,g(z)=P(z)-R(z)[x(1-\lambda)\sigma z-\rho]. \tag{100}$$

Hence (cf. eq. [93]),

$$\sum_{\alpha=1}^{n}\frac{m_\alpha}{k_\alpha}=-\frac{\cos\beta}{1+x(1-\lambda)\cos^2\beta}\left[\frac{P(\cos\beta)}{R(\cos\beta)}-x(1-\lambda)\sigma\cos\beta+\rho\right]. \tag{101}$$

Combining equations (75), (89), and (101), we now obtain

$$\left.\begin{aligned}-cx(1-\lambda)\frac{\cos\beta}{1+x(1-\lambda)\cos^2\beta}(\rho\cos\beta+\sigma)\\ =\frac{\cos\beta[x(1-\lambda)\sigma\cos\beta-\rho]}{1+x(1-\lambda)\cos^2\beta},\end{aligned}\right\} \tag{102}$$

or

$$c=-\frac{x(1-\lambda)\sigma\cos\beta-\rho}{x(1-\lambda)(\rho\cos\beta+\sigma)}, \tag{103}$$

which is our formula for c. With c given by equation (103) we readily verify that

$$\cos\beta+c=\rho\,\frac{1+x(1-\lambda)\cos^2\beta}{x(1-\lambda)(\rho\cos\beta+\sigma)} \tag{104}$$

and

$$\left.\begin{aligned}\frac{1+x(1-\lambda)\cos^2\beta}{\cos\beta+c}(\mu+c)\\ =\frac{1}{\rho}[x(1-\lambda)(\rho\cos\beta+\sigma)\mu-x(1-\lambda)\sigma\cos\beta+\rho]\\ =\frac{1}{\rho}[\rho-x(1-\lambda)\{\sigma(\cos\beta-\mu)-\rho\mu\cos\beta\}].\end{aligned}\right\} \tag{105}$$

Accordingly, equation (69) becomes

$$G(\mu)=\frac{1}{\mu_1^2\ldots\mu_n^2\rho}\frac{P(-\cos\beta)P(\mu)}{R(-\cos\beta)R(\mu)}\frac{\rho-x(1-\lambda)[\sigma(\cos\beta-\mu)-\rho\mu\cos\beta]}{1-\mu\sec\beta}, \tag{106}$$

which is our formula for $G(\mu)$.

The angular distribution of the reflected radiation corresponding to the part $I^{(0)}$ is given by (cf. eq. [61])

$$I^{(0)}(0,\mu)=\tfrac{1}{4}\lambda FG(-\mu). \tag{107}$$

With $G(\mu)$ given by equation (106) we can express $I^{(0)}(0,\mu)$ in the form

$$\left.\begin{aligned}I^{(0)}(0,\mu)=\tfrac{1}{4}\lambda FH^{(0)}(\mu)\,H^{(0)}(\cos\beta)\frac{\cos\beta}{\cos\beta+\mu}\\ \times[1-x(1-\lambda)\{\tfrac{\sigma}{\rho}(\cos\beta+\mu)+\mu\cos\beta\}],\end{aligned}\right\} \tag{108}$$

where we have introduced the function $H^{(0)}(\mu)$ defined by

$$H^{(0)}(\mu)=\frac{(-1)^n}{\mu_1\ldots\mu_n}\frac{P(-\mu)}{R^{(0)}(-\mu)}. \tag{109}$$

In equation (109) we have added a superscript "0" to R to emphasize the fact that this function is defined in terms of the characteristic roots appropriate to the system of equations governing $I_i^{(0)}$.

For $\lambda = 1$ the solution (108) for the angular distribution reduces to

$$I^{(0)}(0, \mu) = \tfrac{1}{4} F H^{(0)}(\mu) H^{(0)}(\cos\beta) \frac{\cos\beta}{\cos\beta + \mu}, \tag{110}$$

and it may be readily verified that this is *identical* with the solution found in paper VIII for the reflection effect in eclipsing binaries.[8]

4. *The solution of a general type of integrodifferential equation of which equation (8) is a special case.*—Equation (8) is typical of a large class of equations which occurs in this theory. It is of the general type

$$\mu \frac{dI}{d\tau} = I - \epsilon\psi(\mu) \int_{-1}^{+1} I(\tau, \mu')\psi(\mu')\, d\mu' - \epsilon F e^{-\tau \sec\beta}\psi(\cos\beta)\psi(\mu), \tag{111}$$

where ϵ is a constant and $\psi^2(\mu)$ is a polynomial of degree $2m$ in μ. It will therefore be convenient to have the solution of this general equation appropriate to the conditions of our problem.

Since $\psi^2(\mu)$ is rational, it suggests that we express $I(\tau, \mu)$ in the form

$$I(\tau, \mu) = \phi(\tau, \mu)\psi(\mu) \tag{111'}$$

and obtain for ϕ the equation

$$\mu \frac{d\phi}{d\tau} = \phi - \epsilon \int_{-1}^{+1} \phi(\tau, \mu')\psi^2(\mu')\, d\mu' - \epsilon F e^{-\tau \sec\beta}\psi(\cos\beta). \tag{112}$$

To solve equation (112) in the nth approximation, we replace it by the system of $2n$ linear equations

$$\mu_i \frac{d\phi_i}{d\tau} = \phi_i - \epsilon \Sigma a_j \psi^2(\mu_j)\phi_j - \epsilon F e^{-\tau \sec\beta}\psi(\cos\beta) \qquad (i = \pm 1, \ldots, \pm n), \tag{113}$$

where ϕ_i denotes $\phi(\tau, \mu_i)$ and the rest of the symbols have their usual meanings. In this connection, it should be noted that, in order that we may be consistent in our scheme of approximation, it is necessary that the order of the approximation

$$n \geqslant m, \tag{114}$$

where it may be recalled that $2m$ is the degree of the polynomial $\psi^2(\mu)$.

Considering first the homogeneous system

$$\mu_i \frac{d\phi_i}{d\tau} = \phi_i - \epsilon \Sigma a_j \psi^2(\mu_j)\phi_j \qquad (i = \pm 1, \ldots, \pm n), \tag{115}$$

[8] It should, however, be pointed out in this equation that for $\lambda = 1$ the characteristic equation allows only $n - 1$ distinct nonvanishing roots for k^2. (In fact, the characteristic equation [26] reduces to the one considered in paper II *independently* of x for $\lambda = 1$.) Accordingly, in this case there exist only $(2n - 2)$ independent integrals of the form

$$I_i = \frac{\text{constant}}{1 \pm \mu_i k_a} e^{\mp k_a \tau} \qquad \begin{pmatrix} i = \pm 1, \ldots, \pm n \\ a = 1, \ldots, n-1 \end{pmatrix}$$

for the homogeneous system (10). On the other hand, when $\lambda = 1$, equation (10) admits the further integral.

$$I_i = b\left(\tau + \frac{1}{1 - \frac{1}{3}x}\mu_i + Q\right) \qquad (i = \pm 1, \ldots, \pm n)$$

with two arbitrary constants b and Q. Nevertheless, it can be shown that the procedure of formally putting $\lambda = 1$ in eq. (108) actually leads to the correct solution.

associated with equation (113), we readily verify that it admits $2n$ linearly independent integrals of the form

$$\phi_i = \frac{\text{constant}}{1 \pm \mu_i k_a} e^{\mp k_a \tau} \qquad \begin{pmatrix} i = \pm 1, \ldots, \pm n \\ a = 1, \ldots, n \end{pmatrix}, \tag{116}$$

where $\pm k_a$, ($a = 1, \ldots, n$) are $2n$ distinct[9] nonvanishing roots of the characteristic equation

$$1 = 2\epsilon \sum_{j=1}^{n} \frac{a_j \psi^2 (\mu_j)}{1 - \mu_j^2 k^2}. \tag{117}$$

To find a particular integral of the nonhomogeneous equation (113), we set

$$\phi_i = \epsilon F \psi (\cos \beta)\, h_i e^{-\tau \sec \beta} \qquad (i = \pm 1, \ldots, \pm n), \tag{118}$$

where the h_i's are constants unspecified for the present. Inserting this form for ϕ_i in equation (113), we find that

$$h_i (1 + \mu_i \sec \beta) = \epsilon \Sigma a_j h_j \psi^2 (\mu_j) + 1 . \tag{119}$$

The constants h_i must therefore be expressible in the form

$$h_i = \frac{\gamma}{1 + \mu_i \sec \beta}, \tag{120}$$

the constant γ in turn being determined by the condition

$$\gamma = \epsilon \gamma \Sigma \frac{a_j \psi^2 (\mu_j)}{1 + \mu_j \sec \beta} + 1, \tag{121}$$

or

$$\gamma = \frac{1}{1 - 2\epsilon \sum_{j=1}^{n} \frac{a_j \psi^2 (\mu_j)}{1 - \mu_j^2 \sec^2 \beta}}. \tag{122}$$

By arguments similar to those adopted in the reduction of analogous equations (VIII, eq. [40] and eq. [40] in the preceding section) it can be shown that the formula for γ can be reduced to the form

$$\gamma = \frac{1}{\mu_1^2 \ldots \mu_n^2} \frac{P(\cos \beta) P(-\cos \beta)}{R(\cos \beta) R(-\cos \beta)}, \tag{123}$$

where it should be noted that

$$R(\mu) = \prod_{a=1}^{n} (1 - k_a \mu) \tag{124}$$

has to be evaluated in terms of the characteristic roots of the system under consideration.

[9] An exceptional case may arise if

$$\sum_{j=1}^{n} a_j \psi^2 (\mu_j) = \frac{1}{2\epsilon},$$

when $k^2 = 0$ will be a root of the characteristic equation. However, this is not likely to happen in practice. And, even if it does, it can be shown that our final solution (141) for the angular distribution of the emergent radiation will continue to be valid.

When we return to equations (118) and (120), it is seen that equation (113) admits the particular integral

$$\phi_i = \frac{\epsilon\gamma F\psi(\cos\beta)}{1+\mu_i \sec\beta} e^{-\tau\sec\beta} \qquad (i=\pm 1, \ldots, \pm n) . \quad (125)$$

Adding to this particular integral the general solution of the homogeneous system (115) which is compatible with the boundedness of the solution for $\tau \to \infty$, we have

$$\phi_i = \epsilon F\psi(\cos\beta) \left[\sum_{a=1}^{n} \frac{M_a e^{-k_a\tau}}{1+\mu_i k_a} + \frac{\gamma e^{-\tau\sec\beta}}{1+\mu_i \sec\beta} \right] \qquad (i=\pm 1, \ldots, \pm n) , \quad (126)$$

where the M_a's ($a = 1, \ldots, n$) are n constants of integration, to be determined from the boundary conditions at $\tau = 0$, namely, that here

$$\phi_{-i} = 0 \qquad (i=1, \ldots, n) . \quad (127)$$

In terms of the function

$$G(\mu) = \sum_{a=1}^{n} \frac{M_a}{1-\mu k_a} + \frac{\gamma}{1-\mu\sec\beta} , \quad (128)$$

the boundary conditions are

$$G(\mu_i) = 0 \qquad (i=1, \ldots, n) . \quad (129)$$

The angular distribution of the emergent radiation can also be expressed in terms of $G(\mu)$, for (cf. eqs. [111] and [126])

$$I(0,\mu) = \epsilon F\psi(\cos\beta)\psi(\mu) G(-\mu) . \quad (130)$$

We shall now show how an explicit formula for $G(\mu)$ can be found without having to solve for the constants M_a.

When we consider the function

$$(1-\mu\sec\beta) R(\mu) G(\mu) , \quad (131)$$

it is seen that it is a polynomial of degree n in μ which vanishes for $\mu = \mu_i$ ($i = 1, \ldots, n$). It cannot therefore differ from $P(\mu)$ except by a constant factor, and the constant factor can be found from a comparison of the coefficients of the highest power of μ. In this manner we find that

$$G(\mu) = (-1)^n k_1 \ldots k_n \left[\sum_{a=1}^{n} \frac{M_a}{k_a} \sec\beta + \gamma \right] \frac{P(\mu)}{R(\mu)} \frac{1}{1-\mu\sec\beta} . \quad (132)$$

On the other hand, since (cf. eq. [128])

$$\gamma = \lim_{\mu\to\cos\beta} (1-\mu\sec\beta) G(\mu) , \quad (133)$$

we have, according to equations (123) and (132),

$$\left.\begin{aligned} \frac{1}{\mu_1^2 \ldots \mu_n^2} \frac{P(-\cos\beta) P(\cos\beta)}{R(-\cos\beta) R(\cos\beta)} = (-1)^n k_1 \ldots k_n & \left[\sum_{a=1}^{n} \frac{M_a}{k_a} \sec\beta + \gamma \right] \\ & \times \frac{P(\cos\beta)}{R(\cos\beta)} . \end{aligned}\right\} \quad (134)$$

In other words,

$$(-1)^n k_1 \ldots k_n \left[\sum_{a=1}^{n} \frac{M_a}{k_a} \sec\beta + \gamma \right] = \frac{1}{\mu_1^2 \ldots \mu_n^2} \frac{P(-\cos\beta)}{R(-\cos\beta)}. \tag{135}$$

In virtue of this relation, equation (132) becomes

$$G(\mu) = \frac{1}{\mu_1^2 \ldots \mu_n^2} \frac{P(\mu) P(-\cos\beta)}{R(\mu) R(-\cos\beta)} \frac{1}{1 - \mu \sec\beta}. \tag{136}$$

Consequently, the formula giving the angular distribution of the reflected radiation can be expressed in the form (cf. eq. [130])

$$I(0, \mu) = \epsilon F \psi(\cos\beta) \psi(\mu) H(\mu) H(\cos\beta) \frac{\cos\beta}{\cos\beta + \mu}, \tag{137}$$

where (cf. eq. [109])

$$H(\mu) = \frac{(-1)^n}{\mu_1 \ldots \mu_n} \frac{P(-\mu)}{R(-\mu)}. \tag{138}$$

5. *The solution of equation (8) in the* n*th approximation.*—To apply the results of the preceding section to the solution of equation (8), we have only to set

$$\epsilon = \tfrac{1}{4} x \lambda \qquad \text{and} \qquad \psi^2(\mu) = 1 - \mu^2. \tag{139}$$

Moreover, according to equation (114), solutions must be sought in approximations *higher* than the first.

The characteristic equation is (cf. eq. [117])

$$1 = \tfrac{1}{2} x \lambda \sum_{j=1}^{n} \frac{a_j (1 - \mu_j^2)}{1 - \mu_j^2 k^2}, \tag{140}$$

and the angular distribution of the reflected radiation corresponding to the part $I^{(1)}$ of I is given by

$$I^{(1)}(0, \mu) = \tfrac{1}{4} x \lambda F \sin\vartheta \sin\beta H^{(1)}(\mu) H^{(1)}(\cos\beta) \frac{\cos\beta}{\cos\beta + \mu}. \tag{141}$$

In equation (141) we have added a superscript "1" to H to emphasize the fact that the $R(\mu)$ occurring in the definition of $H(\mu)$ (cf. eq. [138]) has to be evaluated in terms of the roots of the characteristic equation (140).

6. *Angular distribution of the reflected radiation: numerical results.*—Combining the results of the preceding sections, we can express the angular distribution of the reflected radiation in the form (cf. eqs. [90], [91], [108], and [141])

$$\left.\begin{aligned} I(0, \mu) = \tfrac{1}{4} \lambda F \Big[& H^{(0)}(\mu) H^{(0)}(\mu') \Big\{ 1 - x(1-\lambda) \Big(\frac{\sigma}{\rho}(\mu + \mu') + \mu\mu' \Big) \Big\} \\ & + x(1-\mu^2)^{\frac{1}{2}} (1-\mu'^2)^{\frac{1}{2}} H^{(1)}(\mu) H^{(1)}(\mu') \cos\varphi \Big] \frac{\mu'}{\mu + \mu'}, \end{aligned}\right\} \tag{142}$$

where we have written μ' for $\cos\beta$. For specified values of x and λ the solution becomes determinate in terms of the positive nonvanishing roots $k_a^{(0)}$ and $k_a^{(1)}$ ($a = 1, \ldots, n$) of the characteristic equations

$$1 = \lambda \sum_{j=1}^{n} \frac{a_j [1 + x(1-\lambda)\mu_j^2]}{1 - \mu_j^2 k^2} \tag{143}$$

and

$$1 = \tfrac{1}{2} x\lambda \sum_{j=1}^{n} \frac{a_j (1-\mu_j^2)}{1-\mu_j^2 k^2}, \tag{144}$$

respectively. In particular, the functions $H^{(i)}(\mu)$ $(i = 0 \text{ and } 1)$ have the representation

$$H^{(i)}(\mu) = \frac{1}{\mu_1 \ldots \mu_n} \frac{\prod_{j=1}^{n} (\mu + \mu_j)}{\prod_{\alpha=1}^{n} (1 - k_\alpha^{(i)} \mu)} \qquad (i = 0,1). \tag{145}$$

For the purposes of the practical evaluation of the roots $k_\alpha^{(0)}$ and $k_\alpha^{(1)}$ it is convenient to transform equations (143) and (144) in the following manner (cf. VII, p. 336, n. 6): Letting

$$\Delta_{2m}^{(0)} = \sum_{j=1}^{n} \frac{a_j [1 + x(1-\lambda)\mu_j^2] \mu_j^{2m}}{1-\mu_j^2 k^2} \quad \text{and} \quad \Delta_{2m}^{(1)} = \sum_{j=1}^{n} \frac{a_j (1-\mu_j^2) \mu_j^{2m}}{1-\mu_j^2 k^2}, \tag{146}$$

we readily establish the recursion formulae

$$\Delta_{2m}^{(0)} = \frac{1}{k^2} \left[\Delta_{2m-2}^{(0)} - \left\{ \frac{1}{2m-1} + \frac{x(1-\lambda)}{2m+1} \right\} \right] \qquad (m \leqslant 2n) \tag{147}$$

and

$$\Delta_{2m}^{(1)} = \frac{1}{k^2} \left[\Delta_{2m-2}^{(1)} - \frac{2}{4m^2-1} \right] \qquad (m \leqslant 2n); \tag{148}$$

these, together with the relations

$$\Delta_0^{(0)} = \frac{1}{\lambda} \quad \text{and} \quad \Delta_0^{(1)} = \frac{2}{x\lambda}, \tag{149}$$

determine the Δ's very simply. And in terms of these Δ's the characteristic equations are expressible in the form

$$\sum_{m=0}^{\infty} \Delta_{2m}^{(i)} p_{2m} = 0, \tag{150}$$

where the p_{2m}'s are the coefficients of μ^{2m} in the Legendre polynomial $P_{2n}(\mu)$. It will be noticed that, in contrast to equations (143) and (144), equation (150) does not require an explicit knowledge of the Gaussian weights and divisions.

In Tables 1 through 8 the functions $H^{(0)}(\mu)$ and $H^{(1)}(\mu)$ are tabulated for various values of the parameters which enter into them. Certain other auxiliary quantities, such as the characteristic roots, are also tabulated. Except for the case $x = 0$, all the quantities tabulated are those in the second approximation. However, for the case $x = 0$, the solutions have also been found in the third approximation. It would appear, from an inspection particularly of Table 4, that the solutions in the second approximation provide an accuracy of 1–2 per cent over the entire range of the variables.

A comparison of our results with Ambarzumian's tabulation for the case $x = 1$ indicates that his method of solving his integral equations leads to errors which exceed 5 per cent over certain ranges of the variables.

TABLE 1

THE CHARACTERISTIC ROOTS $k_1^{(0)}$ AND $k_2^{(0)}$ AND $(1-\lambda)\sigma/\rho$ FOR VARIOUS VALUES OF x AND λ

λ	$x=1.0$			$x=0.5$			$x=-0.5$			$x=-1.0$		
	$k_1^{(0)}$	$k_2^{(0)}$	$(1-\lambda)\sigma/\rho$	$k_1^{(0)}$	$k_2^{(0)}$	$(1-\lambda)\sigma/\rho$	$k_1^{(0)}$	$k_2^{(0)}$	$(1-\lambda)\sigma/\rho$	$k_1^{(0)}$	$k_2^{(0)}$	$(1-\lambda)\sigma/\rho$
0.9	0.4401	2.0534	0.1348	0.4847	2.0543	0.1259	0.5633	2.0561	0.1129	0.5987	2.0571	0.1079
0.8	0.6114	2.1394	.1477	0.6637	2.1426	.1402	0.7567	2.1491	.1285	0.7987	2.1525	.1238
0.7	0.7346	2.2299	.1438	0.7864	2.2358	.1380	0.8794	2.2481	.1286	0.9215	2.2545	.1246
0.6	0.8313	2.3243	.1318	0.8785	2.3327	.1277	0.9640	2.3503	.1206	1.0029	2.3595	.1175
0.5	0.9103	2.4219	.1153	0.9507	2.4322	.1124	1.0246	2.4535	.1074	1.0585	2.4645	.1053
0.4	0.9765	2.5223	.0956	1.0090	2.5333	.0938	1.0691	2.5559	.0906	1.0970	2.5675	.0892
0.3	1.0328	2.6249	.0738	1.0570	2.6352	.0728	1.1023	2.6564	.0710	1.1237	2.6672	.0702
0.2	1.0815	2.7291	.0503	1.0972	2.7375	.0499	1.1275	2.7544	.0491	1.1420	2.7629	.0487
0.1	1.1239	2.8347	0.0256	1.1315	2.8396	0.0256	1.1466	2.8494	0.0253	1.1540	2.8543	0.0253

TABLE 2

THE FUNCTION $H^{(0)}(\mu)$ FOR VARIOUS VALUES OF λ AND FOR $x=1.0$ IN THE SECOND APPROXIMATION

μ	$\lambda=0.9$	$\lambda=0.8$	$\lambda=0.7$	$\lambda=0.6$	$\lambda=0.5$	$\lambda=0.4$	$\lambda=0.3$	$\lambda=0.2$	$\lambda=0.1$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.148	1.121	1.100	1.082	1.066	1.051	1.037	1.024	1.012
0.2	1.275	1.221	1.180	1.145	1.115	1.088	1.064	1.041	1.020
0.3	1.387	1.306	1.246	1.197	1.155	1.117	1.084	1.054	1.026
0.4	1.488	1.380	1.302	1.240	1.187	1.141	1.100	1.064	1.031
0.5	1.579	1.445	1.350	1.276	1.214	1.160	1.114	1.072	1.034
0.6	1.663	1.503	1.393	1.307	1.237	1.177	1.125	1.079	1.037
0.7	1.739	1.555	1.430	1.334	1.257	1.191	1.134	1.084	1.040
0.8	1.810	1.602	1.463	1.358	1.274	1.203	1.143	1.089	1.042
0.9	1.876	1.645	1.493	1.380	1.289	1.214	1.150	1.094	1.044
1.0	1.937	1.684	1.520	1.399	1.303	1.223	1.156	1.097	1.046

TABLE 3

THE FUNCTION $H^{(0)}(\mu)$ FOR VARIOUS VALUES OF λ AND FOR $x=0.5$ IN THE SECOND APPROXIMATION

μ	$\lambda=0.9$	$\lambda=0.8$	$\lambda=0.7$	$\lambda=0.6$	$\lambda=0.5$	$\lambda=0.4$	$\lambda=0.3$	$\lambda=0.2$	$\lambda=0.1$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.143	1.115	1.094	1.077	1.061	1.047	1.034	1.022	1.011
0.2	1.265	1.209	1.168	1.135	1.106	1.081	1.058	1.037	1.018
0.3	1.371	1.288	1.229	1.182	1.142	1.107	1.076	1.048	1.023
0.4	1.465	1.356	1.280	1.220	1.170	1.128	1.091	1.057	1.027
0.5	1.551	1.416	1.323	1.252	1.194	1.145	1.102	1.064	1.031
0.6	1.628	1.468	1.361	1.280	1.215	1.159	1.112	1.070	1.033
0.7	1.698	1.515	1.394	1.304	1.232	1.172	1.120	1.075	1.036
0.8	1.763	1.557	1.424	1.325	1.247	1.182	1.127	1.080	1.038
0.9	1.823	1.595	1.450	1.344	1.260	1.192	1.133	1.083	1.039
1.0	1.878	1.629	1.474	1.361	1.272	1.200	1.139	1.087	1.041

TABLE 4

THE FUNCTION $H^{(0)}(\mu)$ IN THE SECOND AND THIRD APPROXIMATIONS FOR VARIOUS VALUES OF λ AND FOR $x=0.0$*

μ	$\lambda=0.95$		$\lambda=0.9$		$\lambda=0.8$		$\lambda=0.7$		$\lambda=0.6$	
	Second Approximation	Third Approximation	Second Approximation	Third Approximation	Second Approximation	Third Approximation	Second Approximation	Third Approximation	Second Approximation	Third Approximation
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.158	1.171	1.138	1.150	1.110	1.120	1.089	1.097	1.072	1.078
0.2	1.297	1.314	1.255	1.270	1.199	1.211	1.158	1.168	1.125	1.133
0.3	1.421	1.439	1.356	1.373	1.272	1.285	1.214	1.224	1.168	1.176
0.4	1.533	1.552	1.445	1.462	1.335	1.348	1.260	1.270	1.202	1.210
0.5	1.636	1.655	1.525	1.541	1.389	1.401	1.299	1.309	1.231	1.242
0.6	1.731	1.749	1.597	1.612	1.437	1.448	1.333	1.342	1.256	1.269
0.7	1.819	1.836	1.662	1.677	1.479	1.490	1.362	1.371	1.277	1.292
0.8	1.901	1.918	1.722	1.736	1.517	1.527	1.388	1.396	1.295	1.312
0.9	1.978	1.994	1.777	1.790	1.551	1.560	1.412	1.419	1.311	1.329
1.0	2.050	2.065	1.828	1.840	1.582	1.591	1.432	1.439	1.326	1.345

μ	$\lambda=0.5$		$\lambda=0.4$		$\lambda=0.3$		$\lambda=0.2$		$\lambda=0.1$	
	Second Approximation	Third Approximation	Second Approximation	Third Approximation	Second Approximation	Third Approximation	Second Approximation	Third Approximation	Second Approximation	Third Approximation
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.056	1.062	1.043	1.047	1.031	1.034	1.020	1.022	1.009	1.010
0.2	1.098	1.104	1.074	1.079	1.052	1.056	1.033	1.036	1.016	1.017
0.3	1.130	1.136	1.097	1.102	1.069	1.072	1.043	1.045	1.021	1.022
0.4	1.155	1.161	1.116	1.120	1.081	1.084	1.051	1.053	1.024	1.025
0.5	1.176	1.182	1.131	1.135	1.092	1.094	1.057	1.059	1.027	1.028
0.6	1.194	1.199	1.143	1.147	1.100	1.103	1.062	1.064	1.029	1.030
0.7	1.209	1.214	1.154	1.157	1.107	1.110	1.067	1.068	1.031	1.032
0.8	1.222	1.227	1.163	1.166	1.113	1.115	1.070	1.072	1.033	1.034
0.9	1.234	1.238	1.171	1.174	1.119	1.121	1.074	1.075	1.034	1.035
1.0	1.244	1.248	1.178	1.181	1.123	1.125	1.076	1.078	1.036	1.036

* The characteristic roots for this case are those which have been tabulated in a different connection by C. U. Cesco, S. Chandrasekhar, and J. Sahade (*Ap. J.*, **100**, 355, 1944; esp. p. 358, Table 1). However, it should be noted that λ, as used in the present paper, is $1-\lambda$, as used in the paper just quoted.

TABLE 5

THE FUNCTION $H^{(0)}(\mu)$ FOR VARIOUS VALUES OF λ AND FOR $x=-0.5$ IN THE SECOND APPROXIMATION

μ	$\lambda=0.9$	$\lambda=0.8$	$\lambda=0.7$	$\lambda=0.6$	$\lambda=0.5$	$\lambda=0.4$	$\lambda=0.3$	$\lambda=0.2$	$\lambda=0.1$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.134	1.105	1.084	1.067	1.052	1.039	1.028	1.018	1.008
0.2	1.246	1.189	1.148	1.116	1.090	1.067	1.047	1.030	1.014
0.3	1.343	1.258	1.199	1.155	1.118	1.088	1.061	1.038	1.018
0.4	1.427	1.316	1.241	1.186	1.141	1.104	1.072	1.045	1.021
0.5	1.502	1.366	1.277	1.211	1.160	1.117	1.081	1.050	1.024
0.6	1.570	1.409	1.307	1.233	1.175	1.128	1.088	1.055	1.026
0.7	1.630	1.447	1.334	1.252	1.188	1.137	1.095	1.058	1.027
0.8	1.686	1.482	1.357	1.268	1.200	1.145	1.100	1.061	1.029
0.9	1.736	1.512	1.377	1.282	1.210	1.152	1.104	1.064	1.030
1.0	1.783	1.540	1.395	1.294	1.218	1.158	1.108	1.066	1.031

TABLE 6

THE FUNCTION $H^{(0)}(\mu)$ FOR VARIOUS VALUES OF λ AND FOR $x=-1.0$ IN THE SECOND APPROXIMATION

μ	$\lambda=0.9$	$\lambda=0.8$	$\lambda=0.7$	$\lambda=0.6$	$\lambda=0.5$	$\lambda=0.4$	$\lambda=0.3$	$\lambda=0.2$	$\lambda=0.1$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.130	1.101	1.079	1.062	1.048	1.036	1.025	1.016	1.007
0.2	1.238	1.180	1.139	1.107	1.082	1.060	1.042	1.026	1.012
0.3	1.330	1.244	1.186	1.142	1.107	1.079	1.054	1.034	1.016
0.4	1.411	1.298	1.225	1.170	1.128	1.093	1.064	1.039	1.018
0.5	1.482	1.344	1.257	1.193	1.144	1.104	1.071	1.044	1.020
0.6	1.545	1.384	1.284	1.212	1.157	1.113	1.077	1.047	1.022
0.7	1.601	1.419	1.307	1.229	1.169	1.121	1.083	1.050	1.023
0.8	1.653	1.450	1.328	1.243	1.178	1.128	1.087	1.053	1.024
0.9	1.700	1.477	1.346	1.255	1.187	1.134	1.090	1.055	1.025
1.0	1.743	1.502	1.362	1.266	1.194	1.139	1.094	1.057	1.026

TABLE 7

THE CHARACTERISTIC ROOTS $k_1^{(1)}$ AND $k_2^{(1)}$ FOR VARIOUS VALUES OF $x\lambda$

$x\lambda$	$k_1^{(1)}$	$k_2^{(1)}$	$x\lambda$	$k_1^{(1)}$	$k_2^{(1)}$	$x\lambda$	$k_1^{(1)}$	$k_2^{(1)}$	$x\lambda$	$k_1^{(1)}$	$k_2^{(1)}$
1.0	1.1212	2.4875	0.5	1.1454	2.7222	−0.1	1.1638	2.9835	−0.6	1.1742	3.1866
0.9	1.1270	2.5357	0.4	1.1491	2.7672	−0.2	1.1661	3.0251	−0.7	1.1759	3.2259
0.8	1.1323	2.5833	0.3	1.1525	2.8116	−0.3	1.1683	3.0662	−0.8	1.1775	3.2646
0.7	1.1370	2.6302	0.2	1.1556	2.8554	−0.4	1.1704	3.1068	−0.9	1.1791	3.3030
0.6	1.1414	2.6765	0.1	1.1585	2.8987	−0.5	1.1723	3.1470	−1.0	1.1805	3.3409

TABLE 8

THE FUNCTION $H^{(1)}(\mu)$ FOR VARIOUS VALUES OF $x\lambda$ IN THE SECOND APPROXIMATION

μ	$x\lambda=1.0$	$x\lambda=0.9$	$x\lambda=0.8$	$x\lambda=0.7$	$x\lambda=0.6$	$x\lambda=0.5$	$x\lambda=0.4$	$x\lambda=0.3$	$x\lambda=0.2$	$x\lambda=0.1$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.040	1.035	1.031	1.027	1.023	1.019	1.015	1.011	1.007	1.003
0.2	1.067	1.060	1.052	1.045	1.038	1.031	1.024	1.018	1.012	1.006
0.3	1.088	1.077	1.067	1.058	1.049	1.040	1.031	1.023	1.015	1.007
0.4	1.103	1.091	1.079	1.068	1.057	1.046	1.036	1.027	1.018	1.009
0.5	1.115	1.101	1.088	1.075	1.063	1.052	1.040	1.030	1.019	1.009
0.6	1.125	1.110	1.095	1.082	1.068	1.056	1.044	1.032	1.021	1.010
0.7	1.133	1.117	1.102	1.087	1.073	1.059	1.046	1.034	1.022	1.011
0.8	1.140	1.123	1.107	1.091	1.076	1.062	1.049	1.036	1.023	1.011
0.9	1.146	1.128	1.111	1.095	1.079	1.064	1.050	1.037	1.024	1.012
1.0	1.151	1.133	1.115	1.098	1.082	1.067	1.052	1.038	1.025	1.012

μ	$x\lambda=-0.1$	$x\lambda=-0.2$	$x\lambda=-0.3$	$x\lambda=-0.4$	$x\lambda=-0.5$	$x\lambda=-0.6$	$x\lambda=-0.7$	$x\lambda=-0.8$	$x\lambda=-0.9$	$x\lambda=-1.0$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.997	0.993	0.990	0.987	0.983	0.980	0.977	0.974	0.971	0.968
0.2	0.994	0.989	0.983	0.978	0.973	0.968	0.963	0.958	0.954	0.949
0.3	0.993	0.985	0.979	0.972	0.966	0.960	0.954	0.948	0.942	0.936
0.4	0.992	0.984	0.976	0.968	0.961	0.954	0.947	0.940	0.933	0.927
0.5	0.991	0.982	0.973	0.965	0.957	0.949	0.941	0.934	0.927	0.920
0.6	0.990	0.980	0.971	0.962	0.954	0.945	0.937	0.929	0.921	0.914
0.7	0.989	0.979	0.970	0.960	0.951	0.942	0.934	0.925	0.917	0.909
0.8	0.989	0.978	0.968	0.958	0.949	0.940	0.931	0.922	0.914	0.906
0.9	0.989	0.978	0.967	0.957	0.947	0.938	0.928	0.920	0.911	0.903
1.0	0.988	0.977	0.966	0.956	0.946	0.936	0.926	0.917	0.908	0.900

II. DIFFUSE REFLECTION IN ACCORDANCE WITH RAYLEIGH'S PHASE FUNCTION

7. *The reduction of the equation of transfer.*—For Rayleigh's form of the phase function the equation of transfer appropriate for the problem of diffuse reflection is (cf. eq. [2])

$$\left.\begin{aligned}\cos\vartheta\,\frac{dI(\tau,\vartheta,\varphi)}{d\tau} &= I(\tau,\vartheta,\varphi)-\frac{3}{16\pi}\int_0^{\pi}\!\!\int_0^{2\pi} I(\tau,\vartheta',\varphi')\,[1+\cos^2\vartheta\cos^2\vartheta' \\ &\quad+\tfrac{1}{2}\sin^2\vartheta\sin^2\vartheta'+2\cos\vartheta\sin\vartheta\cos\vartheta'\sin\vartheta'\cos(\varphi-\varphi') \\ &\quad+\tfrac{1}{2}\sin^2\vartheta\sin^2\vartheta'\cos 2(\varphi-\varphi')\,]\sin\vartheta' d\vartheta' d\varphi'-\tfrac{3}{16}Fe^{-\tau\sec\beta} \\ &\quad\times[1+\cos^2\vartheta\cos^2\beta+\tfrac{1}{2}\sin^2\vartheta\sin^2\beta-2\sin\vartheta\cos\vartheta\sin\beta\cos\beta\cos\varphi \\ &\quad+\tfrac{1}{2}\sin^2\vartheta\sin^2\beta\cos 2\varphi]\,.\end{aligned}\right\}\tag{151}$$

The form of equation (151) suggests that we seek a solution in the form

$$I(\tau,\vartheta,\varphi)=I^{(0)}(\tau,\vartheta)+I^{(1)}(\tau,\vartheta)\cos\varphi+I^{(2)}(\tau,\vartheta)\cos 2\varphi\,. \tag{152}$$

Inserting the foregoing form for $I(\tau, \vartheta, \varphi)$ in equation (151), we find that it breaks up into three equations, one for each of the functions $I^{(0)}$, $I^{(1)}$, and $I^{(2)}$. These are

$$\left.\begin{aligned} \mu \frac{dI^{(0)}}{d\tau} = I^{(0)} - \frac{3}{16}\Big[\int_{-1}^{+1} I^{(0)}(\tau, \mu')(3-\mu'^2)\,d\mu' \\ + \mu^2 \int_{-1}^{+1} I^{(0)}(\tau, \mu')(3\mu'^2 - 1)\,d\mu'\Big] - \tfrac{3}{32} F e^{-\tau \sec\beta}[(3-\cos^2\beta) \\ + \mu^2(3\cos^2\beta - 1)], \end{aligned}\right\} \quad (153)$$

$$\left.\begin{aligned} \mu \frac{dI^{(1)}}{d\tau} = I^{(1)} - \tfrac{3}{8}\mu(1-\mu^2)^{\frac{1}{2}} \int_{-1}^{+1} I^{(1)}(\tau, \mu')\mu'(1-\mu'^2)^{\frac{1}{2}} d\mu' \\ + \tfrac{3}{8} F \sin\beta \cos\beta\, e^{-\tau\sec\beta}\mu(1-\mu^2)^{\frac{1}{2}}, \end{aligned}\right\} \quad (154)$$

and

$$\left.\begin{aligned} \mu \frac{dI^{(2)}}{d\tau} = I^{(2)} - \tfrac{3}{32}(1-\mu^2) \int_{-1}^{+1} I^{(2)}(\tau, \mu')(1-\mu'^2)\,d\mu' \\ - \tfrac{3}{32} F \sin^2\beta\, e^{-\tau\sec\beta}(1-\mu^2). \end{aligned}\right\} \quad (155)$$

8. *The solution of equations (153), (154), and (155) in the* n*th approximation.*—Considering equation (153) first, the equivalent system of linear equations in the nth approximation is

$$\left.\begin{aligned} \mu_i \frac{dI_i^{(0)}}{d\tau} = I_i^{(0)} - \tfrac{3}{16}[\Sigma a_j(3-\mu_j^2) I_j^{(0)} + \mu_i^2 \Sigma a_j (3\mu_j^2 - 1) I_j^{(0)}] \\ - \tfrac{3}{32} F e^{-\tau\sec\beta}[(3-\cos^2\beta) + \mu_i^2(3\cos^2\beta - 1)] \qquad (i = \pm 1, \ldots, \pm n), \end{aligned}\right\} \quad (156)$$

where the various symbols have their usual meanings.

It is seen that the homogeneous system associated with equation (156) is the same as that considered in paper III, §§ 3–5. Accordingly, the complementary function of equation (156) is the same as the general solution (III, eq. [36]) of the homogeneous system. Accordingly, to complete the solution we need only find a particular integral. This can be found in the following manner:

Setting

$$I_i^{(0)} = \tfrac{3}{32} F h_i e^{-\tau\sec\beta} \qquad (i = \pm 1, \ldots, \pm n) \quad (157)$$

in equation (156) (the h_i's are certain constants unspecified for the present), we verify that we must have

$$\left.\begin{aligned} h_i(1+\mu_i\sec\beta) = [\tfrac{3}{16}\Sigma a_j h_j(3-\mu_j^2) + 3 - \cos^2\beta] \\ + \mu_i^2[\tfrac{3}{16}\Sigma a_j h_j(3\mu_j^2 - 1) + 3\cos^2\beta - 1]. \end{aligned}\right\} \quad (158)$$

The foregoing equation implies that the constants h_i must be expressible in the form

$$h_i = \frac{\delta - \mu_i^2\gamma}{1+\mu_i\sec\beta} \qquad (i = \pm 1, \ldots, \pm n), \quad (159)$$

where the constants γ and δ have to be determined in accordance with the relation

$$\left.\begin{aligned} \delta - \mu_i^2\gamma = \{\tfrac{3}{16}[\delta(3E_0 - E_2) - \gamma(3E_2 - E_4)] + 3 - \cos^2\beta\} \\ + \mu_i^2\{\tfrac{3}{16}[\delta(3E_2 - E_0) - \gamma(3E_4 - E_2)] + 3\cos^2\beta - 1\} \end{aligned}\right\} \quad (160)$$

and where the E's have the same meaning as in equation (37). From equation (160) we conclude that the equations which determine γ and δ are

$$\delta = \tfrac{3}{16} [\delta (3E_0 - E_2) - \gamma (3E_2 - E_4)] + 3 - \cos^2 \beta \tag{161}$$

and

$$\gamma = \tfrac{3}{16} [\gamma (3E_4 - E_2) - \delta (3E_2 - E_0)] - (3 \cos^2 \beta - 1) . \tag{162}$$

Solving these equations, we find that

$$\gamma = \frac{1}{1 - \frac{3}{8} \sum_{j=1}^{n} \frac{a_j (3 - \mu_j^2)}{1 - \mu_j^2 \sec^2 \beta}} \tag{163}$$

and

$$\delta = 3\gamma . \tag{164}$$

In reducing the solutions for γ and δ to the foregoing forms, repeated use has been made of the recursion formula (42) which the E's satisfy.

It is seen that the formula (163) for γ bears the same relation to the characteristic equation (III, eq. [30])

$$1 = \frac{3}{8} \sum_{j=1}^{n} \frac{a_j (3 - \mu_j^2)}{1 - \mu_j^2 k^2} \tag{165}$$

as the γ defined in paper VIII, equation (40), bears to the corresponding characteristic equation (10) of the same paper. We can, accordingly, express γ in the form (cf. eqs. [52] and [123])

$$\gamma = \frac{1}{\mu_1^2 \ldots \mu_n^2} \frac{P(\cos \beta) P(-\cos \beta)}{R(\cos \beta) R(-\cos \beta)} , \tag{166}$$

where

$$R(\mu) = \prod_{a=1}^{n-1} (1 - k_a \mu) \tag{167}$$

has naturally to be evaluated in terms of the $(n - 1)$ nonvanishing positive roots of the equation (165).

Returning to equation (157), we now express the particular integral in the form

$$I_i^{(0)} = \tfrac{3}{32} F e^{-\tau \sec \beta} \gamma \frac{3 - \mu_i^2}{1 + \mu_i \sec \beta} \qquad (i = \pm 1, \ldots , \pm n) . \tag{168}$$

Adding to this particular integral the general solution of the homogeneous system which is compatible with the boundedness of the solution for $\tau \rightarrow \infty$, we have

$$\left. \begin{aligned} I_i^{(0)} = \tfrac{3}{32} F \Big[(3 - \mu_i^2) \sum_{a=1}^{n-1} \frac{M_a e^{-k_a \tau}}{1 + \mu_i k_a} + X + \gamma \frac{3 - \mu_i^2}{1 + \mu_i \sec \beta} e^{-\tau \sec \beta} \Big] \\ (i = \pm 1, \ldots , \pm n) , \end{aligned} \right\} \tag{169}$$

where the constants M_a ($a = 1, \ldots , n - 1$) and X have to be determined from the boundary conditions at $\tau = 0$.

At $\tau = 0$ we must require that

$$I_i^{(0)} = 0 \qquad (i = 1, \ldots , n, \tau = 0) . \tag{170}$$

In terms of the function

$$G(\mu) = (3-\mu^2)\sum_{\alpha=1}^{n-1}\frac{M_\alpha}{1-\mu k_\alpha} + X + (3-\mu^2)\frac{\gamma}{1-\mu\sec\beta} \tag{171}$$

the boundary conditions are

$$G(\mu_i) = 0 \qquad (i = 1, \ldots, n). \tag{172}$$

The angular distribution of the reflected radiation corresponding to the part $I^{(0)}$ of I (cf. eq. [152]) is also expressible in terms of $G(\mu)$. We have

$$I^{(0)}(0,\mu) = \tfrac{3}{32}FG(-\mu). \tag{173}$$

As in the earlier cases, we shall now show how an explicit formula for $G(\mu)$ can be found without having the necessity to solve for the constants M_α and X.

Consider the function

$$(1-\mu\sec\beta)R(\mu)G(\mu) = (1-\mu\sec\beta)\prod_{\alpha=1}^{n-1}(1-k_\alpha\mu)G(\mu). \tag{174}$$

This is a polynomial of degree $n+1$ in μ which vanishes for $\mu = \mu_i$, $i = 1, \ldots, n$. Consequently, there must exist a proportionality of the form

$$(1-\mu\sec\beta)R(\mu)G(\mu) \propto P(\mu)(\mu+c), \tag{175}$$

where c is some constant. The constant of proportionality can be found from a comparison of the coefficients of the highest powers of μ on either side. In this manner we obtain

$$G(\mu) = (-1)^n k_1 \ldots k_{n-1}\left[\sec\beta\sum_{\alpha=1}^{n-1}\frac{M_\alpha}{k_\alpha}+\gamma\right]\frac{P(\mu)(\mu+c)}{R(\mu)(1-\mu\sec\beta)}, \tag{176}$$

On the other hand, since

$$\gamma(3-\cos^2\beta) = \lim_{\mu\to\cos\beta}(1-\mu\sec\beta)G(\mu), \tag{177}$$

we have, according to equations (166) and (176),

$$\left.\begin{aligned}\frac{3-\cos^2\beta}{\mu_1^2\ldots\mu_n^2}\frac{P(\cos\beta)P(-\cos\beta)}{R(\cos\beta)R(-\cos\beta)} &= (-1)^n k_1\ldots k_{n-1}\left[\sec\beta\sum_{\alpha=1}^{n-1}\frac{M_\alpha}{k_\alpha}+\gamma\right]\\ &\quad\times\frac{P(\cos\beta)}{R(\cos\beta)}(\cos\beta+c).\end{aligned}\right\} \tag{178}$$

In other words,

$$(-1)^n k_1\ldots k_{n-1}\left[\sec\beta\sum_{\alpha=1}^{n-1}\frac{M_\alpha}{k_\alpha}+\gamma\right] = \frac{3-\cos^2\beta}{\mu_1^2\ldots\mu_n^2}\frac{P(-\cos\beta)}{R(-\cos\beta)}\frac{1}{\cos\beta+c}. \tag{179}$$

In virtue of this relation, equation (176) becomes

$$G(\mu) = \frac{1}{\mu_1^2\ldots\mu_n^2}\frac{P(\mu)P(-\cos\beta)}{R(\mu)R(-\cos\beta)}\frac{(3-\cos^2\beta)(\mu+c)}{(1-\mu\sec\beta)(\cos\beta+c)}. \tag{180}$$

It remains only to determine c.

From equation (171) it follows that

$$X = G(+\sqrt{3}) = G(-\sqrt{3}). \tag{181}$$

With $G(\mu)$ given by equation (180), the foregoing equation reduces to

$$\frac{P(+\sqrt{3})}{R(+\sqrt{3})}\,\frac{c+\sqrt{3}}{\cos\beta-\sqrt{3}}=\frac{P(-\sqrt{3})}{R(-\sqrt{3})}\,\frac{c-\sqrt{3}}{\cos\beta+\sqrt{3}}. \tag{182}$$

Solving for c, we find

$$c=-\sqrt{3}\,\frac{A(\sqrt{3}+\cos\beta)-B(\sqrt{3}-\cos\beta)}{A(\sqrt{3}+\cos\beta)+B(\sqrt{3}-\cos\beta)}, \tag{183}$$

where we have written

$$A=\frac{P(+\sqrt{3})}{R(+\sqrt{3})}\quad\text{and}\quad B=\frac{P(-\sqrt{3})}{R(-\sqrt{3})}. \tag{184}$$

With this value of c we verify that

$$\cos\beta+c=-\frac{(A-B)(3-\cos^2\beta)}{A(\sqrt{3}+\cos\beta)+B(\sqrt{3}-\cos\beta)} \tag{185}$$

and

$$\left.\begin{aligned}(3-\cos^2\beta)\frac{\mu+c}{\cos\beta+c}=\Big[\frac{A}{A-B}&(\sqrt{3}+\cos\beta)(\sqrt{3}-\mu)\\ &-\frac{B}{A-B}(\sqrt{3}-\cos\beta)(\sqrt{3}+\mu)\Big].\end{aligned}\right\} \tag{186}$$

The formula (180) for $G(\mu)$ becomes

$$\left.\begin{aligned}G(\mu)&=\frac{1}{\mu_1^2\ldots\ldots\mu_n^2}[\xi(\sqrt{3}+\cos\beta)(\sqrt{3}-\mu)\\ &+(1-\xi)(\sqrt{3}-\cos\beta)(\sqrt{3}+\mu)]\times\frac{P(-\cos\beta)P(\mu)}{R(-\cos\beta)R(\mu)}\,\frac{1}{1-\mu\sec\beta},\end{aligned}\right\} \tag{187}$$

where

$$\xi=\frac{A}{A-B}. \tag{188}$$

With $G(\mu)$ given by equation (187), the angular distribution of the reflected radiation corresponding to $I^{(0)}$ can be written in the form

$$\left.\begin{aligned}I^{(0)}(0,\mu)&=\tfrac{3}{32}FH^{(0)}(\mu)\,H^{(0)}(\cos\beta)\frac{\cos\beta}{\cos\beta+\mu}\\ &\times[\xi(\sqrt{3}+\cos\beta)(\sqrt{3}+\mu)+(1-\xi)(\sqrt{3}-\cos\beta)(\sqrt{3}-\mu)].\end{aligned}\right\} \tag{189}$$

This completes the solution of equation (153).

Turning our attention next to equations (154) and (155), we observe that both these equations belong to the general type considered in § 4. We can, therefore, write down at once the expressions governing the angular distribution of the emergent radiations $I^{(1)}(0,\mu)$ and $I^{(2)}(0,\mu)$. We have (cf. eq. [137])

$$I^{(1)}(0,\mu)=-\tfrac{3}{8}F\sin\beta\cos\beta\sin\vartheta\cos\vartheta H^{(1)}(\mu)\,H^{(1)}(\cos\beta)\frac{\cos\beta}{\cos\beta+\mu} \tag{190}$$

and

$$I^{(2)}(0,\mu)=\tfrac{3}{32}F\sin^2\beta\sin^2\vartheta H^{(2)}(\mu)\,H^{(2)}(\cos\beta)\frac{\cos\beta}{\cos\beta+\mu}, \tag{191}$$

where $H^{(1)}(\mu)$ and $H^{(2)}(\mu)$ are to be evaluated in terms of the positive nonvanishing roots of the equations

$$1=\frac{3}{4}\sum_{j=1}^{n}\frac{a_j(1-\mu_j^2)\mu_j^2}{1-k^2\mu_j^2} \tag{192}$$

and

$$1 = \frac{3}{16} \sum_{j=1}^{n} \frac{a_j (1 - \mu_j^2)^2}{1 - k^2 \mu_j^2}, \tag{193}$$

respectively.[10]

9. *The angular distribution of the reflected radiation: numerical results.*—Combining the solutions (189), (190), and (191) in accordance with equation (152), we obtain the angular distribution of the reflected radiation. We have

$$\left.\begin{aligned} I(0, \mu) = {} & \tfrac{3}{32} F \left[H^{(0)}(\mu) H^{(0)}(\mu') \left\{ \xi (\sqrt{3} + \mu)(\sqrt{3} + \mu') + (1 - \xi) \right.\right. \\ & \left. \times (\sqrt{3} - \mu)(\sqrt{3} - \mu') \right\} - 4 \mu \mu' (1 - \mu^2)^{\frac{1}{2}} (1 - \mu'^2)^{\frac{1}{2}} H^{(1)}(\mu) H^{(1)}(\mu') \cos \varphi \\ & \left. + (1 - \mu^2)(1 - \mu'^2) H^{(2)}(\mu) H^{(2)}(\mu') \cos 2\varphi \right] \frac{\mu'}{\mu + \mu'}, \end{aligned}\right\} \tag{194}$$

where we have written μ' for $\cos \beta$.

TABLE 9

THE CONSTANTS OCCURRING IN THE FORMULA FOR THE ANGULAR DISTRIBUTION OF THE RADIATION REFLECTED FROM A SEMI-INFINITE PLANE-PARALLEL ATMOSPHERE IN ACCORDANCE WITH RAYLEIGH'S LAW IN VARIOUS APPROXIMATIONS

Second Approximation	Third Approximation	Fourth Approximation
$k_1^{(0)} = 1.870829$	$k_1^{(0)} = 3.08624$	$k_1^{(0)} = 4.337235$
	$k_2^{(0)} = 1.20629$	$k_2^{(0)} = 1.562180$
		$k_3^{(0)} = 1.096117$
$\xi = 0.29926$	$\xi = 0.29646$	$\xi = 0.29561$
$k_1^{(1)} = 2.86760$	$k_1^{(1)} = 4.15155$	
$k_2^{(1)} = 1.13000$	$k_2^{(1)} = 1.46094$	
	$k_3^{(1)} = 1.06316$	
$k_1^{(2)} = 2.79728$	$k_1^{(2)} = 4.02457$	
$k_2^{(2)} = 1.15840$	$k_2^{(2)} = 1.49449$	
	$k_3^{(2)} = 1.07209$	

[10] In terms of the quantities

$$\Delta_{2m}^{(1)} = \sum_{j=1}^{n} \frac{a_j (1 - \mu_j^2) \mu_j^{2m+2}}{1 - k^2 \mu_j^2} \quad \text{and} \quad \Delta_{2m}^{(2)} = \sum_{j=1}^{n} \frac{a_j (1 - \mu_j^2)^2 \mu_j^{2m}}{1 - k^2 \mu_j^2}$$

the equations (192) and (193) can be reduced to the form (cf. eq. [150])

$$\sum_{m=0}^{n} \Delta_{2m}^{(i)} p_{2m} = 0,$$

where the p_{2m}'s have the same meanings as in eq. (150). The Δ's themselves can be evaluated simply from the recurrence formulae

$$\Delta_{2m}^{(1)} = \frac{1}{k^2} \left[\Delta_{2m-2}^{(1)} - \frac{2}{(2m+1)(2m+3)} \right],$$

$$\Delta_{2m}^{(2)} = \frac{1}{k^2} \left[\Delta_{2m-2}^{(2)} - \frac{8}{(2m-1)(2m+1)(2m+3)} \right],$$

and the relations

$$\Delta_0^{(1)} = \tfrac{4}{3} \quad \text{and} \quad \Delta_0^{(2)} = \tfrac{16}{3}.$$

In Table 9 the various constants which occur in the foregoing solution in the first four approximations are collected together. In Table 10 the function $H^{(0)}(\mu)$ in the second, third, and fourth approximations is tabulated, while in Table 11 the functions $H^{(1)}(\mu)$ and $H^{(2)}(\mu)$ are tabulated in the second and the third approximations. An inspection of these tables reveals that the accuracy of the third approximation is within 1 per cent over the entire range in which the functions are defined.

TABLE 10

THE FUNCTION $H^{(0)}(\mu)$ IN SECOND, THIRD, AND FOURTH APPROXIMATIONS (Rayleigh's Case)

μ	Second Approximation	Third Approximation	Fourth Approximation
0	1.000	1.000	1.000
0.1	1.217	1.233	1.242
0.2	1.424	1.448	1.460
0.3	1.626	1.653	1.665
0.4	1.823	1.853	1.864
0.5	2.018	2.048	2.060
0.6	2.210	2.241	2.252
0.7	2.401	2.432	2.443
0.8	2.591	2.622	2.632
0.9	2.779	2.810	2.820
1.0	2.967	2.998	3.008

TABLE 11

THE FUNCTION $H^{(1)}(\mu)$ AND $H^{(2)}(\mu)$ IN THE SECOND AND THIRD APPROXIMATIONS (Rayleigh's Case)

μ	$H^{(1)}(\mu)$		$H^{(2)}(\mu)$	
	Second Approximation	Third Approximation	Second Approximation	Third Approximation
0	1.000	1.000	1.000	1.000
0.1	1.008	1.008	1.011	1.013
0.2	1.014	1.014	1.019	1.021
0.3	1.019	1.018	1.024	1.026
0.4	1.022	1.022	1.028	1.030
0.5	1.025	1.024	1.031	1.033
0.6	1.028	1.027	1.033	1.035
0.7	1.029	1.029	1.035	1.037
0.8	1.031	1.030	1.037	1.038
0.9	1.033	1.032	1.038	1.040
1.0	1.034	1.033	1.039	1.041

As stated in the introduction, applications of the solutions obtained in this paper to the problem of planetary illumination will be found in a forthcoming paper.

I wish to record my indebtedness to Mrs. Frances H. Breen for assistance with the numerical work.

APPENDIX

The methods used in §8 enable us to complete the explicit solution of the constants of integration and the law of darkening for the problem considered in paper III. Thus, considering the function (III, eq. [55])

$$S(\mu) = (3-\mu^2)\sum_{a=1}^{n-1}\frac{L_a}{1-\mu k_a}+Q-\mu\,, \tag{1}$$

we have already shown that (cf. III, eqs. [59] and [60])

$$S(\mu) = \sigma\frac{P(\mu)}{R(\mu)}\,, \tag{2}$$

where

$$\sigma = (-1)^n k_1 \ldots . k_{n-1}\left(1-\sum_{a=1}^{n-1}\frac{L_a}{k_a}\right) \tag{3}$$

is a constant and $P(\mu)$ and $R(\mu)$ are defined as in III, equations (57) and (58). In paper III the constant σ was left undetermined. We shall now show how σ can also be determined.

Setting $\mu = +\sqrt{3}$, respectively, $-\sqrt{3}$ in equations (1) and (2), we have (cf. eq. [186])

$$Q-\sqrt{3} = \sigma\frac{P(+\sqrt{3})}{R(+\sqrt{3})} = \sigma A\,, \tag{4}$$

and

$$Q+\sqrt{3} = \sigma\frac{P(-\sqrt{3})}{R(-\sqrt{3})} = \sigma B\,. \tag{5}$$

Solving these equations, we find

$$Q = \sqrt{3}\,\frac{B+A}{B-A}\,, \tag{6}$$

and

$$\sigma = \frac{2\sqrt{3}}{B-A}\,. \tag{7}$$

With σ thus determined, the constants L_a also become determinate according to III, equation (63).

The law of darkening takes the form

$$I(0,\mu) = \tfrac{3}{4}FS(-\mu) = \frac{3\sqrt{3}}{2(B-A)}\,F\,\frac{P(-\mu)}{R(-\mu)}\,, \tag{8}$$

or, introducing the function $H^{(0)}(\mu)$ as in equation (189), we can write

$$I(0,\mu) = \lambda F H^{(0)}(\mu)\,, \tag{9}$$

where

$$\lambda = (-1)^n \mu_1 \ldots . \mu_n\,\frac{3\sqrt{3}}{2(B-A)}\,. \tag{10}$$

In the second, third, and fourth approximations λ takes the values

$$\left.\begin{aligned} \lambda &= 0.42064 \quad \text{(second approximation)}\,,\\ &= 0.41950 \quad \text{(third approximation)}\,,\\ &= 0.41916 \quad \text{(fourth approximation)}\,. \end{aligned}\right\} \tag{11}$$

STELLAR MODELS WITH PARTIALLY DEGENERATE ISOTHERMAL CORES AND POINT-SOURCE ENVELOPES

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ABSTRACT

Stellar models, consisting of partially degenerate isothermal cores and point-source envelopes, are considered in which the ratio of the mean molecular weights in the core and the envelope takes two values, namely, 1 and 2. The results presented in this paper do not agree with those of a similar calculation attempted by Gamow and Keller and do not support their suggestion that allowing for the degeneracy of the isothermal core devoid of hydrogen will significantly alter the early course of evolution of the main-sequence stars.

1. *Introduction.*—In their discussion of the evolution of the main-sequence stars M. Schönberg and S. Chandrasekhar[1] conclude, on the basis of their study of stellar models with isothermal cores, that there is an upper limit to the fraction of the mass which can exist in the form of isothermal cores devoid of hydrogen. On the strength of this result they say that "it is difficult to escape the conclusion that beyond this point [where the core devoid of hydrogen has increased to its maximum size] the star must evolve through nonequilibrium configurations." But it has recently been claimed by G. Gamow[2] that this is a "paradoxical result" which arises only because of the "arbitrary assumption" made by Schönberg and Chandrasekhar that the isothermal core is gaseous and that, if once this assumption is dropped to allow for the possible degeneracy of the core, the isothermal regions can spread through the whole star. Indeed, G. Gamow and G. Keller[3] believe that they have shown that stellar models consisting of partially degenerate isothermal cores and radiative envelopes can be constructed which will account for the giants and their energy production in terms of the carbon cycle. These conclusions are at such variance with those of Schönberg and Chandrasekhar that we have re-examined the problem *de novo* and find that we cannot substantiate the claims of either Gamow or of Gamow and Keller. But before we enter into the details of the construction of the models, it is of interest to examine the problem in a general way and see why it is that allowing for the possible degeneracy of the core cannot affect in any significant way the conclusions of Schönberg and Chandrasekhar.[4]

According to the calculations of Schönberg and Chandrasekhar, the central temperature and the density of the configuration which incloses the maximum fraction (~ 10 per cent) of the mass in the isothermal core (assumed gaseous) are given by

$$T_c = 0.713\, \frac{\mu_c H}{k}\, \frac{GM}{R} \tag{1}$$

and

$$\rho_c = 3210\, \frac{3M}{4\pi R^3}, \tag{2}$$

[1] *Ap. J.*, **96**, 161, 1942. [2] *Phys. Rev.*, **67**, 120, 1945. [3] *Rev. Mod. Phys.*, **17**, 125, 1945.

[4] I am indebted to Dr. S. Chandrasekhar for the discussion which follows.

where the various symbols have their usual meaning. It is now clear that the assumption of the gaseous nondegenerate nature of the core will be a valid one if the pressure computed according to the perfect gas formula

$$P_{\text{nondeg}} = \frac{k}{\mu_c H} \rho_c T_c \tag{3}$$

exceeds that given by the degenerate formula[5]

$$P_{\text{deg}} = \frac{K_1}{\mu_c^{5/3}} \rho_c^{5/3} , \tag{4}$$

where

$$K_1 = \frac{1}{20}\left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{m_e H^{5/3}} = 9.91 \times 10^{12} \tag{5}$$

is the unrelativistic degenerate constant. (It will be noted that we are ignoring the relativistic modification of the formula giving the pressure of a degenerate case. This is not of importance in the present context. But it is of importance in another connection, to which we shall presently make reference.)

The validity of the assumption concerning the nondegeneracy of the core requires, then, that

$$\frac{k}{\mu_c H} \rho_c T_c > \frac{K_1}{\mu_c^{5/3}} \rho_c^{5/3} . \tag{6}$$

Substituting for ρ_c and T_c from equations (1) and (2), the foregoing inequality reduces to

$$\left(\frac{M}{\odot}\right)^{1/3} \frac{R}{R_\odot} > \frac{2.00}{\mu_c^{5/3}} . \tag{7}$$

On the other hand, if a temperature of 20,000,000 K° is accepted as a criterion for the operation of the carbon cycle, we should have

$$2 \times 10^7 = 0.713 \frac{\mu_c H}{k} \frac{GM}{R} . \tag{8}$$

Eliminating R between the relations (7) and (8), we find

$$\frac{M}{\odot} > \frac{1.95}{\mu_c^2} = 0.5 \qquad (\mu_c = 2) . \tag{9}$$

In other words, for $T_c = 2.0 \times 10^7$ degrees (K), degeneracy cannot be a significant factor in the evolution of stars of masses greater than $\frac{1}{2}\odot$ *during* the stage of the burning of the hydrogen. For stars of small mass ($M < 0.5\ \odot$), incipient degeneracy will occur already during the early stages. This is, of course, well known from the study of the M dwarfs themselves and is not of any special relevance in our present context.

Turning next to a somewhat different aspect of the same problem,[6] we first recall that the maximum mass in hydrostatic equilibrium which can be degenerate is $6.6\odot\mu_e^{-2}$. *Under no circumstances can degeneracy spread over a larger mass.* For $\mu_e \sim \mu_c = 2$ this is $1.65\odot$; and, accordingly, for stars of large mass ($M > 3 - 4\odot$), degeneracy can *never* spread over more than a fraction of its mass. This, again, is a well-known result; but it should be emphasized in this connection that, in order that we may have $1.65\odot$ in the

[5] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure*, p. 421, Chicago: University of Chicago Press, 1939.

[6] Chandrasekhar, *op. cit.*, p. 438. Here μ_e is the mean molecular weight appropriate for computing the pressure of the electron gas. Since we are dealing with situations in which no hydrogen is present, we need not distinguish between μ_c and μ_e.

degenerate core, the density throughout the core must (normally!) exceed 10^8 gm/cm^3. Such configurations must be, accordingly, of white-dwarf dimensions and can therefore have no bearing on the structure and constitution of the giant stars.

When the two lines of arguments presented in the preceding paragraphs are combined, it is evident that degeneracy as a factor in the evolution of the main-sequence stars with masses exceeding $0.75\odot$ can play no significant role during the early stages, when hydrogen is being burned. This situation is slightly different for stars of small masses ($M < 0.5\odot$). But, as we have already pointed out, these stars are already incipiently degenerate on the main sequence. Evolution of these stars is, therefore, principally one along a sequence in which the transition is directly to the white-dwarf stages. Since degeneracy is an important factor, temperature is irrelevant to the situation, and the isothermal character of the core devoid of hydrogen plays a minor role. It is thus apparent that, where the isothermal character of the core is significant, degeneracy ceases to play a role. There is thus a mutual exclusiveness of the two factors.

In spite of what we have said in the preceding paragraphs, it is definitely of interest to study stellar models in which we make allowance for the possible degeneracy of the core. These models are studied in this paper. But we may draw attention, even at this stage, to the fact that our results are in disagreement with those of Gamow and Keller. We are unable to follow their method of fitting in detail, but we feel that there can be no doubt about the correctness of our results.[7]

2. *The equations of the models.*—In the absence of relativity effects the Fermi-Dirac equation of state of the partially degenerate isothermal core with negligible radiation pressure can be expressed parametrically in the form

$$\rho = \frac{4\pi\mu_e H\,(2mkT)^{3/2}}{h^3} F_{1/2}(\phi)\ , \tag{10}$$

and

$$P = \frac{4\pi\,(2m)^{3/2}\,(kT)^{5/2}}{h^3}\,\mathfrak{F}_{3/2}(\phi)\ , \tag{11}$$

where m is the mass of the electron; h, the Planck constant; H, the mass of the proton; k, the Boltzmann gas constant; μ_e, the molecular weight of an electron gas; and $F_{1/2}(\phi)$, the Fermi-Dirac[8] functions defined by

$$F_\nu(\phi) = \int_0^\infty \frac{u^\nu du}{e^{-\phi+u}+1} \tag{12}$$

and

$$\tfrac{2}{3}F_{3/2}(\phi) = \mathfrak{F}_{3/2}(\phi)\ , \tag{13}$$

where ϕ is a measure of the degree of degeneracy.

With the foregoing equation of state the equation of equilibrium

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho \tag{14}$$

can be reduced to the form

$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2\frac{d\phi}{d\eta}\right) = -F_{1/2}(\phi)\ , \tag{15}$$

where

$$r = \alpha_1\eta\ , \qquad \text{and} \qquad \alpha_1 = \left[\frac{h^3}{G\,(4\pi\mu_e H)^2\,(2m)^{3/2}\,(kT)^{1/2}}\right]^{1/2}. \tag{16}$$

[7] One error to which we may draw attention is that the formula (18) quoted by Gamow and Keller for the pressure of a degenerate gas is incorrect by a factor 8, but this may be an error in the type.

[8] J. McDougall and E. C. Stoner, *Phil. Trans.*, R.S., A, **237**, 67, 1938.

The mass of the material inclosed up to a radius η is given by

$$M(\eta) = 4\pi \int_0^r \rho r^2 d r = 4\pi a_1^3 \int_0^\eta \rho \eta^2 d\eta . \tag{17}$$

In order to express ρ in terms of ϕ and η we make use of equations (10) and (15); thus

$$\rho = -\frac{4\pi\mu_e H (2mkT)^{3/2}}{h^3} \frac{1}{\eta^2} \frac{d}{d\eta}\left(\eta^2 \frac{d\phi}{d\eta}\right). \tag{18}$$

Hence the required mass relation is

$$M(\eta) = -\frac{(4\pi)^2 a_1^3 H \mu_e (2mkT)^{3/2}}{h^3} \eta^2 \frac{d\phi}{d\eta}. \tag{19}$$

The standard equations of equilibrium for the point-source envelope,

$$\frac{k}{\mu H} \frac{d}{d r}(\rho T) = -\frac{GM(r)\rho}{r^2}; \qquad \frac{dM(r)}{d r} = 4\pi\rho r^2 , \tag{20}$$

and

$$\frac{a}{3} \frac{d}{d r}(T^4) = -\frac{\kappa_0 L}{4\pi c r^2} \frac{\rho^2}{T^{3.5}}, \tag{21}$$

where the law of opacity is the conventional law of Kramers,

$$\kappa = \kappa_0 \rho T^{-3.5} , \tag{22}$$

can be reduced to the nondimensional forms

$$\frac{d}{d\xi}(\sigma\theta) = -\frac{5}{2}\frac{\sigma\psi}{\xi^2}; \qquad \frac{d\psi}{d\xi} = \sigma\xi^2 . \tag{23}$$

$$\frac{d\theta}{d\xi} = \frac{-Q\sigma^2}{\xi^2 \theta^{6.5}} \tag{24}$$

by the transformation

$$r = R\xi ; \qquad \rho = \rho_0 \sigma ; \qquad T = T_0 \theta ; \qquad M(r) = M\psi , \tag{25}$$

with

$$R^3 \rho_0 = \frac{M}{4\pi}; \qquad RT_0 = \frac{2}{5} \frac{\mu H}{k} GM \tag{26}$$

and

$$Q = \frac{\kappa_0 L}{16\pi} \frac{3}{ac} \frac{\rho_0^2}{RT_0^{7.5}}. \tag{27}$$

According to equations (20), (23), and (25), the mass of the material inclosed up to a radius r_1 is given by

$$M(r_1) = -\frac{1}{G} \frac{r_1^2}{\rho} \frac{dP}{dr} = -\tfrac{2}{5} M \frac{\xi^2}{\sigma} \frac{d(\sigma\theta)}{d\xi}. \tag{28}$$

3. *The equations of fit.*—Let the interface occur at $\xi = \xi_i$ and $\eta = \eta_i$. We then have two sets of equations for the absolute value of the radius, mass, and pressure at the interface. Our equations of fit are derived from the requirement that the two sets agree. In this manner we find

$$-\tfrac{2}{5} M \frac{\xi^2}{\sigma} \frac{d(\sigma\theta)}{d\xi} = -\frac{(4\pi)^2 a_1^3 H \mu_e (2mkT)^{3/2}}{h^3} \eta^2 \frac{d\phi}{d\eta}, \tag{29}$$

$$R\xi = \left[\frac{h^3}{G(4\pi\mu_e H)^2 (2m)^{3/2} (kT)^{1/2}}\right]^{1/2} \eta . \tag{30}$$

$$\rho_0 \sigma = \frac{4\pi\mu_e H (2mkT)^{3/2}}{h^3} F_{1/2}(\phi)\ x , \tag{31}$$

and

$$\frac{k}{\mu H}\rho_0 T_0 \sigma\theta = \frac{4\pi(2m)^{3/2}(kT)^{5/2}}{h^3}\tfrac{2}{3}F_{3/2}(\phi) \tag{32}$$

where, in equation (31)

$$x = \frac{\rho_{\rm env}}{\rho_{\rm core}} \simeq \frac{\mu_{\rm env}}{\mu_{\rm core}}. \tag{33}$$

Equation (33) allows for a discontinuity of the density at $\xi = \xi_i$ to correspond with the difference in the mean molecular weights of the core and the envelope.

This system of equations can be reduced to one involving only two homology invariant combinations of σ, θ, and their derivatives σ', θ', and ξ. Raise equation (30) to the third power, divide by (29) and multiply by 4π times (31). We are left with

$$-\frac{5}{2}\frac{\sigma^2\xi}{(\sigma\theta)'} = -\frac{\eta}{\phi'}F_{1/2}(\phi)\,x\,, \tag{34}$$

which we re-write in the form

$$-\frac{5}{2}\frac{\sigma^2\xi}{(\sigma\theta)'} = u(\xi) = U(\eta) = -\frac{\eta}{\phi'}F_{1/2}(\phi)\,x\,. \tag{35}$$

Divide (29) by (32), multiply by (31), and divide by (30). We then have

$$-\frac{(\sigma\theta)'}{\sigma\theta}\xi = -\frac{F_{1/2}(\phi)}{\tfrac{2}{3}F_{3/2}(\phi)}\eta\phi' x \tag{36}$$

which we re-write in the form

$$-\frac{2}{5}\frac{(\sigma\theta)'}{\sigma\theta}\xi = v(\xi) = V(\eta) = -\frac{2}{5}\frac{F_{1/2}(\phi)}{\tfrac{2}{3}F_{3/2}(\phi)}\eta\phi' x\,. \tag{37}$$

4. *The construction of the models.*—Our fitting conditions are those which are usually used, namely, that the mass, radius, and pressure are continuous at the interface; and we have also made allowance for the difference in the mean molecular weights of the core and envelope by making the density discontinuous at the interface (cf. eq. [33]). However, it should be pointed out in this connection that it is not strictly correct to use the perfect gas equation for $\xi \geq \xi_i$ and change over abruptly to the Fermi-Dirac equation of state interior to ξ_i. But this cannot be avoided if we want to use the available integrations of the point-source envelope. It seems reasonable to suppose that for each value of the degree of degeneracy there would be a finite number of such models possible. With the available integrations of the point-source envelope we found that this supposition was true and that for any degenerate core and point-source envelope there were, at most, two such models. For each value of the degree of degeneracy a certain mass was obtained; and in order to determine the values of the physical characteristics for any desired mass, we must interpolate across these curves.

In order to determine the physical characteristics of any composite configuration, it is necessary to determine if there are any solutions which, besides satisfying the equations describing the core and the envelope, also satisfy the interfacial conditions. The procedure in such an investigation is to make use of the homology invariant quantities u and v[9] and transfer our curves to the u, v plane. In addition to the homology invariant quantities u and v which describe the point-source envelope, we introduce analogous functions U and V to describe the isothermal core. In this manner we are able to transfer the available isothermal and point-source solutions to a common plane. Each intersection between the two sets of curves then represents a possible composite configuration of the

[9] Chandrasekhar, *op. cit.*, pp. 146 ff.

type desired. Obviously, we cannot choose the mass of the core as an independent variable but must determine it as a result of the fitting conditions and the equations describing the core and envelope.[10]

For the fitting we had two sets of integrations of the point-source envelope available. One set of these integrations is due to Miss I. Nielsen and is for values of log $Q = \bar{3}.2494$, $\bar{3}.1494$, $\bar{3}.0494$, $\bar{4}.9494$, $\bar{4}.8494$, $\bar{4}.7494$, and $\bar{4}.6494$; the other set is due to M. Schönberg and is for values of log $Q = \bar{3}.7894$, $\bar{3}.1894$, $\bar{3}.2894$, $\bar{3}.3894$, $\bar{3}.4394$, $\bar{3}.4894$, $\bar{3}.5894$, and $\bar{3}.9394$.[11] The integrations by G. Wares[12] for $\phi_0 = 0$, 2, 3, 5, 10, and 20 were used for the isothermal core.

A graphical method was used to determine the homology invariant functions u and v for each intersection between the point-source envelopes and isothermal cores; the values for ξ_i, ψ_i, and η_i were then determined by interpolation in the tables.

5. *The physical characteristics of the models.*—The mass, radius, luminosity, and ratio of the central to the mean density can be readily derived from equations (10), (16), (19), (25), and (27). The formulae are given for $\kappa_0 = 4 \times 10^{25}$ and in terms of the sun's radius and mass, with T in units of 2×10^7 degrees K.

Radius of the core:

$$\frac{r}{R_\odot}\,\mu_e T^{0.25} = 0.02236\,\eta_i\,. \tag{38}$$

Mass of the core:

$$\frac{M(\eta)}{M_\odot}\,\mu_e^2 T^{-0.75} = -1.9351\,\eta_i^2\phi_i' \times 10^{-2}\,. \tag{39}$$

Radius of the configuration:

$$\frac{R}{R_\odot}\,\mu_e T^{0.25} = \frac{0.02236\,\eta_i}{\xi_i}\,. \tag{40}$$

Mass of the configuration:

$$\frac{M}{M_\odot}\,\mu_e^2 T^{-0.75} = -\frac{1.9351 \times 10^{-2}\,\eta_i^2\phi_i'}{\psi_i}\,. \tag{41}$$

Luminosity of the configuration:

$$\frac{L}{L_\odot}\,\mu_e^3 T^{-4.25} = \frac{1}{\theta_i^{7.5}}\,\frac{\eta_i\sigma_i^2}{\xi_i}\,\frac{Q \times 3.359 \times 10^{-4}}{[F_{1/2}(\phi_i)]^2}\,. \tag{42}$$

Ratio of the central to the mean density:

$$\frac{\rho_c}{\bar{\rho}} = \frac{0.3336\,\eta_i^3\psi_i}{\xi_i^3(-\eta_i^2\phi_i')}\,F_{1/2}(\phi_0)\,. \tag{43}$$

The quantities (40), (41), (42), and (43) are tabulated in Table 1 for $\mu_{\text{core}} | \mu_{\text{env}} = 1$ and in Table 2 for $\mu_{\text{core}} | \mu_{\text{env}} = 2$.

[10] This is where our method differs from that used by G. Gamow and G. Keller.

[11] I am indebted to Dr. Schönberg for making these integrations available.

[12] "Partially Degenerate Stellar Models" (a dissertation submitted to the Faculty of the Division of the Physical Sciences in candidacy for the degree of Doctor of Philosophy, Department of Astronomy and Astrophysics, University of Chicago Libraries, 1940).

TABLE 1

PHYSICAL CHARACTERISTICS OF THE COMPOSITE MODEL FOR $x = 1$ AND VARIOUS VALUES OF ϕ_0

$Q\times 10^3$	ξ_i	ψ_i	$\mu_e T^{0.25} R/R_\odot$	$\mu_e^2 T^{-0.75} M/M_\odot$	$\mu_e^3 T^{-4.25} L/L_\odot$	$\rho_c/\bar{\rho}$
			(a) $\phi_0=0$			
1.120	0.1466	0.1172	0.3974	0.4773	0.246×10^{-2}	51.5
1.410	.1678	.1987	0.4797	0.5831	$.942\times10^{-2}$	74.1
1.547	.1698	.2150	0.4993	0.5996	$.121\times10^{-1}$	81.3
1.776	.1742	.2532	0.5598	0.6662	$.243\times10^{-1}$	103
1.947	.1753	.2710	0.5929	0.6983	$.516\times10^{-1}$	117
2.451	.1764	.3112	0.6758	0.7642	$.728\times10^{-1}$	158
2.750	.1749	.3262	0.7224	0.7973	.103	185
3.086	.1718	.3417	0.7870	0.8412	.153	227
3.885	.1654	.3614	0.9160	0.9295	.320	324
6.157	.1551	.3950	1.169	1.058	.947	588
8.698	.1350	.4043	1.720	1.339	$.442\times10$	149×10
6.157	0.0875	0.2249*	17.41	10.77	0.989×10^{5}	192×10^{3}
			(b) $\phi_0=2$			
1.120	0.1638	0.1530	0.2241	0.3370	0.132×10^{-3}	48.2
1.410	.1928	.2641	.2581	.3822	$.532\times10^{-3}$	65.0
1.547	.1974	.2893	.2662	.3884	$.714\times10^{-3}$	70.2
1.776	.2028	.3330	.2886	.4118	$.132\times10^{-2}$	84.4
1.947	.2050	.3532	.2995	.4215	$.175\times10^{-2}$	92.3
2.451	.2069	.3981	.3318	.4468	$.349\times10^{-2}$	118
2.750	.2060	.4152	.3488	.4585	$.471\times10^{-2}$	134
3.086	.2030	.4322	.3720	.4721	$.661\times10^{-2}$	157
3.885	.1992	.4578	.4122	.4971	$.116\times10^{-1}$	204
6.157	.1848	.4900	.5056	.5397	$.296\times10^{-1}$	346
8.698	0.1689	0.5013	0.6304	0.6015	0.728×10^{-1}	602
			(c) $\phi_0=3$			
1.120	0.1763	0.1824	0.1884	0.3273	0.548×10^{-4}	46.9
1.410	.2131	.3216	.2117	.3582	$.250\times10^{-3}$	60.7
1.547	.2200	.3547	.2182	.3650	$.357\times10^{-3}$	65.3
1.776	.2256	.3994	.2327	.3784	$.630\times10^{-3}$	76.4
1.947	.2288	.4222	.2398	.3845	$.833\times10^{-3}$	82.3
2.451	.2312	.4685	.2614	.4007	$.161\times10^{-2}$	102
2.750	.2309	.4875	.2728	.4078	$.218\times10^{-2}$	114
3.086	.2267	.5010	.2888	.4172	$.288\times10^{-2}$	133
3.885	.2249	.5284	.3109	.4288	$.476\times10^{-2}$	161
6.157	.2112	.5619	.3723	.4577	$.118\times10^{-1}$	258
8.698	0.1976	0.5787	0.4316	0.4800	0.223×10^{-1}	384
			(d) $\phi_0=5$			
1.120	0.2116	0.2758	0.1508	0.3481	0.232×10^{-4}	44.6
1.547	.2814	.5330	.1664	.3657	$.264\times10^{-4}$	57.1
1.776	0.2874	0.5760	0.1737	0.3706	0.452×10^{-4}	64.3

* This intersection corresponds to one for which ψ_i has already reached its maximum and thus is not a physically reliable solution.

TABLE 1—*Continued*

$Q\times10^3$	ξ_i	ψ_i	$\mu_e T^{0.25}R/R_\odot$	$\mu_e^2 T^{-0.75}M/M_\odot$	$\mu_e^3 T^{-4.25}L/L_\odot$	$\rho_c/\bar{\rho}$
	(*d*) $\phi_0=5$—*Continued*					
1.947	0.2923	0.5993	0.1773	0.3736	0.614×10^{-4}	67.4
2.451	.2943	.6372	.1881	.3800	$.107\times10^{-3}$	79.2
2.750	.2945	.6538	.1937	.3827	$.142\times10^{-3}$	85.9
3.086	.2926	.6662	.2002	.3859	$.181\times10^{-3}$	94.1
3.885	.2896	.6888	.2120	.3906	$.278\times10^{-3}$	110
6.157	.2782	.7188	.2388	.4001	$.587\times10^{-3}$	154
8.698	0.2655	0.7353	0.2628	0.4057	0.947×10^{-3}	203
	(*e*) $\phi_0=10$					
0.8900	0.2830	0.4468	0.1077	0.4441	0.698×10^{-5}	34.6
0.7070	.3837	.6912	.1020	.4318	$.529\times10^{-4}$	30.2
0.7070	0.4263	0.7820	0.1018	0.4316	0.164×10^{-3}	30.1
	(*f*) $\phi_0=20$					
0.8900	0.2512	0.3505	0.08810	0.6968	0.328×10^{-6}	33.8
0.7070	.3208	.5214	.08224	.6665	$.103\times10^{-5}$	28.8
0.5616	.3602	.5925	.07845	.6620	$.177\times10^{-5}$	25.2
0.4461	0.3984	0.6588	0.07528	0.6556	0.299×10^{-5}	22.5

TABLE 2

PHYSICAL CHARACTERISTICS OF THE COMPOSITE MODEL FOR $x=\frac{1}{2}$ AND VARIOUS VALUES OF ϕ_0

$Q\times10^3$	ξ_i	ψ_i	$\mu_e T^{0.25} R/R_\odot$	$\mu_e^2 T^{-0.75} M/M_\odot$	$\mu_e^3 T^{-4.25} L/L_\odot$	$\rho_c/\bar{\rho}$
			(a) $\phi_0=0$			
1.120	0.0616	0.0202	0.7058	1.319	0.807×10^{-3}	104
1.410	.0720	.0448	0.8942	1.584	$.216\times10^{-2}$	176
1.547	.0731	.0511	0.9465	1.637	$.280\times10^{-2}$	203
1.776	.0744	.0674	1.116	1.834	$.630\times10^{-2}$	297
1.947	.0738	.0752	1.222	1.938	$.926\times10^{-2}$	370
2.451	.0704	.0922	1.562	2.260	$.258\times10^{-1}$	660
2.750	.0682	.1004	1.790	2.459	$.474\times10^{-1}$	914
3.086	.0631	.1074	2.274	2.909	.121	158×10
3.086	.0451	.0898*	7.597	8.509	$.260\times10^{2}$	202×10^{2}
2.750	.0435	.0775*	10.74	12.38	$.146\times10^{3}$	391×10^{2}
2.451	0.0438	0.0702*	14.46	17.09	0.791×10^{3}	692×10^{2}
			(b) $\phi_0=2$			
1.120	0.0624	0.0205	0.3998	0.8641	0.229×10^{-4}	107
1.410	.0741	.0463	0.4806	1.035	$.693\times10^{-4}$	155
1.547	.0758	.0532	0.4984	1.037	$.857\times10^{-4}$	173
1.776	.0782	.0712	0.5640	1.101	$.167\times10^{-3}$	235
1.947	.0785	.0800	0.5954	1.110	$.219\times10^{-3}$	275
2.451	.0768	.0996	0.7003	1.179	$.447\times10^{-3}$	421
2.750	.0745	.1080	0.7699	1.224	$.659\times10^{-3}$	539
3.086	.0725	.1190	0.8581	1.279	$.102\times10^{-2}$	715
3.885	0.0678	0.1363	1.055	1.388	0.236×10^{-2}	122×10
			(c) $\phi_0=3$			
1.120	0.0626	0.0206	0.3371	0.9128	0.728×10^{-5}	96.3
1.410	.0750	.0468	.3967	0.9793	$.211\times10^{-4}$	146
1.547	.0768	.0542	.4110	0.9755	$.257\times10^{-4}$	163
1.776	.0798	.0728	.4555	1.008	$.471\times10^{-4}$	215
1.947	.0800	.0818	.4806	1.016	$.606\times10^{-4}$	251
2.451	.0798	.1034	.5437	1.032	$.110\times10^{-3}$	358
2.750	.0785	.1134	.5843	1.049	$.153\times10^{-3}$	437
3.086	.0774	.1258	.6308	1.060	$.215\times10^{-3}$	543
3.885	0.0734	0.1448	0.7382	1.100	0.407×10^{-3}	840
			(d) $\phi_0=5$			
1.120	0.0627	0.0206	0.2781	1.027	0.150×10^{-5}	94.8
1.410	.0761	.0478	.3158	1.028	$.400\times10^{-5}$	139
1.547	.0782	.0554	.3247	1.018	$.479\times10^{-5}$	152
1.776	.0818	.0748	.3528	1.022	$.812\times10^{-5}$	194
1.947	.0826	.0846	.3670	1.012	$.101\times10^{-4}$	221
2.451	.0838	.1086	.4017	0.9902	$.168\times10^{-4}$	296
2.750	.0842	.1214	.4195	0.9781	$.218\times10^{-4}$	342
3.086	.0831	.1343	.4460	0.9727	$.288\times10^{-4}$	413
3.885	0.0816	0.1582	0.4904	0.9529	0.455×10^{-4}	558

TABLE 2—*Continued*

$Q\times10^3$	ξ_i	ψ_i	$\mu_e T^{0.25} R/R_\odot$	$\mu_e^2 T^{-0.75} M/M_\odot$	$\mu_e^3 T^{-4.25} L/L_\odot$	$\rho_c/\bar{\rho}$
	(e) $\phi_0=10$					
1.120	0.0629	0.0207	0.2196	1.405	0.154×10^{-6}	92.8
1.410	.0771	.0486	.2456	1.385	$.392\times10^{-6}$	131
1.547	.0792	.0563	.2524	1.365	$.466\times10^{-6}$	145
1.776	.0836	.0768	.2693	1.340	$.757\times10^{-6}$	180
1.947	.0848	.0872	.2773	1.310	$.905\times10^{-6}$	200
2.451	.0872	.1135	.2956	1.238	$.142\times10^{-5}$	256
2.750	.0885	.1280	.3046	1.209	$.178\times10^{-5}$	287
3.086	.0884	.1431	.3171	1.175	$.221\times10^{-5}$	334
3.885	.0886	.1710	.3366	1.113	$.323\times10^{-5}$	421
6.157	.0878	.2244	.3722	1.006	$.600\times10^{-5}$	630
8.698	0.0868	0.2718	0.4022	0.9289	0.976×10^{-5}	862
	(f) $\phi_0=20$					
1.120	0.0630	0.0209	0.1813	2.202	0.161×10^{-7}	93.5
1.410	.0778	.0492	.2014	2.175	$.407\times10^{-7}$	130
1.547	.0796	.0568	.2067	2.136	$.487\times10^{-7}$	143
1.776	.0844	.0776	.2188	2.079	$.757\times10^{-7}$	174
1.947	.0854	.0880	.2256	2.028	$.905\times10^{-7}$	195
2.451	.0885	.1152	.2382	1.905	$.138\times10^{-6}$	245
2.750	.0900	.1302	.2444	1.851	$.172\times10^{-6}$	272
3.086	.0901	.1460	.2530	1.783	$.215\times10^{-6}$	313
3.885	.0910	.1757	.2658	1.674	$.299\times10^{-6}$	386
6.157	.0925	.2356	.2847	1.470	$.518\times10^{-6}$	545
8.698	0.0942	0.2903	0.2971	1.330	0.841×10^{-6}	681

6. *Conclusions.*—An examination of the tables reveals that when $\mu_c/\mu_e = 1$ these composite configurations have radii, masses, and luminosities smaller than those of the sun, while the ratio $\rho_c/\bar{\rho}$ is higher. For $\mu_c/\mu_e = 2$ the radii and masses increase, but the radii in most cases still remain less than that of the sun; the masses and ratio $\rho_c/\bar{\rho}$ are larger than the sun, while the luminosity remains much smaller.

With the data given in Tables 1 and 2 we can derive the course of evolution of a given mass as the degree of degeneracy at the center varies. Thus it is found that $M/M_\odot = 0.1264\times10^{-5}\,\mu_e^{-5}\,T^{3/4}$ is included among the solutions found with $\phi_0 = 0$, 2, 3, and 5 and for $x = 1$. Interpolating in Table 1, we obtain the results given in Table 4; from this table we can follow the variation in mass and radius for a given central temperature. Table 5 is also derived from Table 3 and shows the variation in temperature and radius for a given mass. Tables 3, 4, 5, 6, 7, and 8 are for $x = 1$, while Tables 9, 10, 11, 12, 13, and 14 show similar results for $x = \frac{1}{2}$.

It will be seen that our values lie far below those given by Gamow and Keller[13] even if we allow for the difference in κ_0.

In a later paper we shall combine the results of this and the earlier investigations[14] for a discussion of stellar evolution.

[13] *Op. cit.*, Fig. 8.

[14] *Ap. J.*, **100**, 343, 1944; **102**, 216, 1945.

TABLE 3

PHYSICAL CHARACTERISTICS FOR A STAR OF MASS $M/M_\odot = 0.1264 \times 10^{-5}\mu_e^{-2}T^{3/4}$ (T IN DEGREES)
($T=2\times 10^7$; $x=1$; $\mu_e^2 M/M_\odot = 0.378$)

ϕ_0	$\mu_e R/R_\odot$	ψ_i	ξ_i	$L\mu_e^3/L_\odot$
0	0.366	....	....	....
2	.256	0.2567	0.1909	0.507×10^{-3}
3	.233	.3994	.2256	$.630\times 10^{-3}$
5	0.185	0.6277	0.2938	0.942×10^{-4}

TABLE 4

MASS RADIUS RELATIONS FOR STARS OF DIFFERENT MASSES WITH GIVEN CENTRAL TEMPERATURE AND $x=1$
(Derived from Table 3)

T	10^6	5×10^6	10×10^6
$\mu_e^2 M/M_\odot$	0.040	0.134	0.225
ϕ_0	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$
0	0.775	0.518	0.436
2	.541	.362	.304
3	.492	.329	.277
5	0.392	0.262	0.220

TABLE 5

MASS RADIUS RELATIONS FOR STARS OF DIFFERENT CENTRAL TEMPERATURES AND GIVEN MASS AND FOR $x=1$
(Derived from Table 3)

T	73.1×10^6	8.55×10^6	3.39×10^6
M	$M_\odot$	$\frac{1}{5}M_\odot$	$\frac{1}{10}M_\odot$
ϕ_0	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$
0	0.265	0.453	0.571
2	.185	.316	.399
3	.168	.288	.362
5	0.134	0.229	0.289

TABLE 6

PHYSICAL CHARACTERISTICS FOR A STAR OF MASS $M/M_\odot = 0.1598 \times 10^{-5}\mu_e^{-2}\,T^{3/4}$ (T IN DEGREES)
($T = 2\times 10^7$; $x = 1$; $\mu_e^2 M/M_\odot = 0.478$)

ϕ_0	$\mu_e R/R_\odot$	ψ_i	ξ_i	$L\mu_e^3/L_\odot$
0	0.397	0.1172	0.1466	0.246×10^{-2}
2	.378	.4362	.2024	$.736\times 10^{-2}$
3	0.432	0.5787	0.1976	0.223×10^{-1}

TABLE 7

MASS RADIUS RELATIONS FOR STARS OF DIFFERENT MASSES WITH GIVEN CENTRAL TEMPERATURE AND $x=1$

(Derived from Table 6)

T	10^6	5×10^6	10×10^6
$\mu_e^2 M/M_\odot$	0.050	0.169	0.284
ϕ_0	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$
0	0.840	0.562	0.472
2	.800	.535	.450
3	0.913	0.610	0.513

TABLE 8

MASS RADIUS RELATIONS FOR STARS OF DIFFERENT CENTRAL TEMPERATURES AND GIVEN MASS AND FOR $x=1$

(Derived from Table 6)

T	53.6×10^6	6.26×10^6	2.49×10^6
M	$M_\odot$	$\frac{1}{5}M_\odot$	$\frac{1}{10}M_\odot$
ϕ_0	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$
0	0.311	0.531	0.669
2	.296	.506	.637
3	0.338	0.577	0.727

TABLE 9

PHYSICAL CHARACTERISTICS FOR A STAR OF MASS $M/M_\odot = 0.4584 \times 10^{-5}\mu_e^{-2}T^{3/4}$ (T IN DEGREES)

($T = 2\times10^7$; $x=\frac{1}{2}$; $\mu_e^2 M/M_\odot = 1.371$)

ϕ_0	$\mu_e R/R_\odot$	ψ_i	ξ_i	$L\mu_e^3/L_\odot$
0	0.740	0.0247	0.0635	0.105×10^{-2}
2	1.026	.1338	.0685	$.217\times10^{-2}$
10	0.254	.0588	.0797	$.492\times10^{-6}$
20	0.293	0.2732	0.0937	0.735×10^{-6}

TABLE 10

MASS RADIUS RELATIONS FOR STARS OF DIFFERENT MASSES WITH GIVEN CENTRAL TEMPERATURE AND $x=\frac{1}{2}$

(Derived from Table 9)

T	10^6	5×10^6	10×10^6
$\mu_e^2 M/M_\odot$	0.145	0.485	0.815
ϕ_0	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$
0	1.565	1.046	0.880
2	2.170	1.451	1.220
10	0.538	0.360	0.302
20	0.620	0.414	0.349

TABLE 11

MASS RADIUS RELATIONS FOR STARS OF DIFFERENT CENTRAL TEMPERATURES AND GIVEN MASS AND FOR $x = \frac{1}{2}$

(Derived from Table 9)

T	15.1×10^6	1.55×10^6	0.609×10^6
M	$M_\odot$	$\frac{1}{5}M_\odot$	$\frac{1}{10}M_\odot$
ϕ_0	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$
0	0.822	1.402	1.771
2	1.140	1.945	2.456
10	0.283	0.482	0.609
20	0.326	0.556	0.702

TABLE 12

PHYSICAL CHARACTERISTICS FOR A STAR OF MASS $M/M_\odot = 0.3320 \times 10^{-5}\mu_e^{-2}T^{3/4}$ (T IN DEGREES)

($T = 2 \times 10^7$; $x = \frac{1}{2}$; $\mu_e^2 M/M_\odot = 0.993$)

ϕ_0	$\mu_e R/R_\odot$	ψ_i	ξ_i	$L\mu_e^3/L_\odot$
2	0.456	0.0384	0.0705	0.550×10^{-4}
3	.421	.0585	.0777	$.307\times10^{-4}$
5	.399	.1068	.0837	$.163\times10^{-4}$
10	0.379	0.2348	0.0876	0.685×10^{-5}

TABLE 13

MASS RADIUS RELATIONS FOR STARS OF DIFFERENT MASSES WITH GIVEN CENTRAL TEMPERATURE AND $x=\frac{1}{2}$

(Derived from Table 12)

T	10^6	5×10^6	10×10^6
$\mu_e^2 M/M_\odot$	0.105	0.351	0.590
ϕ_0	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$
2	0.964	0.644	0.542
3	.891	.596	.501
5	.844	.564	.474
10	0.801	0.536	0.450

TABLE 14

MASS RADIUS RELATIONS FOR STARS OF DIFFERENT CENTRAL TEMPERATURES AND GIVEN MASS AND FOR $x=\frac{1}{2}$

(Derived from Table 12)

T	20.2×10^6	2.36×10^6	0.937×10^6
M	$M_\odot$	$\frac{1}{5}M_\odot$	$\frac{1}{10}M_\odot$
ϕ_0	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$	$\mu_e R/R_\odot$
2	0.455	0.778	0.980
3	.420	.719	.906
5	.398	.681	.858
10	0.378	0.646	0.814

I wish to express my sincere thanks to Dr. S. Chandrasekhar for suggesting this problem and for many helpful discussions.

THE SPECTRUM OF PROCYON: A TYPICAL STAR OF CLASS F*

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ABSTRACT

A table of identifications, covering the wave-length interval λλ 3800–6768, has been prepared from measures of high-dispersion McDonald spectrograms of the type dF4 star, Procyon. Many of the metals are represented by strong lines due to both the neutral and the singly ionized atoms. Most of the rare earths are weakly but certainly present. Bands of *CN* and *CH*, despite the numerous obscurations of the rotational lines by blending and masking, are faintly recognizable.

The value of an extensive list of lines in a stellar spectrum, compiled from spectrograms of high quality and dispersion, is well recognized. Apart from the general concern in knowing which of the less common atoms and ions can be identified, it is of some interest, in the case of the F-type spectra, to know what molecular lines, if any, are present. Furthermore, in making studies of the radial velocities and physical conditions of stars, the need is frequently felt for a list of lines in a typical spectrum which are essentially free from blending effects.

Between 1941 and 1943, eight spectrograms of Procyon (α Canis Minoris), spectral type dF4, were taken by Drs. O. Struve and P. Swings with the coudé spectrograph of the McDonald Observatory. These plates have a linear dispersion ranging from 1.4 A/mm at λ 3800 to 16.3 A/mm at λ 6768. Subsequently, this entire region was measured, for the most part on as many as three different plates of the same interval, by Dr. Struve. Late in 1944 the results of these measures were placed in the hands of the writer, and he was granted the privilege of making and discussing the identifications.

Among the studies of other F-type spectra, some of the more extensive are those of α Persei (cF4),[1] δ Cephei (cF2–cG4),[2] 7 ε Aurigae (cF2),[3] α Carinae (cF0),[4] and γ Cygni (cF7p).[5] These stars are nontypical F types in the sense that all five are supergiants; δ Cephei exhibits a periodically variable spectrum, and γ Cygni is a star exceptionally rich in rare earths. Procyon itself has previously been studied by means of lower-dispersion spectra in the region λλ 4250–4703 by S. Albrecht[6] and in the region λλ 6562–7593 by F. E. Roach.[7] The present re-examination of this spectrum is considered justified on the basis of our better, high-dispersion spectrograms, together with the great amount of laboratory material on atomic, as well as molecular, spectra which has appeared since the time of Albrecht's paper.

* *Contributions from the McDonald Observatory, University of Texas*, No. 122.

[1] Theodore Dunham, Jr., *Contr. Princeton Obs.*, No. 9, 1929.

[2] C. J. Krieger, *Ap. J.*, **79**, 98, 1934.

[3] E. B. Frost, O. Struve, and C. T. Elvey, *Pub. Yerkes Obs.*, Vol. **7**, Part II, 1932; William W. Morgan, *Pub. Yerkes Obs.*, Vol. **7**, Part III, 1935; P. Swings and O. Struve, *Ap. J.*, **94**, 307, 1941.

[4] Jesse L. Greenstein, *Ap. J.*, **95**, 161, 1942.

[5] F. E. Roach, *Ap. J.*, **96**, 272, 1942.

[6] *Ap. J.*, **80**, 86, 1934.

[7] *Ap. J.*, **80**, 233, 1934.

Procyon is a close visual binary, magnitudes 0.5 and 10.8,[8] with a period of forty years. Its parallax of 0".32 fixes the absolute magnitude of the system at +3.0. Albrecht's[9] supposition that the brighter component is a spectroscopic binary of very short period was subsequently shown by W. Schaub[10] to be incorrect.

THE TABLE OF IDENTIFICATIONS

The absorption lines measured in the Procyon spectrum, some thirty-six hundred in number, are listed in Table 1. The wave lengths given are the simple averages of those obtained from the plates measured; no real difference in the wave lengths of a given line measured on different plates is apparent. These wave lengths were computed to three decimal places. Although this accuracy would be justified for many lines in the violet and blue regions, all the wave lengths are given only to the nearest 0.01 A. The intensities of a given line, as estimated from several plates, are in close agreement; the averaged intensities are given in the second column and are on the usual system, the faintest lines, whose reality in some cases may be doubtful, being assigned an intensity of 0. Unidentified lines of 0 intensity measured on only one plate have been omitted. The predicted lines are designated by the usual "Pr." The customary letters "b," "n," and "nn," appearing after some of the stellar intensities, mean, respectively, "broad," "nebulous," and "very nebulous." In the third column, headed "N," is given the number of plates on which a specific line was measured. The value of N may be taken as an indication of the accuracy of the entered wave length. It is believed that essentially all the faint lines listed in the table are real, although somewhat greater faith may be placed on the reality of those faint lines measured on two or more plates. In a given region of the spectrum the number of plates actually measured is indicated by the N-number appearing opposite the higher intensities in the second column. Owing to slight differences in exposure and plate quality, lines falling in the wings of very strong features sometimes do not appear masked on all the plates measured; for this reason the reader should not be disturbed by the frequent disparity of the N-values recorded for neighboring strong lines.

In making the identifications, a preliminary survey of the wave-length table was first made by means of the *M.I.T. Wavelength Tables*. Then the evidence for the presence of all the chemical elements and their ions was carefully considered. The final identifications were entered in the table, considering each multiplet of a given element or ion whenever the existence of a term classification rendered this possible, largely by means of the *Revised Multiplet Table of Astrophysical Interest*,[11] parts of which were very kindly lent by the author, C. E. Moore, before its publication. So far as has been possible, the contributors to a line have been listed in the order of their relative importance. Some identifications are somewhat doubtful, represent very minor contributions, or differ appreciably in wave length; such entries have been inclosed in parentheses.

Despite the many compilations of atomic and molecular spectral data which have appeared during recent years, there still are some strong lines which remain completely unidentified. Indeed, for 8 per cent of the lines in Table 1 no identifications whatever have been given, and, in addition, many of the entered identifications could be greatly improved. Most of these lines are also present in the spectrum of the sun, and the stronger ones can be found in a few spectra as early as type A. The variation of intensity with spectral type for the unidentified lines of intensity 3 or greater in the Procyon spectrum

[8] A special magnitude determination by G. P. Kuiper, *Ap. J.*, **85**, 253, 1937.

[9] *Loc. cit.*

[10] *Zs. f. Ap.*, **9**, 198, 1934.

[11] *Contr. Princeton Obs.*, No. 20, 1945.

TABLE 1

THE SPECTRUM OF PROCYON (α CANIS MINORIS) IN THE REGION λλ 3800–6768

λ	Int.	N	Identification
3799.56..	2	1	*Fe* I 9.55 (50)
3800.62..	1	1	*Mn* I 0.55 (4), *Zr* II 0.73 (5)
3801.72..	2n	1	*Fe* I 1.68 (3), *Fe* I 1.80 (1), *Fe* I 1.98 (2), *Mn* II 1.63 (3), *Mn* I 1.91 (4)
3802.30..	1	1	*Fe* I 2.28 (1)
3803.51..	1	1	*V* I 3.47 (25)
3804.02..	3	1	*Fe* I 4.01 (2)
3804.55..	1	1	*V* I 4.59 (3), (*Mn* II 4.48 (0))
3804.78..	1	1	*Cr* I 4.80 (50)
3805.35..	3	1	*Fe* I 5.34 (12)
3805.78..	0n	1	(*Co* I 5.78 (2))
3806.19..	2n	1	*Fe* I 6.20 (2)
3806.70..	4	1	*Mn* I 6.72 (20), *Fe* I 6.70 (10)
3807.14..	2	1	*Ni* I 7.14 (35)
3807.54..	4	1	*Fe* I 7.53 (7)
3808.00..	1	1	*Cr* I 7.93 (15), *Co* I 8.10 (10), *Ce* II 8.12 (300)
3808.28..	2	1	*Fe* I 8.29 (1)
3808.72..	3	1	*Fe* I 8.73 (4)
3809.04..	1	1	*Fe* I 9.04 (1)
3809.55..	3	1	*Mn* I 9.59 (10), *V* I 9.60 (15), *Cr* II 9.54 (1)
3810.25..	0n	1	*Fe* II 0.21 (Pr)
3810.76..	2	1	*Fe* I 0.76 (2)
3811.05..	0	1	*Fe* I 1.05 (1), *Co* I 1.06 (5)
3811.86..	5	1	*Fe* I 1.89 (2), *Fe* I 1.81 (Pr)
3812.25..	1n	1	*Cr* I 2.25 (12)
3812.95..	4n	1	*Fe* I 2.96 (40), *Fe* I 3.06 (5)
3813.38..	1	1	*Ti* II 3.39 (2)
3813.59..	0	1	*Fe* I 3.64 (2), *V* I 3.49 (60)
3813.88..	1	1	*Fe* I 3.89 (2), *Fe* I 3.93 (Pr)
3814.15..	2	1	*Fe* II 4.12 (4)
3814.56..	3	1	*Ti* II 4.58 (4), *Fe* I 4.53 (5), *Cr* I 4.62 (12), *Co* I 4.46 (5)
3814.80..	2	1	*Ti* I 4.86 (4), *Nd* II 4.72 (60)
3815.39..	2	1	*V* II 5.38 (200), *Cr* I 5.43 (30)
3815.83..	7	1	*Fe* I 5.84 (100)
3816.19..	0	1	*Cr* I 6.17 (20), (*Pr* II 6.17 (125), (*CN* 1, 1 R 6.24)
3816.35..	3	1	*Fe* I 6.34 (4), *Co* I 6.32 (15), *Co* I 6.46 (15)
3816.86..	2n	1	*Co* I 6.88 (5), *Mn* I 6.75 (5), *Fe* I 6.92 (Pr)
3817.40..	0	1	*Ce* II 7.46 (25)
3817.61..	3	1	*Zr* II 7.59 (12), *Fe* I 7.65 (3), *Ti* I 7.64 (5)
3818.32..	2	1	*Y* II 8.34 (60), (*Pr* II 8.28 (125))
3818.58..	1	1	(*Cr* I 8.48 (25))
3818.84..	0	1	(*Zr* II 8.78 (1))
3819.55..	3n	1	*Cr* I 9.56 (40), *Fe* I 9.50 (Pr), *Eu* II 9.67 (6000)
3820.44..	10	1	*Fe* I 0.43 (250)
3820.84..	0	1	*Cr* I 0.87 (7), *CN* 0, 0 R 0.78
3821.18..	5	1	*Fe* I 1.18 (10)
3821.88..	4	1	*Fe* I 1.83 (3), *Fe* II 1.92 (Pr)
3822.30..	1	1	*CN* 0, 0 R 2.29
3822.85..	1	1	*V* I 2.89 (15), *Fe* II 2.74 (3)
3823.51..	3	1	*Mn* I 3.51 (20), *Cr* I 3.52 (12)
3823.86..	2	1	*Mn* I 3.89 (10)
3824.09..	2	1	*Fe* I 4.07 (1)
3824.33..	3	1	*Fe* I 4.31 (2), *CN* 1, 1 R 4.36
3824.49..	4	1	*Fe* I 4.44 (50)
3824.93..	3	1	*Fe* II 4.91 (4)
3825.41..	1	1	*Fe* I 5.40 (1), *Cr* I 5.39 (20)
3825.88..	15	1	*Fe* I 5.88 (200)
3826.42..	0	1	*Cr* I 6.42 (40)
3826.62..	1	1	*Fe* I 6.63 (Pr)
3826.85..	1	1	*Fe* I 6.84 (1)
3827.10..	1	1	*Fe* II 7.08 (4)
3827.29..	0	1	*Zr* II 7.27 (1)
3827.59..	3	1	*Fe* I 7.57 (1), (*Fe* II 7.67 (Pr))
3827.84..	7	1	*Fe* I 7.82 (75)
3828.14..	1	1	*CN* 0, 0 R 8.18, *CN* 1, 1 R 8.18, *Ti* I 8.18 (3)
3828.52..	2	1	*Fe* I 8.51 (1), *V* I 8.56 (60)
3829.30..	15nn	1	*Mg* I 9.36 (40)
3829.69..	2n	1	*Fe* I 9.77 (2), *Mn* I 9.68 (5), *CN* 0, 0 R 9.62
3830.04..	0	1	*Cr* I 0.03 (50)
3830.80..	2	1	*Fe* I 0.76 (1), *Fe* I 0.85 (1)
3831.73..	3n	1	*Ni* I 1.69 (20), *CN* 1, 1 R 1.85
3832.34..	15nn	1	*Mg* I 2.30 (80)
3832.88..	1	1	*Y* II 2.89 (100), *Ni* I 2.87 (5)
3833.12..	1	1	*Sc* II 3.06 (3), *Ti* I 3.19 (4), (*Fe* II 3.02 (Pr))
3833.34..	2	1	*Fe* I 3.31 (5)
*	.....	1	*H* 9 5.39, *Mg* I 8.29 (100), *Fe* I 4.22 (100)
3839.28..	3	1	*Fe* I 9.26 (7)
3839.61..	1	1	*Fe* I 9.63 (2)
3839.78..	2	1	*Mn* I 9.78 (8)
3840.46..	6	2	*Fe* I 0.44 (80)
3840.76..	1	1	*V* I 0.75 (60), (*Cr* I 0.70 (4))
3841.08..	6	2	*Fe* I 1.05 (80), *Mn* I 1.08 (10)
3841.25..	2	1	*Cr* I 1.28 (50)
3841.50..	0	1	*Co* I 1.46 (5)
3841.86..	1	2	(*CN* 0, 0 R 1.73)
3842.05..	2	2	*Co* I 2.05 (30), *Cr* I 2.03 (10)
3842.61..	0	1	*Cr* II 2.66 (1)
3842.95..	2n	2	*Fe* I 2.98 (1), *Fe* I 2.90 (Pr), *CN* 0, 0 R 2.99
3843.05..	3	1	*Zr* II 3.03 (30), *Sc* II 3.00 (4)
3843.27..	2	2	*Fe* I 3.26 (8)
3843.71..	2n	2	*Fe* I 3.72 (Pr), *Co* I 3.69 (4)
3843.98..	1	2	*Mn* I 3.98 (7)
3844.24..	2	2	*CN* 0, 0 R 4.23, *Ni* I 4.28 (3)
3844.40..	1	2	*V* I 4.44 (20)
3844.58..	0	2	*Ni* I 4.58 (3), (*Gd* II 4.58 (125))
3844.92..	1	2	*CN* 1, 1 R 5.02, (*V* I 4.89 (4))
3845.18..	3	2	*Fe* I 5.17 (5), *Fe* II 5.18 (3), *Fe* I 5.22 (Pr)

* No measures, owing to a defect in the plate.

TABLE 1—*Continued*

λ	Int.	N	Identification
3845.48	2	2	*Co* I 5.47 (60), *CN* 0, 0 R 5.45
3845.72	2	2	*Fe* I 5.69 (1)
3846.00	2	2	*Fe* I 6.00 (1)
3846.42	2	2	*Fe* I 6.41 (2), *Ti* I 6.44 (6)
3846.81	4n	2	*Fe* I 6.80 (8)
3846.89	1b	1	*Fe* I 6.95 (1), *CN* 1, 1 R 6.96
3847.32	2	2	*V* II 7.32 (100), *V* I 7.32 (20)
3847.80	1	1	*CN* 0, 0 R 7.84
3847.88	0n	2	*CN* 1, 1 R 7.92
3848.28	2	2	*Mg* II 8.24 (7), *Fe* I 8.30 (Pr), (*Nd* II 8.23 (50))
3848.54	1	2	*Ce* II 8.60 (150), *Nd* II 8.52 (80)
3848.99	2	2	*Cr* I 8.98 (40), *CN* 0, 0 R 9.01
3849.36	2	2	*Cr* I 9.36 (50)
3849.56	2	2	*Cr* I 9.53 (40), *Ni* II 9.58 (2)
3849.98	4	2	*Fe* I 9.97 (40), *Cr* I 0.04 (50)
3850.48	0n	2	*Mg* II 0.40 (6)
3850.82	6	2	*Fe* I 0.82 (12)
3851.30	1	2	*CN* 0, 0 R 1.28
3851.75	1n	2	*Nd* II 1.75 (60), (Pr II 1.62 (200))
3852.15	1	2	*Cr* I 2.22 (30)
3852.40	0	1	*CN* 0, 0 R 2.39, *CN* 1, 1 R 2.39, *Gd* II 2.47 (150)
3852.57	6	2	*Fe* I 2.57 (6)
3853.00	1	2	*Ti* I 3.04 (10)
3853.15	1n	1	*Cr* I 3.18 (12), *Ce* II 3.16 (125), *Zr* II 3.07 (2)
3853.41	2n	2	*Fe* I 3.46 (1)
3853.70	2	2	*Ti* I 3.72 (10), *Si* II 3.66 (3)
3853.99	0	1	*CN* 1, 1 R 4.06
3854.24	2	1	*Cr* I 4.22 (50), (*Sm* II 4.21 (300))
3854.36	2	2	*Fe* I 4.38 (1), *Ce* II 4.32 (100)
3854.55	0	1	*CN* 0, 0 R 4.57
3854.87	0	1	*Cr* II 4.86 (2), *CN* 1, 1 R 4.85, *Pr* II 4.90 (100)
3855.33	2n	2	*Fe* I 5.33 (1), *V* I 5.37 (30), *Cr* I 5.29 (12)
3855.58	1	1	*Cr* I 5.57 (30), *CN* 0, 0 R 5.62, *CN* 1, 1 R 5.62
3855.86	2	2	*V* I 5.84 (60), *Fe* I 5.85 (1)
3856.04	2	2	*Si* II 6.02 (8)
3856.40	8	2	*Fe* I 6.37 (50)
3857.04	1n	1	*Cr* II 6.93 (2)
3857.26	0n	1	*Cr* II 7.19 (1)
3857.64	2	2	*Cr* I 7.63 (20), *CN* 0, 0 R 7.69
3857.90	0	1	*CN* 1, 1 R 7.90
3858.31	7	2	*Ni* I 8.30 (40)
3858.87	4	2	*Cr* I 8.90 (15)
3859.24	4	1	*Fe* I 9.21 (10), *Mg* I 9.24 (1)
3859.50	0	1	(*Sc* II 9.36 (Pr))
3859.93	10n	1	*Fe* I 9.91 (300)
3860.56	1	1	*CN* 0, 0 R 0.63
3860.97	0	1	*Fe* II 0.92 (3)
3861.16	1	2	*Co* I 1.16 (20)
3861.37	3	2	*Fe* I 1.34 (2)
3861.61	2	2	*Fe* I 1.60 (1), *CN* 0, 0 R 1.57
3862.00	1n	2	....
3862.59	6	2	*Si* II 2.59 (6)
3863.06	2	2	*Ni* I 3.07 (5)
3863.39	2	2	*CN* 0, 0 R 3.39, *Fe* II 3.41 (1)
3863.74	5	2	*Fe* I 3.74 (2), *V* II 3.81 (60), *Fe* I 3.70 (Pr)
3863.98	1	2	*Fe* II 3.95 (1), *CN* 1, 1 P 4.03
3864.13	0	1	*Mo* I 4.12 (1000)
3864.30	2	2	*Fe* I 4.31 (Pr), *CN* 0, 0 R 4.30
3864.49	1b	1	*La* II 4.49 (100), *CN* 0, 0 P 4.53, CN 1, 1 P 4.53
3864.89	2	2	*V* I 4.86 (35)
3865.17	0	2	*CN* 0, 0 R 5.15
3865.53	11	2	*Fe* I 5.53 (30), *Cr* II 5.59 (75)
3865.98	1n	2	*CN* 0, 0 R 5.99, *Cr* II 6.01 (5), *CN* 1, 1 P 5.99
3866.15	1b	1	*CN* 0, 0 P 6.20, *CN* 1, 1 P 6.11
3866.47	1	2	*Ti* I 6.45 (15), *Cr* II 6.54 (7)
3866.82	2n	2	*CN* 0, 0 R 6.82, *V* II 6.74 (60)
3867.23	4	2	*Fe* I 7.22 (7)
3867.58	1	1	*V* I 7.60 (15), *CN* 0, 0 R 7.62, *CN* 1, 1 P 7.65
3867.94	3	2	*Fe* I 7.92 (1)
3868.04	1	1	*CN* 1, 1 P 8.00, *CN* 1, 1 P 8.12
3868.23	1b	1	*Fe* I 8.24 (1)
3868.44	1b	2	*Ti* I 8.40 (10), *CN* 0, 0 R 8.41, *CN* 1, 1 P 8.45
3868.70	0	1	*CN* 0, 0 P 8.61, *CN* 1, 1 P 8.72
3869.05	0	2	*CN* 1, 1 P 9.04, (*Nd* II 9.04 (30))
3869.14	1	1	*CN* 0, 0 R 9.18, *CN* 1, 1 P 9.13
3869.34	0	1	*CN* 0, 0 P 9.37, *CN* 1, 1 P 9.33, *Ti* I 9.28 (5)
3869.60	6	2	*Fe* I 9.56 (2), *Fe* I 9.61 (1)
3869.88	1	2	*CN* 0, 0 R 9.92, *CN* 1, 1 P 9.83, *CN* 1, 1 P 9.92
3870.17	2	2	*Cr* I 0.27 (25), *CN* 0, 0 P 0.11, *CN* 1, 1 P 0.15
3870.50	2	2	*Ca* I 0.51 (2), *Co* I 0.53 (4)
3870.83	2	2	*CN* 0, 0 P 0.84, *CN* 1, 1 P 0.88
3871.19	1b	2	*CN* 1, 1 P 1.24, *CN* 1, 1 P 1.13, (*V* I 1.08 (8))
3871.36	1b	2	*CN* 0, 0 R 1.37†, Band head *CN* (1, 1) 1.44
3871.64	1b	1	*La* II 1.64 (200), *Ni* I 1.60 (3), *CN* 0, 0 P 1.53
3871.75	4b	2	*Fe* I 1.75 (4)
3872.10	1	2	*Dy* II 2.10 (600), *CN* 0, 0 R 2.06

† Wave length computed by formula.

TABLE 1—*Continued*

λ	Int.	N	Identification	λ	Int.	N	Identification
3872.50..	8	2	*Fe* I 2.50 (60)	3882.52..	1	2	*CN* 0, 0 P 2.58, *CN* 0, 0 P 2.52, *Ce* II 2.45 (75)
3872.77..	2	2	*Fe* II 2.76 (Pr), *CN* 0, 0 R 2.74, (*V* I 2.75 (4))	3882.95..	2n	2	*CN* 0, 0 P 3.10, *CN* 0, 0 P 2.99, *CN* 0, 0 P 2.85, *Ti* I 2.89 (20)
3872.93..	2	2	*Fe* I 2.92 (1), *Fe* II 2.98 (Pr), *CN* 0, 0 P 2.96	3883.30..	5	2	*Fe* I 3.28 (4), *Cr* I 3.29 (60)
3873.13..	4	2	*Co* I 3.12 (60)	3883.68..	1	2	*Cr* I 3.66 (20)
3873.29..	1	1	*Ti* I 3.20 (10), *CN* 0, 0 R 3.37	3883.97..	1	2	....
3873.59..	1	2	*CN* 0, 0 P 3.53, *Cr* II 3.51 (2)	3884.35..	6	2	*Fe* I 4.36 (3)
3873.77..	4	2	*Fe* I 3.76 (8)	3884.64..	1	2	*Fe* I 4.66 (1), *Co* I 4.60 (10)
3873.98..	3b	2	*Co* I 3.95 (40)	3884.85..	1	2	*V* II 4.85 (50)
3874.03..	1b	2	*Fe* I 4.05 (1)	3885.19..	4	2	*Cr* I 5.22 (40), *Fe* I 5.15 (1), *Cr* I 5.08 (20)
3874.51..	2	2	*Cr* I 4.57 (40), *Cr* II 4.41 (Pr)	3885.52..	4	2	*Fe* I 5.51 (5)
3874.80..	1n	2	*CN* 0, 0 P 4.76, *Cr* II 4.76 (Pr)	3885.72..	1	1	*Fe* I 5.76 (Pr)
3875.08..	1b	2	*V* I 5.08 (35), *Cr* I 5.14 (10)	3885.94..	1	2	*Fe* I 5.94 (Pr)
3875.28..	2b	2	*Ti* I 5.26 (20), *CN* 0, 0 P 5.34	3886.30..	8	2	*Fe* I 6.28 (40)
3875.55..	0	1	*Sm* II 5.54 (200)	3886.80..	2	2	*Cr* I 6.79 (50)
3875.79..	3	2	*Ca* I 5.81 (4), *CN* 0, 0 P 5.77	3887.08..	6	2	*Fe* I 7.05 (15)
3876.05..	4	2	*Fe* I 6.04 (4)	3887.85..	1nn	2	*Ti* I 8.02 (4), (*Nd* II 7.87 (30))
3876.31..	1	2	*CN* 0, 0 P 6.32				
3876.47..	0	1	*CN* 0, 0 P 6.45	3888.51..	4	2	*Fe* I 8.52 (20), *Fe* I 8.42 (1)
3876.66..	1n	2	*Fe* I 6.67 (1)	3888.81..	1	1	*Fe* I 8.82 (3)
3876.86..	2n	2	*Co* I 6.83 (20), *CN* 0, 0 P 6.84	3889.14..	35nn	2	*H* 8 9.05
				3889.69..	1	2	*Ni* I 9.67 (15)
3876.96..	1b	2	*CN* 0, 0 P 6.97, (*Ce* II 6.97 (15))	3890.02..	1	1	*Ce* II 9.99 (300), *Fe* I 9.93 (1), *Sm* II 0.08 (200)
3877.24..	1n	1	*Pr* II 7.22 (200), *Cr* II 7.26 (2)	3890.23..	0	1	*V* I 0.18 (25), *Mg* I 0.24 (3)
				3890.40..	1	2	*Fe* I 0.39 (1)
3877.42..	1nn	1	*CN* 0, 0 P 7.48, *CN* 0, 0 P 7.35	3890.65..	1	1	(*Nd* II 0.58 (50))
				3890.86..	4	2	*Fe* I 0.84 (2)
3878.01..	10	2	*Fe* I 8.02 (60)	3891.19..	1	2	(*Sm* II 1.21 (100)), (*V* I 1.23 (2))
3878.30..	3	2	*V* II 8.28 (20), *CN* 0, 0 P 8.30, *Ce* II 8.37 (150)	3891.51..	1	2	....
3878.66..	12n	2	*Fe* I 8.58 (100), *Fe* I 8.68 (8), *V* II 8.72 (300), *Fe* I 8.74 (2), *Mg* I 8.58 (1)	3891.80..	1b	2	*Ba* II 1.78 (50)
				3891.94..	2b	2	*Fe* I 1.93 (3)
				3892.29..	1	2	(*Cr* II 2.14 (4))
3879.22..	2n	2	*Cr* I 9.22 (50), *CN* 0, 0 9.28, *CN* 0, 0 P 9.18	3892.58..	1	2	(*Mn* I 2.62 (1))
				3892.94..	3n	2	*Fe* I 2.98 (1), *Fe* I 2.89 (1), *V* I 2.86 (25)
3879.61..	1n	2	*CN* 0, 0 P 9.58, *CN* 0, 0 P 9.68	3893.37..	8	2	*Fe* I 3.39 (7), *Fe* I 3.32 (1), *Mg* I 3.38 (3)
3879.97..	0n	2	*CN* 0, 0 P 9.96, *CN* 0, 0 P 0.06	3893.63..	1	1	....
3880.38..	1n	2	*CN* 0, 0 P 0.42, *CN* 0, 0 P 0.33	3893.96..	3b	1	*Fe* I 3.92 (2)
				3894.04..	8n	2	*Co* I 4.07 (60), *Cr* I 4.04 (40), *Fe* I 4.00 (2)
3880.77..	1	2	*CN* 0, 0 P 0.76, *CN* 0, 0 P 0.67, (*Nd* II 0.78 (40))	3894.50..	1	2	*Fe* I 4.49 (1)
3881.07..	0	1	*CN* 0, 0 P 1.08, *CN* 0, 0 P 1.00	3894.68..	0n	2	*Gd* II 4.71 (200), *Nd* II 4.63 (40)
3881.22..	0	1	*Cr* I 1.21 (40)	3894.99..	2	2	*Co* I 4.98 (20)
3881.32..	1	1	*CN* 0, 0 P 1.37, *CN* 0, 0 P 1.30	3895.22..	1	2	*Ti* I 5.24 (30), *Cr* II 5.16 (2), *Cr* II 5.12 (1)
3881.60..	0	2	*CN* 0, 0 P 1.59, *CN* 0, 0 P 1.65	3895.47..	1	2	*Fe* I 5.45 (1)
				3895.67..	9	2	*Fe* I 5.66 (25), *Mg* I 5.66 (10)
3881.91..	4	2	*Co* I 1.87 (25), *Cr* I 1.86 (10), *Zr* II 1.97 (7), (*Ni* II 1.92 (1))	3896.14..	4b	2	*V* II 6.16 (60), *Fe* II 6.11 (Pr), *V* I 6.16 (6)
				3896.48..	2	2	....
3882.08..	0	1	*CN* 0, 0 P 2.10, *CN* 0, 0 P 2.17, *Ti* I 2.15 (15)	3896.78..	1n	2	*Ce* II 6.80 (100)
				3897.28..	0	1	(*Ti* I 7.29 (1))
3882.31..	4	2	*Ti* II 2.28 (Pr), *Ti* I 2.31 (10)	3897.46..	4	2	*Fe* I 7.45 (2)

TABLE 1—*Continued*

λ	Int.	N	Identification
3897.90..	4	2	*Fe* I 7.90 (8)
3898.02..	5	2	*Fe* I 8.01 (10)
3898.46..	1nn	2	*Co* I 8.48 (4), *Ti* I 8.49 (8), *Cr* II 8.49 (1)
3899.08..	8	2	*V* II 9.14 (200), *Fe* I 9.04 (2)
3899.35..	1	2	....
3899.73..	10	2	*Fe* I 9.71 (30)
3900.04..	0b	1	(*Cr* II 9.95 (2))
3900.20..	1b	2	*V* I 0.18 (6), *Nd* II 0.23 (60)
3900.55..	11	2	*Ti* II 0.55 (70), *Fe* I 0.52 (2)
3900.82..	1n	2	(*Cr* II 0.81 (1))
3901.15..	0n	2	*V* I 1.15 (6)
3901.57..	1	2	....
3901.87..	2	2	*Nd* II 1.85 (50)
3902.25..	1	2	*V* I 2.25 (50)
3902.39..	0	1	*Gd* II 2.40 (150)
3902.67..	1	2	....
3902.96..	9	2	*Fe* I 2.95 (20), *Cr* I 2.92 (50), (*Mo* I 2.97 (50))
3903.26..	4	2	*V* II 3.27 (250), *Cr* I 3.16 (25)
3903.58..	0	1	(*Sm* II 3.42 (500))
3903.90..	8	2	*Fe* I 3.90 (5)
3904.18..	1	1	(*Cr* II 4.15 (1))
3904.44..	1n	2	....
3904.62..	1	1	*Ni* I 4.64 (Pr)
3904.80..	4	2	*Ti* I 4.78 (40)
3905.18..	0	2	*Fe* I 5.19 (Pr)
3905.55..	12	2	*Si* I 5.53 (100), *Cr* II 5.64 (25)
3905.90..	1	2	*Nd* II 5.89 (100), (*Cr* II 5.88 (Pr))
3906.04..	1	2	*Fe* II 6.04 (5)
3906.32..	1	2	*Co* I 6.29 (10)
3906.49..	7	2	*Fe* I 6.48 (8)
3906.75..	4	2	*Fe* I 6.75 (2), (*V* I 6.75 (6))
3906.98..	1	1	*Fe* I 6.96 (Pr)
3907.13..	1n	2	*Eu* II 7.10 (3000)
3907.48..	3	2	*Fe* I 7.46 (1), *Sc* I 7.48 (75)
3907.70..	1	2	*Ti* II 7.65 (Pr), *Cr* I 7.78 (7)
3907.94..	5	2	*Fe* I 7.94 (4)
3908.42..	1nn	2	*Pr* II 8.43 (200), *Ce* II 8.41 (125), *Ce* II 8.54 (100), (*Fe* II 8.54 (Pr))
3908.76..	4	2	*Cr* I 8.76 (100)
3908.93..	2	2	*Ni* I 8.93 (8)
3909.26..	1	2	*Cr* II 9.25 (Pr), *Ce* II 9.31 (35)
3909.47..	0	2	....
3909.67..	4	2	*Fe* I 9.66 (1)
3909.85..	4b	2	*Fe* I 9.83 (3), *V* I 9.89 (20)
3909.93..	2b	1	*Co* I 9.93 (15)
3910.34..	1	2	....
3910.52..	2	2	*Fe* I 0.54 (Pr)
3910.86..	3	2	*Fe* I 0.84 (3)
3911.01..	2	2	*Fe* I 1.00 (1)
3911.20..	0	1	*Ti* I 1.18 (8), *Nd* II 1.17 (60)
3911.32..	1n	1	*Cr* II 1.32 (3)
3911.46..	0	1	(*Cr* II 1.54 (2))
3911.77..	1	2	*Sc* I 1.81 (100), *Fe* I 1.70 (1)
3911.98..	2	2	*Cr* I 1.95 (10)
3912.30..	3	2	*Ti* II 2.32 (Pr), *Ni* I 2.31 (8), *V* I 2.21 (10)
3912.94..	1	2	*Pr* II 2.90 (125), *Ni* I 2.98 (5)
3913.24..	0	1	*Cr* II 3.20 (2)
3913.47..	8b	2	*Ti* II 3.46 (60)
3913.62..	4b	2	*Fe* I 3.64 (4)
3913.98..	0	1	*Cr* II 3.96 (1)
3914.30..	5	2	*V* II 4.33 (250), *Ti* I 4.33 (35), *Fe* I 4.27 (1), (*Zr* II 4.36 (7))
3914.51..	4	2	*Fe* II 4.48 (2), *Ni* I 4.51 (2), (*Fe* I 4.43 (Pr))
3914.96..	0n	2	*Fe* I 4.73 (1), *Cr* I 4.96 (4)
3915.24..	2	2	(*Cr* II 5.30 (Pr))
3915.48..	1	2	(*Ti* II 5.43 (Pr)), (*Cr* II 5.58 (1))
3915.61..	1	2	(*Sc* II 5.60 (Pr))
3915.89..	3n	2	*Zr* II 5.94 (25), *Cr* I 5.84 (40), *La* II 6.04 (300)
3916.25..	1	2	*Cr* I 6.24 (25)
3916.41..	4	2	*V* II 6.42 (200)
3916.73..	5	2	*Fe* I 6.73 (6)
3917.18..	6	2	*Fe* I 7.18 (8), *Co* I 7.12 (8)
3917.37..	1	1	(*Sm* II 7.44 (200))
3917.66..	1n	1	*Cr* I 7.60 (15)
3917.93..	1n	1	....
3918.36..	7n	1	*Fe* I 8.42 (4), *Fe* I 8.32 (3), (*Fe* II 8.51 (Pr)), (*Mn* I 8.32 (3))
3918.64..	6	1	*Fe* I 8.64 (6)
3919.12..	7	1	*Cr* I 9.16 (100), *Fe* I 9.07 (3)
3919.76..	1nn	1	*Ce* II 9.81 (100), *Ti* I 9.82 (5)
3920.27..	8	1	*Fe* I 0.26 (20)
3920.64..	1	1	*Fe* I 0.64 (1)
3920.85..	3	1	*Fe* I 0.84 (1)
3921.05..	4	1	*Cr* I 1.02 (50), *Zr* II 1.02 (Pr)
3921.25..	1	1	*Fe* I 1.27 (1)
3921.54..	1	1	*La* II 1.54 (200), *Ti* I 1.42 (30)
3921.75..	1	1	*Mn* I 1.76 (3), *Ce* II 1.73 (100)
3922.07..	1	1	*Fe* I 2.09 (Pr)
3922.42..	1	1	*V* I 2.43 (12), *Sm* II 2.40 (800)
3922.70..	1	1	*Co* I 2.76 (7)
3922.93..	10	1	*Fe* I 2.91 (25)
3923.43..	1n	1	*Ti* II 3.39 (Pr), *Sc* II 3.50 (2)
3924.12..	1nn	1	*Mn* I 4.08 (3), *Ni* I 4.18 (Pr)
3924.52..	3	1	*Ti* I 4.53 (50)
3925.20..	4	1	*Fe* I 5.20 (1)
3925.64..	5	1	*Fe* I 5.65 (4)
3925.99..	6	1	*Fe* I 5.95 (6), *Fe* I 6.00 (1)
3926.26..	0	1	*Ti* I 6.32 (10)
3926.46..	2	1	*Mn* I 6.47 (10), *V* II 6.50 (10)
3927.44..	1nn	1	....
3927.93..	8b	1	*Fe* I 7.92 (30)
3928.05..	5b	1	*Fe* I 8.08 (1)
3928.30..	1	1	*Sm* II 8.28 (400)

TABLE 1—*Continued*

λ	Int.	N	Identification
3928.65	3	1	*Cr* I 8.64 (60)
3928.89	1	1	....
3929.18	3b	1	*Fe* I 9.21 (1), *La* II 9.22 (300), *Ti* II 9.15 (Pr), *Fe* I 9.11 (1)
3929.32	2b	1	(*Zr* II 9.54 (8)), (*Cr* II 9.52 (1))
3929.72	2b	1	*V* II 9.73 (50)
3929.89	1b	1	*Ti* I 9.88 (40)
3930.32	5	1	*Fe* I 0.30 (25)
3931.13	1	1	*Fe* I 1.12 (3), *Ce* II 1.09 (125)
3933.76	100nn	1	*Ca* II 3.66 (400)
3935.83	0	1	*Fe* I 5.82 (8), *Fe* II 5.94 (6)
3936.12	0	1	*Zr* II 6.07 (7), *Cr* II 6.13 (1)
3936.43	1	1	....
3936.67	0	1	(*Fe* I 6.77 (Pr))
3937.31	3	1	*Fe* I 7.33 (3)
3937.58	0	1	*Cr* II 7.61 (1)
3937.98	2nn	1	*Ti* I 8.00 (2)
3938.36	6n	1	*Mg* I 8.40 (0), *Fe* II 8.29 (2)
3938.93	0	1	*Fe* II 8.97 (4)
3939.65	1	1	....
3940.04	2	1	*Fe* I 0.04 (1)
3940.31	1	1	*Ti* II 0.32 (Pr), *Ce* II 0.34 (100)
3940.89	5	1	*Fe* I 0.88 (5), *Co* I 0.89 (12)
3941.27	4	1	*Fe* I 1.28 (3)
3941.50	3	1	*Cr* I 1.49 (60), *Mo* II 1.48 (10)
3941.76	2b	1	*Co* I 1.73 (20)
3941.85	1b	1	*Zr* II 1.92 (3), *Ni* I 1.86 (1)
3942.15	1	1	*Ce* II 2.15 (125)
3942.42	5	1	*Fe* I 2.44 (6)
3942.78	2n	1	*Ce* II 2.75 (150), *Mn* I 2.86 (2)
3943.11	3	1	....
3943.35	3	1	*Fe* I 3.34 (2)
3943.53	2	1	....
3944.02	10n	1	*Al* I 4.01 (10)
3944.33	1	1	....
3944.71	3	1	*Fe* I 4.75 (2), *Dy* II 4.69 (600)
3944.91	3	1	*Fe* I 4.89 (3)
3945.18	5	1	*Fe* I 5.12 (4), *Co* I 5.33 (15), *Fe* II 5.21 (Pr)
3945.56	0	1	*Cr* I 5.50 (9)
3945.86	2n	1	*Cr* I 5.97 (10)
3946.52	1n	1	*Sm* II 6.51 (200)
3947.00	5	1	*Fe* I 7.00 (4)
3947.39	0	1	*Fe* I 7.39 (1)
3947.53	4	1	*Fe* I 7.53 (5)
3947.74	3	1	*Ti* I 7.77 (40)
3948.10	5	1	*Fe* I 8.10 (6)
3948.75	7n	1	*Fe* I 8.78 (10), *Ti* I 8.67 (60)
3948.89	1b	1	*Ca* I 8.90 (6), (*Ti* II 8.93 (Pr))
3949.15	5	1	*Fe* I 9.16 (1), *La* II 9.10 (600)
3949.60	1n	1	*Cr* I 9.64 (8)
3949.96	6	1	*Fe* I 9.95 (10)
3950.36	5	1	*Y* II 0.35 (200)
3950.74	1n	1	*Fe* I 0.80 (Pr)
3951.15	6	1	*Fe* I 1.16 (9), *Cr* I 1.10 (10)
3951.59	1n	1	(*Y* II 1.59 (5))
3951.95	5	1	*V* II 1.97 (500)
3952.16	0	1	*Nd* II 2.20 (100)
3952.34	0	1	*Co* I 2.33 (8), *Cr* I 2.40 (15)
3952.65	8	1	*Fe* I 2.61 (8), *Fe* I 2.70 (1)
3952.95	3	1	*Co* I 2.92 (25)
3953.17	4	1	*Fe* I 3.16 (4), *Cr* I 3.16 (18)
3953.52	2	1	*Pr* I 3.52 (125), *Nd* II 3.52 (60), *Fe* I 3.50 (Pr)
3953.68	0	1	*Ce* II 3.66 (12)
3953.87	3	1	*Fe* I 3.86 (1)
3954.52	2	1	....
3954.72	2	1	*Fe* I 4.72 (1)
3955.33	7	1	*Fe* I 5.35 (3)
3955.67	1	1	(*Fe* I 5.77 (Pr))
3955.96	5	1	*Fe* I 5.96 (2)
3956.33	3b	1	*Ti* I 6.34 (60), *Ce* II 6.28 (150)
3956.45	4b	1	*Fe* I 6.46 (9)
3956.69	5	1	*Fe* I 6.68 (12)
3957.02	5	1	*Fe* I 7.03 (4), *Ca* I 7.05 (10)
3957.40	1	1	....
3957.65	2	1	*Fe* I 7.62 (1), *Fe* II 7.66 (Pr), *Gd* II 7.68 (150)
3957.95	1	1	*Co* I 7.93 (15)
3958.21	4	1	*Ti* I 8.21 (80), *Zr* II 8.24 (50)
3958.75	1	1	....
3959.24	1n	1	(*Sc* II 9.36 (Pr))
3959.53	0n	1	*Fe* I 9.45 (Pr)
3959.88	1n	1	(*Cr* II 9.73 (1))
3960.28	3	1	*Fe* I 0.28 (1)
3960.61	0	1	(*Cr* I 0.76 (5))
3960.88	1	1	*Fe* II 0.90 (3), *Ce* II 0.91 (125)
3961.16	3	1	*Fe* I 1.15 (2)
3961.53	10	1	*Al* I 1.52 (10)
3962.11	2	1	*Ni* I 2.12 (3), (*Cr* I 2.19 (3))
3962.38	3	1	*Fe* I 2.35 (2), *Fe* I 2.40 (Pr)
3962.70	0	1	*Fe* I 2.65 (Pr)
3962.86	2	1	*Ti* I 2.85 (35)
3963.11	4	1	*Fe* I 3.11 (6)
3963.44	1	1	*Fe* I 3.43 (Pr)
3963.69	3	1	*Cr* I 3.69 (100)
3964.24	2n	1	*Ti* I 4.27 (35)
3964.54	3	1	*Fe* I 4.52 (3), *Fe* II 4.57 (Pr)
3964.92	1n	1	*Pr* II 4.82 (250)
3965.18	0n	1	*Pr* II 5.26 (150)
3965.52	3	1	*Fe* I 5.51 (1), *Fe* I 5.43 (1)
3966.06	5	1	*Fe* I 6.07 (10)
3966.61	4	1	*Fe* I 6.63 (10), *Fe* I 6.53 (1)
3967.43	0	1	*Fe* I 7.42 (8)
3968.60	80nn	1	*Ca* II 8.47 (350)
3969.28	1	1	*Fe* I 9.26 (30), *Fe* II 9.38 (Pr), *Fe* II 9.40 (Pr)
3970.13	3n	1	*He* 0.08, *Cr* I 9.75 (70), *Fe* I 0.39 (4)
3971.32	3	1	*Fe* I 1.32 (9), *Cr* I 1.26 (20)
3971.63	1	1	*Ce* II 1.68 (100)
3971.84	0b	1	*Fe* I 1.82 (1)
3971.98	1b	1	*Eu* II 1.98 (4000)

TABLE 1—*Continued*

λ	Int.	N	Identification
3972.17..	2	1	*Ni* I 2.17 (10), Pr II 2.16 (100)
3972.56..	1n	1	*Co* I 2.51 (6), *Ca* I 2.57 (1)
3972.88..	1b	1	*Fe* I 2.92 (1)
3973.20..	1n	1	*Co* I 3.14 (10), *Nd* II 3.27 (80)
3973.64..	9	1	*V* II 3.64 (300), *Fe* I 3.66 (3), *Ca* I 3.71 (12), *Ni* I 3.56 (25)
3973.91..	1	1	....
3974.18..	3	1	*Fe* II 4.16 (3)
3974.44..	2	1	*Fe* I 4.40 (1)
3974.72..	3n	1	*Co* I 4.73 (10), *Ni* I 4.65 (10), *Fe* I 4.76 (1), *Fe* I 4.64 (Pr)
3975.16..	1n	1	*Fe* I 5.21 (1), *Fe* II 5.03 (2)
3975.40..	0	1	(*Co* I 5.32 (3))
3975.66..	0	1	*Ti* I 5.69 (Pr)
3975.86..	2	1	*Fe* I 5.84 (1)
3976.13..	2	1	....
3976.38..	1	1	*Fe* I 6.39 (1)
3976.64..	5	1	*Cr* I 6.66 (100), *Fe* I 6.62 (4)
3976.87..	3	1	*Fe* I 6.86 (1)
3977.16..	1	1	*Co* I 7.18 (3)
3977.76..	7	1	*Fe* I 7.74 (12), *V* II 7.73 (60)
3978.15..	1n	1	....
3978.42..	2n	1	*Fe* I 8.46 (1)
3978.67..	3	1	*Cr* I 8.68 (18), *Co* I 8.65 (10), *Ce* II 8.65 (125)
3979.16..	1n	1	*Fe* I 9.12 (Pr)
3979.54..	3n	1	*Cr* II 9.51 (20), *Co* I 9.52 (10), *Fe* I 9.63 (1), (*Nd* II 9.48 (60))
3979.98..	1	1	....
3980.62..	1	1	*Fe* I 0.65 (1)
3981.09..	1n	1	*Fe* I 1.10 (1), *Cr* I 1.23 (15)
3981.63..	1	1	*Fe* II 1.61 (Pr)
3981.78..	4	1	*Fe* I 1.78 (7), *Ti* I 1.76 (70)
3982.01..	4	1	*Ti* II 2.00 (tr), *Zr* II 2.01 (3)
3982.26..	0	1	(*Nd* II 2.36 (20)
3982.59..	6	1	*Y* II 2.59 (150)
3983.03..	1nn	1	(*Ce* II 2.90 (60))
3983.33..	1n	1	*Fe* I 3.35 (1)
3983.56..	1b	1	....
3983.95..	6	1	*Fe* I 3.96 (10), *Cr* I 3.91 (100)
3984.16..	1	1	*Ni* I 4.14 (8)
3984.34..	2	1	*Cr* I 4.34 (25), *V* I 4.34 (6), *Ti* I 4.31 (3)
3984.67..	3	1	*Ce* II 4.68 (100), *V* I 4.60 (6), *Zr* II 4.76 (4)
3984.95..	1	1	*Fe* I 4.94 (Pr)
3985.38..	5	1	*Fe* I 5.39 (3), *Fe* I 5.32 (Pr)
3985.82..	1n	1	*V* II 5.78 (30), *Fe* I 5.93 (Pr)
3986.19..	4	1	*Fe* I 6.18 (5)
3986.36..	1	1	*Fe* I 6.30 (Pr), *Mn* I 6.40 (2)
3986.76..	7	1	*Mg* I 6.75 (1)
3987.09..	2n	1	*Co* I 7.12 (6), *Ni* I 7.09 (2)
3987.62..	4	1	*Ti* II 7.63 (Pr)
3988.04..	1b	1	*Yb* I 7.99 (1000)
3988.23..	1b	1	....
3988.50..	3	1	*La* II 8.51 (500)
3989.02..	5	1	*Sc* II 9.06 (2)
3989.44..	0	1	*Ce* II 9.44 (30)
3989.80..	9	1	*Ti* I 9.76 (80), *Fe* I 9.86 (2)
3990.08..	0n	1	*Cr* I 9.99 (15), *Sm* II 0.08 (200), (*Nd* II 0.10 (60))
3990.38..	4	1	*Fe* I 0.38 (2)
3991.12..	4	1	*Zr* II 1.14 (40), *Cr* I 1.12 (80)
3991.42..	2	1	(*Co* I 1.53 (4))
3991.73..	3	1	*Cr* I 1.67 (25), *Nd* II 1.74 (80), *Co* I 1.68 (6)
3992.28..	2	1	*Fe* I 2.40 (1), *Ce* II 2.39 (125), *Cr* I 2.11 (4)
3992.83..	2	1	*Cr* I 2.84 (30)
3993.12..	1	1	....
3993.65..	1n	1	....
3993.97..	2b	1	*Cr* I 3.97 (15), *Ce* II 3.82 (200), *Ni* I 3.95 (3)
3994.11..	4b	1	*Fe* I 4.12 (2), (*Fe* I 4.27 (Pr))
3994.61..	2n	1	*Co* I 4.54 (6), *Nd* II 4.68 (80), *Ti* I 4.68 (4)
3995.30..	7	1	*Co* I 5.31 (60), *Fe* I 5.20 (1)
3995.75..	2	1	*La* II 5.74 (400)
3996.00..	4	1	*Fe* I 6.00 (4)
3996.28..	2	1	*Fe* I 6.26 (Pr), *Fe* II 6.36 (Pr)
3996.76..	0	1	*Fe* I 6.79 (Pr)
3996.99..	2	1	*Fe* I 6.97 (2)
3997.11..	2	1	*V* II 7.13 (200)
3997.41..	5	1	*Fe* I 7.39 (15), *Fe* I 7.49 (Pr)
3997.92..	3	1	*Co* I 7.90 (40)
3998.07..	4	1	*Fe* I 8.05 (10)
3998.50..	0	1	*Fe* I 8.48 (Pr)
3998.64..	5	1	*Ti* I 8.64 (100)
3998.99..	3	1	*Zr* II 8.98 (30)
3999.26..	1	1	*Ce* II 9.24 (500), *V* II 9.20 (30), *Ti* I 9.34 (7)
3999.65..	1	1	*Cr* I 9.68 (7)
3999.95..	1n	1	*Fe* I 0.02 (1)
4000.28..	3	1	*Fe* I 0.27 (1)
4000.48..	4	1	*Fe* I 0.47 (2), (*Dy* II 0.45 (800))
4000.83..	1	1	....
4001.14..	1	1	....
4001.42..	2	1	*Cr* I 1.44 (25)
4001.68..	5	1	*Fe* I 1.67 (5)
4002.08..	3	1	*Fe* II 2.07 (2)
4002.52..	2n	1	*Fe* II 2.55 (3), *Fe* I 2.66 (1), *Cr* II 2.48 (5), *Ti* I 2.47 (9)
4002.94..	4	1	*V* II 2.94 (80)
4003.29..	2	1	*Cr* II 3.33 (25)
4003.49..	1	1	....
4003.76..	3	1	*Fe* I 3.76 (2), *Ti* I 3.79 (10), *Ce* II 3.77 (100)
4004.25..	0nn	1	*Fe* II 4.15 (Pr)
4004.89..	7	2	*Fe* I 4.83 (1), *Fe* I 4.98 (1)
4005.27..	8	2	*Fe* I 5.25 (25)
4005.71..	6	2	*V* II 5.71 (800)
4005.97..	0	1	*Ti* I 5.95 (6)

TABLE 1—*Continued*

λ	Int.	N	Identification
4006.16..	1b	2	*Fe* I 6.16 (Pr), *Ni* I 6.14 (3)
4006.33..	4b	2	*Fe* I 6.31 (3)
4006.70..	3n	2	*Fe* I 6.63 (2), *Fe* I 6.77 (1)
4006.99..	1	1	(*Cr* II 7.04 (Pr))
4007.28..	4	2	*Fe* I 7.28 (6), *Fe* I 7.23 (1)
4007.66..	0n	1	*Fe* II 7.72 (Pr), *Cr* II 7.56 (2)
4007.95..	2nn	2	*Ti* I 8.05 (9)
4008.19..	1b	2	*V* II 8.17 (20)
4008.46..	0	1	(*Sc* II 8.41 (Pr))
4008.89..	4	2	*Ti* I 8.93 (35)
4009.17..	2	2	....
4009.50..	1	1	*Fe* I 9.55 (Pr)
4009.72..	5	2	*Fe* I 9.71 (10)
4009.93..	0	1	*Ni* I 9.98 (3)
4010.18..	1	2	*Fe* I 0.18 (1)
4010.41..	1	1	....
4010.59..	2	2	....
4010.80..	1	2	*Fe* I 0.77 (1)
4010.97..	1	2	....
4011.42..	3	2	*Fe* I 1.41 (1)
4011.72..	2	2	*Fe* I 1.71 (1)
4011.97..	0	1	*Fe* I 1.90 (Pr)
4012.40..	10n	2	*Ti* II 2.37 (4), *Cr* II 2.50 (30), *Ce* II 2.39 (300)
4013.24..	1n	2	*Ti* I 3.24 (Pr)
4013.64..	2	2	*Fe* I 3.64 (2), *Ti* I 3.59 (12)
4013.83..	4	2	*Fe* I 3.82 (2), *Fe* I 3.80 (1)
4014.24..	1	2	*Fe* I 4.28 (1)
4014.54..	6	2	*Fe* I 4.53 (10), *Sc* II 4.49 (5)
4014.93..	1	2	*Ce* II 4.90 (125)
4015.13..	0	2	*Fe* II 5.20 (Pr)
4015.46..	1b	1	*Ti* I 5.38 (12), *Ni* II 5.50 (1)
4015.58..	4n	2	....
4015.96..	1	2	(*Ce* II 5.88 (20))
4016.19..	0	1	*Ti* I 6.26 (6)
4016.43..	4	2	*Fe* I 6.43 (2), *Fe* I 6.54 (1)
4016.79..	1	2	*V* II 6.82 (20)
4017.15..	6	2	*Fe* I 7.16 (6), *Fe* I 7.09 (1)
4017.52..	4n	2	(*Ni* I 7.56 (6))
4017.79..	1	2	*Ti* I 7.77 (15)
4018.10..	5	2	*Mn* I 8.10 (20)
4018.28..	4	2	*Fe* I 8.28 (4), *Zr* II 8.38 (10)
4018.59..	0	2	*Fe* II 8.49 (Pr)
4018.85..	1b	2	(*Nd* II 8.83 (30))
4019.08..	2n	2	*Fe* I 9.05 (1), *Ni* I 9.06 (3)
4019.58..	1n	1	....
4020.06..	1b	2	*Fe* I 0.02 (Pr)
4020.28..	2b	2	....
4020.48..	1b	2	*Fe* I 0.49 (1), *Sc* I 0.40 (75)
4020.91..	2	2	*Co* I 0.90 (20)
4021.33..	1	2	*Nd* II 1.33 (80)
4021.62..	1	2	*Fe* I 1.62 (1)
4021.89..	6	2	*Fe* I 1.87 (12), *Ti* I 1.81 (25)
4022.27..	2	2	*Cr* I 2.26 (18)
4022.50..	0	2	*Fe* I 2.45 (1)
4022.75..	2	2	*Fe* I 2.74 (1)
4023.02..	1	2	*Nd* II 3.00 (80)
4023.39..	5	2	*V* II 3.39 (600)
4023.69..	2	2	*Sc* I 3.69 (100)
4024.07..	3n	2	*Fe* I 4.11 (1)
4024.55..	3	2	*Fe* II 4.55 (5), *Ti* I 4.57 (35), *Zr* II 4.45 (12)
4024.74..	4	2	*Fe* I 4.74 (6)
4025.13..	6	2	*Ti* II 5.14 (2), *Ni* I 5.11 (3)
4025.46..	1	2	*Cr* I 5.44 (5), *Ni* I 5.44 (1)
4025.83..	3	2	....
4026.16..	1	2	*Cr* I 6.17 (18)
4026.46..	2	2	*Ti* I 6.54 (25), *Mn* I 6.44 (4)
4027.05..	1n	2	*Co* I 7.03 (10), *Cr* I 7.10 (20)
4027.41..	1	2	*Ti* I 7.43 (4)
4027.68..	2	2	....
4027.96..	1	2	....
4028.35..	6	2	*Ti* II 8.33 (7), *Ce* II 8.41 (150)
4028.76..	2	2	....
4029.08..	0n	2	....
4029.40..	1	2	(*Ni* I 9.32 (Pr))
4029.65..	5	2	*Fe* I 9.64 (3), *Ti* II 9.64 (Pr), *Zr* II 9.68 (20)
4029.88..	0	2	....
4030.20..	3	2	*Fe* I 0.19 (3)
4030.48..	4	2	*Fe* I 0.50 (6), *Ti* I 0.51 (25)
4030.79..	9	3	*Mn* I 0.76 (200)
4031.11..	0b	1	*Cr* I 1.13 (7)
4031.27..	1n	3	*Fe* I 1.24 (2), *Ce* II 1.34 (150), *Zr* II 1.35 (2)
4031.48..	1b	1	*Fe* II 1.46 (1)
4031.73..	3	3	*Fe* I 1.72 (Pr), *La* II 1.68 (300), *Ti* I 1.75 (3)
4031.97..	4	3	*Fe* I 1.96 (4)
4032.47..	3	3	*Fe* I 2.47 (1)
4032.65..	4	3	*Fe* I 2.63 (4), *Ti* I 2.63 (3)
4033.08..	10	3	*Mn* I 3.07 (150)
4033.60..	1	3	....
4033.90..	0b	2	*Ti* I 3.88 (6), *Pr* II 3.86 (75)
4034.13..	1	3	*Zr* II 4.10 (5), (*Cr* I 3.95 (3))
4034.49..	8	3	*Mn* I 4.49 (100)
4034.90..	0n	3	*Ti* I 4.88 (5)
4035.21..	1	3	*Fe* I 5.25 (Pr)
4035.67..	9	3	*V* II 5.63 (400), *Mn* I 5.73 (15)
4036.04..	0	1	*Fe* I 5.99 (Pr), (*Ni* I 5.96 (Pr))
4036.39..	1n	2	*Fe* I 6.38 (Pr)
4036.77..	2	3	*V* II 6.78 (60)
4037.12..	1	3	....
4037.36..	0	2	*Gd* II 7.34 (200), *Cr* I 7.29 (10)
4037.59..	1	1	....
4037.65..	1	2	*Fe* I 7.72 (1), *Ce* II 7.66 (25)
4038.00..	1	2	*Cr* II 8.03 (25)
4038.21..	1	1	*Ni* I 8.27 (Pr)
4038.62..	0b	1	*Fe* I 8.62 (−)
4038.79..	2	3	....
4039.08..	1	3	*Cr* I 9.10 (20)
4039.56..	1	2	*V* II 9.57 (20)
4039.92..	0	1	*Fe* I 9.94 (1)
4040.09..	3	3	(*Fe* I 0.09 (1))
4040.65..	4	3	*Fe* I 0.65 (4)

TABLE 1—*Continued*

λ	Int.	N	Identification
4040.80..	1b	2	*Ce* II 0.76 (300), *Nd* II 0.80 (100)
4041.35..	7	3	*Mn* I 1.36 (50), *Fe* I 1.29 (1)
4041.67..	2	3	*Cr* I 1.79 (6), *Fe* II 1.64 (Pr), (*Sm* II 1.68 (200))
4041.97..	0	3	*Fe* I 1.91 (—)
4042.27..	0	2	*Cr* I 2.25 (8)
4042.58..	1	2	*Ce* II 2.58 (200), (*V* I 2.64 (5))
4042.90..	1	3	*La* II 2.91 (300)
4043.32..	0	3	....
4043.95..	8	3	*Fe* I 3.90 (5), *Fe* II 4.01 (2), *Fe* I 3.99 (Pr)
4044.20..	0	2	(*K* I 4.14 (8))
4044.59..	7	3	*Fe* I 4.61 (6)
4044.84..	0	1	*Pr* II 4.82 (60)
4045.13..	3	3	*Fe* I 5.14 (1), *Mn* I 5.13 (2)
4045.39..	1	2	*Co* I 5.39 (20)
4045.64..	3	1	*Zr* II 5.63 (15), *Fe* I 5.60 (Pr)
4045.84..	17	3	*Fe* I 5.82 (60)
4046.01..	2	1	*Fe* I 6.08 (Pr), (*Dy* I 5.98 (400))
4046.42..	1	3	*Fe* I 6.46 (Pr), *V* II 6.27 (50), *Ce* II 6.34 (100)
4046.76..	1	2	*Cr* I 6.76 (6), *Fe* II 6.81 (Pr), *Ni* I 6.76 (2)
4047.03..	1	2	....
4047.29..	1	3	*Fe* I 7.32 (1)
4047.67..	0	2	(*Y* I 7.63 (80))
4048.04..	1	2	*Cr* II 8.02 (Pr)
4048.33..	0	2	....
4048.74..	7	3	*Mn* I 8.76 (15), *Cr* I 8.78 (20), *Zr* II 8.68 (25)
4048.98..	0	1	*Fe* II 8.83 (3), *Mn* I 9.00 (2)
4049.10..	0	2	*Cr* II 9.14 (18)
4049.35..	3	3	*Fe* I 9.33 (1)
4049.60..	0	1	(*Gd* II 9.44 (150))
4049.81..	1	3	*Cr* I 9.78 (5), *Gd* II 9.90 (200)
4050.10..	1	1	*La* II 0.08 (200), *Cr* I 0.02 (4)
4050.35..	1	3	*Zr* II 0.32 (15)
4050.69..	3	3	(*Fe* I 0.69 (1))
4051.09..	1	3	*V* II 1.06 (20), *Nd* II 1.14 (60), *V* I 0.96 (10)
4051.35..	0	3	*V* II 1.34 (100), *V* I 1.35 (12), (*Ru* I 1.40 (125))
4051.94..	5	3	*Fe* I 1.92 (2), *Cr* II 1.97 (12)
4052.31..	2	2	*Fe* I 2.31 (1)
4052.47..	4	3	*Fe* I 2.47 (1), *Mn* I 2.47 (2)
4052.70..	3	3	*Fe* I 2.72 (1), *Fe* I 2.66 (1)
4052.96..	1	2	*Co* I 2.91 (3), *Ti* I 2.93 (2)
4053.27..	3	3	(*Fe* I 3.27 (1))
4053.49..	1	2	*V* II 3.59 (60), *Cr* II 3.45 (1), *Ce* II 3.51 (100)
4053.83..	4	3	*Ti* II 3.81 (3), *Fe* I 3.82 (1)
4054.13..	3	3	*Cr* II 4.11 (8), *Fe* I 4.18 (1)
4054.44..	1	3	....
4054.85..	5	3	*Fe* I 4.88 (3), *Fe* I 4.83 (1)
4055.05..	4	3	*Fe* I 5.04 (3), *Ti* I 5.01 (20)
4055.54..	5	3	*Mn* I 5.54 (20)
4056.03..	1b	2	*Cr* II 6.07 (4), *Fe* I 5.98 (1)
4056.22..	2b	3	*Ti* II 56.21 (1)
4056.42..	2b	3	*Fe* I 6.53 (1), (*Pr* II 6.54 (80))
4056.80..	1n	1	*Cr* I 6.79 (5)
4056.92..	1n	2	....
4057.33..	2b	2	*Fe* I 7.35 (2), *Ni* I 7.35 (2)
4057.52..	9b	3	*Mg* I 7.50 (5), *Fe* II 7.46 (2)
4057.91..	1	3	*Mn* I 7.95 (4), *Cr* I 7.81 (8), (*Pb* I 7.82 (2000))
4058.23..	5	3	*Fe* I 8.23 (4), *Co* I 8.18 (8), *Ti* I 8.14 (7)
4058.77..	4b	2	*Fe* I 8.77 (3), *Cr* I 8.77 (20)
4058.90..	4b	3	*Mn* I 8.93 (10), (*Ca* I 8.91 (1))
4059.37..	1	3	*Mn* I 9.39 (5)
4059.72..	3	3	*Fe* I 9.73 (3)
4060.22..	1n	3	*Ti* I 0.26 (20)
4060.48..	0n	2	(*Cr* I 0.62 (8))
4060.75..	1	3	....
4061.10..	3	3	(*Nd* II 1.08 (200))
4061.43..	1	2	....
4061.72..	1	3	*Mn* I 1.74 (5), *Fe* II 1.79 (1), *Cr* II 1.77 (Pr)
4061.98..	3	3	(*Fe* I 1.96 (1))
4062.46..	5	3	*Fe* I 2.45 (10)
4062.74..	0	3	*Pr* II 2.82 (125)
4063.00..	1	2	....
4063.29..	4	3	*Fe* I 3.29 (3)
4063.61..	13	3	*Fe* I 3.60 (45)
4064.03..	1	3	*Fe* I 4.05 (Pr), *V* I 3.93 (10), *Cr* II 3.94 (Pr)
4064.43..	3n	3	*Fe* I 4.45 (2), *Ti* II 4.35 (1), *Ni* I 4.37 (2)
4065.06..	2	3	*V* II 5.07 (100), *Ti* I 5.09 (15)
4065.38..	3	3	*Fe* I 5.39 (2)
4065.55..	1	1	(*Ti* I 5.60 (0))
4065.75..	1	3	*Cr* I 5.72 (12)
4066.11..	1n	2	*Cr* II 6.16 (Pr), *Fe* I 6.01 (Pr)
4066.37..	1	3	*Fe* II 6.33 (2), *Co* I 6.36 (15)
4066.60..	2	3	*Fe* I 6.59 (1)
4066.99..	5	3	*Fe* I 6.98 (6), *Ni* II 7.05 (3)
4067.28..	4	3	*Fe* I 7.28 (4)
4067.68..	0	2	*Fe* I 7.60 (Pr)
4067.99..	6	3	*Fe* I 7.98 (8)
4068.33..	1	3	(*Ti* I 8.14 (3))
4068.54..	1n	2	*Co* I 8.54 (8), (*Ti* I 8.66 (1))
4068.88..	0	1	*Ce* II 8.84 (75)
4069.06..	1nn	3	*Fe* I 9.08 (1), *Ti* I 8.98 (4)
4069.33..	1	2	*Nd* II 9.27 (80), (*Ni* I 9.24 (2))
4069.60..	2	2	....
4070.00..	1	3	*Fe* II 0.03 (Pr), *Fe* II 9.88 (1)
4070.27..	2	3	*Mn* I 0.28 (5)
4070.78..	6	3	*Fe* I 0.77 (5)
4071.10..	1	3	*Zr* II 1.09 (4)
4071.53..	2b	3	*Fe* I 1.52 (1), *V* I 1.54 (8), (*Ti* I 1.47 (2))
4071.75..	12	3	*Fe* I 1.74 (40)

TABLE 1—*Continued*

λ	Int.	N	Identification
4072.52	4	3	*Fe* I 2.52 (2), *Cr* II 2.56 (4)
4072.91	1	3	*Ni* I 2.91 (2), (*Ce* II 2.92 (20))
4073.14	1b	2	....
4073.48	1	2	*Ce* II 3.48 (200)
4073.77	4	3	*Fe* I 3.76 (4)
4074.07	1	3	....
4074.31	1n	2	*Ti* I 4.36 (3)
4074.77	5	3	*Fe* I 4.79 (5), *Fe* I 4.69 (Pr)
4075.09	2	3	(*Nd* II 5.12 (60))
4075.35	1	2	(*Nd* II 5.27 (50))
4075.69	0	3	*Ce* II 5.71 (150), *Cr* II 5.63 (Pr)
4075.94	4	3	*Cr* II 5.97 (4), *Cr* I 5.92 (6), (*Fe* II 5.95 (Pr))
4076.24	2	3	*Fe* I 6.23 (1)
4076.51	1b	1	*Fe* I 6.50 (1)
4076.60	5b	3	*Fe* I 6.64 (8)
4076.81	4	3	*Fe* I 6.81 (1), *Fe* I 6.88 (1), *Cr* II 6.87 (3)
4077.13	1	1	*Fe* II 7.16 (3), *Cr* I 7.09 (12), (*Ti* I 7.15 (4))
4077.35	2	3	*La* II 7.35 (300), (*Y* I 7.37 (300))
4077.73	12	3	*Sr* II 7.71 (400)
4078.39	5	3	*Fe* I 8.36 (4), *Ti* I 8.47 (30)
4078.83	1	2	....
4079.23	3	3	*Mn* I 9.24 (12), *Fe* I 9.19 (Pr)
4079.42	3	3	*Mn* I 9.42 (10)
4079.85	4	3	*Fe* I 9.85 (4)
4080.22	4	3	*Fe* I 0.23 (2)
4080.52	0	3	(*Cr* I 0.56 (2))
4080.89	3	3	*Fe* I 0.89 (1)
4081.25	2	3	*Ce* II 1.22 (150), *Fe* II 1.42 (Pr)
4081.75	1	3	*Cr* I 1.74 (5)
4082.11	3	3	*Fe* I 2.12 (1)
4082.44	2	3	*Fe* I 2.43 (2), *Ti* I 2.46 (20)
4082.94	4	3	*Mn* I 2.94 (12)
4083.24	1	3	*Ce* II 3.23 (200)
4083.61	7b	3	*Mn* I 3.63 (12), *Fe* I 3.55 (1)
4083.72	4b	3	*Fe* I 3.78 (1)
4084.14	0	1	*Fe* I 4.15 (Pr)
4084.50	6	3	*Fe* I 4.50 (6)
4085.02	5	3	*Fe* I 5.01 (4)
4085.30	5	3	*Fe* I 5.31 (4)
4085.70	1	2	*Zr* II 5.68 (5), *Gd* II 5.65 (200), *V* II 5.67 (10)
4086.02	2nn	3	*Fe* I 5.98 (1)
4086.33	2n	3	*Co* I 6.30 (15), *Cr* II 6.14 (8)
4086.72	3	3	*La* II 6.72 (300)
4087.11	3	3	*Fe* I 7.10 (1)
4087.28	1	1	*Fe* II 7.27 (Pr)
4087.61	1n	2	*Cr* II 7.63 (2)
4087.80	0n	1	*Fe* I 7.80 (Pr)
4088.00	0	2	....
4088.19	1	1	....
4088.58	2	3	*Fe* I 8.57 (1)
4088.81	1	2	*Fe* II 8.75 (Pr), *Cr* II 8.90 (1)
4089.23	3	3	*Fe* I 9.22 (1)
4089.54	0	1	*Cr* II 9.49 (2)
4089.66	1	2	(*Cr* I 9.63 (2))
4090.07	2n	3	*Fe* I 0.08 (1)
4090.53	2	3	*Zr* II 0.52 (10), *V* I 0.58 (25)
4090.97	3	3	*Fe* I 0.98 (1)
4091.28	1	2	....
4091.56	3	3	*Fe* I 1.56 (1)
4091.81	0	2	....
4092.34	6	3	*Co* I 2.39 (25)
4092.65	4	3	*Ca* I 2.63 (8), *V* I 2.69 (50)
4093.09	1	2	*Cr* I 3.06 (4)
4093.96	1n	2	*Ce* II 3.96 (30), (*Mg* II 3.90 (1))
4094.39	1	3	....
4094.68	0	1	(*CN* 1, 2 R 4.64)
4094.94	3	3	*Ca* I 4.93 (12)
4095.32	1	2	*Fe* I 5.27 (Pr)
4095.44	1	2	*V* I 5.49 (25)
4095.62	0	1	*Fe* I 5.65 (Pr)
4096.00	4b	3	*Fe* I 5.98 (4)
4096.11	2b	2	*Fe* I 6.11 (1), *Fe* I 6.22 (Pr)
4096.65	1	3	*Zr* II 6.63 (4)
4097.09	3	3	*Fe* I 7.10 (1), *Fe* I 7.02 (Pr), (*Fe* I 6.95 (Pr))
4097.37	1	2	....
4097.62	1	3	*Cr* I 7.65 (5)
4098.19	4	3	*Fe* I 8.18 (4), *Cr* I 8.18 (7)
4098.56	4	3	*Ca* I 8.53 (15)
4098.79	0	1	(*CN* 2, 3 R 8.86), (*Ce* II 8.98 (15))
4099.05	1	2	*Fe* I 9.08 (1), *Cr* I 9.02 (6)
4099.35	1	1	*Ti* I 9.17 (8), *La* II 9.54 (150)
4099.78	1	2	*V* I 9.80 (60)
4100.17	2	2	*Fe* I 0.17 (3)
4100.76	4	2	*Fe* I 0.74 (3)
4101.26	0	1	*Fe* I 1.27 (1)
4101.72	35	2	*H*δ 1.74
4102.56	0	1	(*Y* I 2.38 (350))
4102.94	3	3	*Si* I 2.93 (25)
4103.32	0	1	*Dy* II 3.31 (600)
4103.61	1n	2	*Fe* I 3.62 (Pr)
4103.78	0	1	*Cr* I 3.85 (4)
4104.15	3	3	*Fe* I 4.13 (3), *Fe* II 4.18 (Pr)
4104.41	0	2	*Fe* I 4.47 (Pr)
4104.84	1n	2	*Cr* I 4.87 (10), *V* I 4.78 (15), *Fe* I 4.97 (1), *Co* I 4.74 (4)
4105.12	0	3	*V* I 5.17 (60), *Fe* I 5.06 (Pr), *Ce* II 05.00 (50)
4105.65	1	1	(*CN* 1, 2 R 5.67)
4106.27	1	3	*Fe* I 6.26 (1)
4106.45	2	3	*Fe* I 6.44 (1)
4106.96	1n	3	....
4107.49	6	3	*Fe* I 7.49 (12)
4107.85	0	1	(*CN* 1, 2 R 7.82)
4108.11	1n	3	*Fe* I 8.14 (Pr)
4108.54	2	3	*Ca* I 8.55 (10)
4108.82	1	2	....
4109.06	4	3	*Fe* I 9.07 (1)

TABLE 1—*Continued*

λ	Int.	N	Identification
4109.48..	2	3	*Nd* II 9.46 (200), (*Mg* II 9.54 (3)), (*Cr* I 9.58 (8))
4109.81..	5	3	*Fe* I 9.81 (9), *V* I 9.79 (50)
4110.03..	0	1	*Zr* II 0.05 (3)
4110.26..	1	2	*Ca* II 0.33 (0)
4110.55..	3	3	*Co* I 0.53 (25)
4110.88..	0	1	*Cr* I 0.87 (8)
4110.97..	4n	3	*Cr* II 1.01 (18), *Fe* I 1.06 (1)
4111.37..	2	3	*Cr* I 1.36 (6), *Ce* II 1.39 (60)
4111.78..	3	3	*V* I 1.78 (100)
4112.09..	0	1	*Fe* I 2.08 (Pr), *Fe* II 1.90 (1), (*CN* 1, 2 R 2.03)
4112.32..	3	3	*Fe* I 2.35 (1)
4112.66..	0	1	*Ti* I 2.71 (20), *Cr* II 2.59 (1)
4112.97..	5	3	*Fe* I 2.97 (3)
4113.21..	1	3	*Zn* I 3.21 (12), *Cr* II 3.24 (5)
4113.58..	1n	1	*V* I 3.52 (12)
4113.79..	1	2	*Ce* II 3.73 (30)
4114.06..	1n	2	*CN* 1, 2 R 4.11
4114.46..	5	3	*Fe* I 4.45 (5)
4114.94..	3	3	*Fe* I 4.96 (1)
4115.17..	2	3	*V* I 5.18 (60)
4115.44..	0	2	*Ce* II 5.37 (150)
4115.96..	1	3	*Ni* I 5.98 (3), *Fe* I 5.90 (Pr)
4116.49..	0	3	*V* I 6.47 (50)
4116.74..	1	2	(*V* I 6.70 (4))
4116.97..	1	3	*Fe* I 6.97 (1), *Ce* II 7.01 (75)
4117.28..	0	3	*Fe* I 7.32 (1), *Ce* II 7.29 (20)
4117.85..	3	3	*Fe* I 7.87 (1), *Fe* I 7.71 (1)
4118.18..	1	3	*V* I 8.18 (8), *Ce* II 8.14 (200)
4118.56..	6	3	*Fe* I 8.55 (15)
4118.83..	7n	3	*Co* I 8.77 (50), *Fe* I 8.90 (1)
4119.45..	2nn	3	*Fe* II 9.53 (Pr), (*Fe* I 9.67 (Pr))
4119.87..	1	3	*Ce* II 9.88 (20), *Ce* II 9.78 (20)
4120.22..	4	3	*Fe* I 0.21 (5)
4120.66..	1	2	*Cr* I 0.61 (12)
4120.81..	1	2	*Ce* II 0.83 (150)
4121.33..	6	3	*Co* I 1.32 (60)
4121.82..	5	3	*Fe* I 1.81 (5), *Cr* I 1.82 (10)
4122.16..	0	3	*Cr* I 2.16 (8), *Ti* I 2.14 (10), *CN* 1, 2 R 2.19
4122.54..	3	1	*Fe* I 2.52 (4)
4122.63..	6n	3	*Fe* II 2.64 (4)
4123.25..	2	3	*La* II 3.23 (400), *Ti* I 3.29 (5)
4123.48..	1	2	*Cr* I 3.39 (10), *Ce* II 3.49 (20)
4123.58..	1n	1	*V* I 3.57 (60), *Ti* I 3.56 (10)
4123.78..	4	3	*Fe* I 3.75 (1)
4123.91..	0	1	*Ce* II 3.87 (150), *Nd* II 3.88 (40)
4124.48..	1	3	*CN* 2, 3 R 4.52
4124.82..	3n	3	*Fe* II 4.79 (1), *Y* II 4.91 (15)
4125.20..	0	1	(*Fe* I 5.23 (Pr))
4125.36..	0	2	....
4125.63..	3	3	*Fe* I 5.62 (1)
4125.88..	2	3	*Fe* I 5.88 (2)
4126.20..	4	3	*Fe* I 6.19 (3)
4126.52..	1	3	*Cr* I 6.52 (20)
4126.87..	1	3	*Fe* I 6.88 (1)
4127.31..	0	3	*Ce* II 7.37 (150), *Cr* I 7.30 (7)
4127.65..	4	3	*Fe* I 7.61 (7), *Cr* I 7.64 (8)
4127.80..	3	2	*Fe* I 7.81 (3)
4128.11..	5	3	*Si* II 8.05 (8), *V* I 8.07 (60), (*Mn* II 8.14 (Pr))
4128.41..	1	2	(*Y* I 8.30 (300))
4128.75..	4	3	*Fe* II 8.74 (3)
4129.18..	3	3	*Cr* I 9.21 (20), *Fe* I 9.22 (1), *Ti* I 9.17 (4)
4129.48..	1	3	*Fe* I 9.47 (Pr)
4129.72..	2	3	*Eu* II 9.73 (5000)
4130.03..	2	2	*Fe* I 0.04 (1)
4130.24..	2	1	(*CN* 0, 1 R 0.28)
4130.36..	1	3	*Gd* II 0.38 (300)
4130.67..	3	3	*Ba* II 0.65 (80) *Ce* II 0.71 (100)
4130.92..	1	3	*Si* II 0.88 (10)
4131.16..	1	2	*Fe* II 1.17 (Pr), *Ce* II 1.10 (100), *Ti* I 1.24 (4)
4131.38..	0	3	*Cr* I 1.36 (10)
4132.04..	9	3	*Fe* I 2.06 (25), *V* I 2.02 (60)
4132.43..	1	1	*Cr* II 2.41 (7), (*CN* 0, 1 R 2.42)
4132.55..	2	3	*Fe* I 2.54 (Pr)
4132.91..	5	3	*Fe* I 2.90 (8)
4133.36..	1	3	*Nd* II 3.36 (50)
4133.63..	1	3	*CN* 1, 2 R 3.67
4133.85..	4	3	*Fe* I 3.87 (2), *Ce* II 3.80 (500)
4134.41..	4	3	*Fe* I 4.43 (1), *V* I 4.49 (60), *Fe* I 4.34 (1)
4134.69..	5	3	*Fe* I 4.68 (12)
4135.03..	1	3	....
4135.32..	1n	2	*Nd* II 5.32 (50)
4135.46..	0	1	*CN* 1, 2 R 5.47, *Ce* II 5.44 (20)
4135.74..	1n	2	*Fe* I 5.77 (1), *Cr* II 5.77 (Pr)
4136.53..	4	3	*Fe* I 6.51 (1)
4137.00..	5	3	*Fe* I 7.00 (7)
4137.39..	2	2	*Fe* I 7.42 (Pr), *CN* 1, 2 R 7.29, *Ti* I 7.28 (10)
4137.67..	1	3	*Ce* II 7.65 (400)
4137.99..	0	2	*Fe* I 7.98 (Pr)
4138.37..	2n	3	*Fe* II 8.40 (Pr)
4139.06..	0n	3	*CN* 1, 2 R 9.05
4139.41..	0	1	(*Co* I 9.45 (3)), (*Ti* I 9.48 (1))
4139.94..	4	3	*Fe* I 9.93 (2)
4140.42..	3	3	*Fe* I 0.44 (1)
4140.78..	0	2	*CN* 0, 1 R 0.79, *CN* 1, 2 R 0.79
4141.06..	1	3	....
4141.43..	0	1	*Fe* I 1.35 (1)
4141.86..	3	3	*Fe* I 1.86 (1)
4142.28..	2b	3	*Cr* I 2.19 (7), *Ni* I 2.32 (4), *Ni* I 2.18 (2), (*Ti* II 2.2[illegible] (Pr))
4142.43..	3b	1	*Ce* II 2.40 (150), *Cr* I 2.47 (5), *Ti* I 2.48 (2)

TABLE 1—*Continued*

λ	Int.	N	Identification
4142.58..	2b	3	*Fe* I 2.62 (1)
4143.03..	1n	3	*Ti* I 3.05 (7), *Fe* II 3.07 (Pr), *Pr* II 3.14 (150), *Ni* I 2.97 (2), (*V* I 2.84 (200))
4143.44..	8	3	*Fe* I 3.42 (15), (*Fe* I 3.51 (Pr))
4143.88..	9	3	*Fe* I 3.87 (30)
4144.52..	1n	3	*Nd* II 4.55 (20), (*CN* 2, 3 R 4.48)
4145.13..	1n	3	*Fe* I 5.21 (1), *Ce* II 5.00 (60)
4145.57..	0	2	....
4145.77..	2	3	*Cr* II 5.77 (25)
4146.06..	3	3	*Fe* I 6.07 (2)
4146.50..	1	3	*Cr* I 6.47 (4)
4146.66..	0	1	*Cr* I 6.70 (6)
4146.99..	2	3	....
4147.34..	2	3	*Fe* I 7.35 (Pr), *Fe* II 7.26 (Pr)
4147.69..	6	3	*Fe* I 7.67 (10)
4148.25..	0n	3	*Fe* I 8.26 (Pr)
4148.78..	1n	2	*Ni* I 8.75 (Pr), (*CN* 0, 1 R 8.74)
4149.25..	4	3	*Zr* II 9.22 (75)
4149.36..	5	3	*Fe* I 9.37 (5)
4149.83..	1n	3	*Fe* I 9.77 (1), *Ce* II 9.94 (50)
4150.28..	3	3	*Fe* I 0.26 (4)
4150.44..	1	3	*Ni* I 0.37 (2), *Ti* I 0.56 (3)
4150.97..	2	3	*Zr* II 0.97 (10), *Ti* I 0.96 (10), *Cr* II 1.00 (5)
4151.36..	0	2	....
4151.74..	1	1	*Fe* II 1.79 (Pr), *Fe* II 1.60 (Pr)
4152.00..	2b	2	*Fe* I 1.96 (1), *La* II 1.98 (250), *Ce* II 1.97 (200), *Fe* I 2.08 (Pr)
4152.12..	7b	3	*Fe* I 2.17 (4)
4152.71..	1n	3	*La* II 2.78 (100), *Cr* I 2.78 (10), (*CN* 0, 1 R 2.56)
4153.02..	0	1	*Cr* I 3.07 (9), *Fe* II 2.98 (Pr)
4153.37..	1n	2	(*CH* 0, 0 R 3.37)
4153.90..	4	3	*Fe* I 3.91 (10)
4154.06..	2	2	*Fe* I 4.11 (1)
4154.51..	5	2	*Fe* I 4.50 (12)
4154.81..	4	2	*Fe* I 4.81 (9)
4155.38..	1n	1	(*Sm* II 5.32 (100))
4155.95..	1n	1	*Nd* II 6.08 (250)
4156.29..	4	3	*Zr* II 6.24 (15)
4156.78..	5	3	*Fe* I 6.80 (12), *Fe* I 6.67 (1)
4157.19..	1	3	....
4157.78..	5	3	*Fe* I 7.79 (8)
4158.39..	1	3	*Co* I 8.42 (4), *Fe* II 8.45 (Pr)
4158.81..	5	3	*Fe* I 8.80 (5)
4159.19..	4	3	....
4159.64..	0	3	*Ti* I 9.63 (9), *V* I 9.69 (8)
4160.08..	0	3	....
4160.35..	2	3	*Fe* II 0.28 (Pr)
4160.63..	1	3	*Fe* II 0.62 (Pr), *Fe* I 0.56 (1)
4160.78..	0	1	*Fe* I 0.78 (Pr)
4161.16..	4	3	*Zr* II 1.20 (20), *Fe* I 1.08 (1)
4161.53..	4	3	*Ti* II 1.52 (1), *Fe* I 1.49 (1)
4161.81..	3	3	*Sr* II 1.80 (30)
4162.24..	0	1	(*CN* 2, 3 R 2.30)
4162.58..	0n	2	(*CH* 0, 0 R 2.68, 2.48), (*CN* 1, 2 R 2.69)
4162.82..	0	1	(*Fe* I 2.91 (Pr))
4163.45..	0	1	*CN* 0, 1 R 3.42, *Fe* I 3.36 (Pr)
4163.65..	9	3	*Ti* II 3.64 (40), *Fe* I 3.68 (1), *Cr* I 3.62 (20)
4164.06..	1	2	*V* II 4.02 (15), (*CN* 1, 2 R 4.04)
4164.15..	1	1	*Ti* I 4.13 (4), *Pr* II 4.19 (100)
4164.32..	1	2	*Fe* I 4.26 (Pr), (*CH* 0, 0 R 4.31)
4164.74..	1	2	*Fe* I 4.80 (1)
4165.07..	0	2	*CN* 0, 1 R 5.13
4165.41..	3n	3	(*Fe* I 5.42 (1))
4165.57..	2b	2	*Cr* I 5.52 (15), *Ce* II 5.61 (200)
4166.04..	1	3	*Ba* II 6.00 (20), (*CN* 2, 3 R 6.08)
4166.30..	1	1	*Ti* I 6.31 (6)
4166.95..	1	1	*Ni* I 6.97 (3), *CN* 0, 1 R 6.83
4167.29..	9	3	*Mg* I 7.27 (10)
4167.91..	4n	3	*Fe* I 7.86 (2)
4168.34..	0	1	*CN* 0, 1 R 8.48, (*Cr* I 8.31 (2))
4168.62..	2	3	*Fe* I 8.62 (1)
4168.95..	3	3	*Fe* I 8.95 (1)
4169.32..	1	3	*Ti* I 9.33 (7)
4169.76..	2n	3	*Fe* I 9.77 (1)
4170.19..	1	2	*Cr* I 0.20 (15), *CN* 0, 1 R 0.12
4170.59..	1	2	*Cr* II 0.58 (Pr)
4170.97..	8	3	*Fe* I 0.91 (5), *Ti* I 1.02 (8)
4171.72..	1	3	*Fe* I 1.70 (2), *Cr* I 1.68 (12)
4171.92..	5	3	*Ti* II 1.90 (30), *Fe* I 1.90 (2), *Cr* II 1.92 (3)
4172.10..	4	3	*Fe* I 2.13 (5)
4172.50..	2	1	(*CH* 0, 0 R 2.56)
4172.69..	7	3	*Fe* I 2.75 (4), *Fe* I 2.64 (1)
4172.97..	1	3	*Fe* I 2.98 (Pr), *Ti* II 3.05 (Pr)
4173.46..	9n	3	*Fe* II 3.45 (8), *Fe* I 3.32 (2), *Ti* II 3.54 (1)
4173.68..	3	1	(*Y* II 3.76 (−))
4173.97..	4	2	*Fe* I 3.93 (2), *Ti* II 4.09 (2)
4174.36..	1	3	*Fe* I 4.42 (1)
4174.54..	0	1	*Ti* I 4.47 (3)
4174.91..	5	3	*Fe* I 4.92 (5)
4175.22..	0	2	*Cr* I 5.23 (8)
4175.64..	4	3	*Fe* I 5.64 (10)
4175.90..	1	2	*Cr* I 5.94 (15), *Fe* I 5.91 (Pr)
4176.08..	0	2	(*Ce* II 6.08 (12))
4176.57..	6	3	*Fe* I 6.57 (7)
4176.89..	1	1	*CN* 2, 3 P 6.95
4177.05..	1	3	*Fe* I 7.08 (1)
4177.61..	9	3	*Fe* I 7.60 (4), *Y* II 7.54 (125)
4178.06..	2	3	(*Sm* II 8.02 (100)), (*CH* 0, 0 R 8.00)

TABLE 1—*Continued*

λ	Int.	N	Identification
4178.40..	1	3	*V* II 8.39 (60)
4178.54..	0	1	*CN* 2, 3 P 8.57
4178.87..	5	3	*Fe* II 8.86 (8)
4179.37..	5	3	*Cr* II 9.43 (12), *V* I 9.42 (15), *Cr* I 9.26 (30)
4179.82..	1	3	*Zr* II 9.81 (15)
4180.12..	1	1	(*CN* 0, 1 P 0.10)
4180.39..	1	3	*Fe* I 0.40 (Pr)
4180.75..	1	3	(*Fe* II 0.97 (Pr)), (*CH* 0, 0 R 0.83)
4181.21..	1	2	*Ti* II 8.17 (Pr), *Fe* I 1.19 (Pr), (*CN* 1, 2 R 1.35)
4181.58..	1	2	*Fe* I 1.55 (Pr), *Cr* II 1.50 (1)
4181.78..	5	3	*Fe* I 1.76 (15)
4181.96..	2	2	....
4182.39..	4	3	*Fe* I 2.38 (4)
4182.77..	2	3	*Fe* I 2.77 (2), *Fe* II 2.69 (Pr)
4183.01..	0	3	*Fe* I 3.02 (1)
4183.43..	3	3	*V* II 3.44 (250)
4184.00..	4	3	....
4184.32..	3	3	*Ti* II 4.33 (0)
4184.54..	0	1	*Ni* I 4.48 (4)
4184.89..	5	3	*Fe* I 4.90 (10), *Cr* I 4.90 (12)
4185.34..	0	2	*Cr* I 5.34 (10)
4185.54..	1	2	*Cr* II 5.50 (Pr)
4185.81..	0	1	*CN* 1, 2 P 5.77, (*CN* 0, 1 P 5.77)
4186.12..	1	3	*Ti* I 6.12 (25)
4186.35..	0	1	*Cr* I 6.36 (15), *CN* 0, 1 R 6.32
4186.63..	2	3	*Ce* II 6.60 (600), *Zr* II 6.70 (12), *CH* 0, 0 R 6.61
4187.05..	6	3	*Fe* I 7.04 (20)
4187.60..	2	3	*Fe* I 7.59 (1)
4187.81..	6	3	*Fe* I 7.80 (20)
4188.16..	0	2	*Sm* II 8.13 (200)
4188.72..	4	3	....
4189.04..	1	3	*CH* 0, 0 R 9.09, 8.88
4189.55..	3	3	*Fe* I 9.56 (2)
4190.20..	1n	3	*Cr* I 0.16 (15), *Ti* II 0.29 (1), *CN* 1, 2 P 0.12
4190.81..	0n	3	*V* II 0.89 (10), *Co* I 0.71 (20), *CN* 1, 2 P 0.74, (*CN* 0, 1 P 0.82)
4191.43..	5	3	*Fe* I 1.44 (15)
4191.68..	3	3	*Fe* I 1.68 (2), *Cr* I 1.75 (10)
4192.07..	2	3	*Cr* I 2.10 (15), *Ni* II 2.07 (1)
4192.22..	0	1	*La* II 2.35 (100)
4192.60..	1	3	*CH* 0,0 R 2.56, *CN* 0,1 R 2.57
4192.90..	1	2	*CN* 1, 2 P 2.93, *Ce* II 3.09 (50), *CN* 1, 2 P 2.90
4193.30..	1n	2	*CN* 1, 2 P 3.41, *CN* 1, 2 P 3.38, (*CN* 0, 1 P 3.22), (*Mg* II 3.44 (2))
4193.69..	1n	3	*Cr* I 3.66 (40), *CN* 0, 1 R 3.73
4193.84..	0	1	*CN* 1, 2 P 3.86, *Ce* II 3.87 (35), *CN* 1, 2 P 3.83
4194.41..	0n	1	*Fe* I 4.49 (Pr), *CN* 1, 2 P 4.27, *CN* 1, 2 P 4.24, (*CN* 0, 1 P 4.36)

λ	Int.	N	Identification
4194.70..	0	1	*CN* 1, 2 P 4.67, *CH* 0, 0 R 4.71, *CN* 1, 2 P 4.64
4194.86..	1n	2	*CH* 0, 0 R 4.86, *CN* 0, 1 R 4.88, *Cr* I 4.95 (20), *CN* 1, 2 P 5.03, *CN* 1, 2 P 5.00, (*Dy* I 4.83 (500))
4195.35..	5	3	*Fe* I 5.34 (5), *Cr* II 5.41 (10)
4195.62..	3	3	*Fe* I 5.62 (3)
4195.98..	0	1	*CN* 0, 1 R 5.92, *CN* 1, 2 P 5. 95, *CN* 1, 2 P 5.92
4196.22..	5	3	*Fe* I 6.22 (4)
4196.57..	2	3	*Fe* I 6.53 (1), *La* II 6.55 (250)
4196.66..	0	1	*Ti* II 6.64 (Pr), *CN* 1, 2 P 6.75
4197.15..	1n	3	Band head *CN* (1, 2) 7.16, *Fe* I 7.10 (Pr)
4197.25..	1n	1	*Cr* I 7.23 (20)
4197.67..	0	2	(*CN* 0, 1 P 7.67)
4198.08..	3	3	*Ti* II 7.95 (Pr)
4198.30..	8	3	*Fe* I 8.31 (20), *Fe* I 8.27 (1)
4198.64..	5	3	*Fe* I 8.64 (4)
4199.11..	6	3	*Fe* I 9.10 (20)
4199.54..	0	3	....
4199.94..	3n	3	*Fe* I 9.97 (1)
4200.46..	1	3	*Ni* I 0.46 (5), *Ti* II 0.40 (Pr)
4200.92..	4	3	*Fe* I 0.93 (3)
4201.24..	1	2	....
4201.53..	0	1	(*CN* 0, 1 P 1.64)
4201.72..	1	3	*Fe* I 1.73 (1), *Ni* I 1.72 (5)
4202.04..	9	3	*Fe* I 2.03 (30)
4202.37..	2	3	*V* II 2.35 (150)
4202.74..	2	3	*Fe* I 2.76 (1)
4203.08..	1	2	(*Ce* II 2.94 (150))
4203.21..	1	1	....
4203.38..	0	1	*Fe* I 3.30 (1), *Ti* I 3.46 (8), (*CN* 0, 1 P 3.44)
4203.56..	2	3	*Fe* I 3.57 (1), *Cr* I 3.59 (18)
4203.96..	6	3	*Fe* I 3.99 (10), *Fe* I 3.95 (1)
4204.24..	1	1	*Cr* I 4.19 (8), *CN* 0, 1 P 4.30
4204.49..	1	3	*Cr* I 4.47 (12)
4204.74..	2	3	*CH* 0, 0 R 4.75, *V* II 4.69 (10)
4205.06..	4	3	*V* II 5.08 (250), *Eu* II 5.05 (6000)
4205.38..	1	1	*Mn* II 5.37 (Pr)
4205.51..	5	3	*Fe* I 5.55 (2), *Fe* II 5.48 (Pr)
4205.94..	0n	2	*Ti* II 5.92 (Pr), *CN* 0, 1 P 5.93, (*Zr* II 5.91 (2))
4206.18..	0	1	*Ca* II 6.21 (−)
4206.32..	0	1	*Mn* II 6.38 (0)
4206.70..	5	3	*Fe* I 6.70 (3)
4207.13..	4	3	*Fe* I 7.13 (4)
4207.41..	2	3	*CN* 0, 1 P 7.43, *Cr* II 7.35 (4)
4207.82..	1	3	....
4208.34..	0	3	*Cr* I 8.36 (15)
4208.60..	5	3	*Fe* I 8.61 (3)
4208.99..	4	3	*Zr* II 8.99 (30)
4209.37..	1	3	*Cr* I 9.37 (20)
4209.80..	2nn	3	*Cr* I 9.76 (15), *V* I 9.86 (20), *V* II 9.74 (10)

TABLE 1—*Continued*

λ	Int.	N	Identification
4210.36. .	7	3	*Fe* I 0.35 (15), *Fe* I 0.40 (Pr)
4210.93. .	1	3	*CH* 0, 0 R 0.96
4211.37. .	1	3	*Cr* I 1.35 (15)
4211.87. .	4	3	*Zr* II 1.88 (12), (*Fe* II 1.80 (Pr))
4212.15. .	0	1	*Fe* I 2.04 (Pr), *Gd* II 2.02 (200)
4212.29. .	1n	1	*CN* 0, 1 P 2.24, *CN* 0, 1 P 2.40
4212.63. .	2	3	*CH* 0, 0 R 2.66, 2.61
4213.13. .	1	3	*CN* 0, 1 P 3.14, *Cr* I 3.18 (10)
4213.35. .	0	1	*Fe* I 3.42 (Pr), *CN* 0, 1 P 3.27
4213.64. .	5	3	*Fe* I 3.65 (5)
4214.26. .	0	2	*CN* 0, 1 P 4.26, (*CN* 0, 1 P 4.36)
4214.56. .	0	1	*CN* 0, 1 P 4.57, (*CN* 0, 1 P 4.66)
4214.83. .	0	1	*CN* 0, 1 P 4.85, (*CN* 0, 1 P 4.93)
4215.05. .	1	3	*Gd* II 5.02 (150), (*Dy* I 5.17 (125))
4215.52. .	15	3	*Sr* II 5.52 (300)
4215.99. .	1	1	*Fe* I 5.97 (1), Band head *CN* (0, 1) 6.04
4216.20. .	5	3	*Fe* I 6.19 (8)
4216.58. .	1	3	. . . .
4216.87. .	0	1	*Cr* II 6.87 (1)
4217.14. .	1n	2	*Gd* II 7.20 (100), *Cr* II 7.07 (1)
4217.23. .	1	1	*CH* 0, 0 R 7.26, 7.19, *Ti* II 7.34 (Pr)
4217.57. .	5	3	*Fe* I 7.55 (7), *Cr* I 7.63 (30), *La* II 7.56 (200)
4218.21. .	2nn	3	*Fe* I 8.23 (Pr), *Ti* II 8.18 (Pr)
4218.41. .	1n	1	(*CH* 1, 1 R 8.38)
4218.74. .	1	3	*CH* 0, 0 R 8.72
4219.37. .	6	3	*Fe* I 9.36 (12)
4219.76. .	0	2	*Fe* I 9.74 (Pr)
4220.05. .	2	3	*Fe* I 0.05 (Pr), *V* II 0.05 (10)
4220.34. .	3	3	*Fe* I 0.35 (4)
4220.82. .	0n	2	(*Sm* II 0.66 (200))
4221.14. .	0	1	(*Dy* I 1.10 (250))
4221.50. .	1n	3	*Cr* I 1.57 (25)
4222.22. .	5	3	*Fe* I 2.22 (12)
4222.61. .	1	2	*Ce* II 2.60 (300), *Cr* I 2.73 (20)
4222.99. .	1	1	*Pr* II 2.98 (150)
4223.10. .	1	2	*CH* 1, 1 R 3.08
4223.49. .	0n	3	*CH* 0, 0 R 3.57, 3.47, *Cr* I 3.47 (7)
4223.74. .	1	2	*Fe* I 3.73 (Pr)
4224.17. .	5	3	*Fe* I 4.18 (6)
4224.51. .	4	3	*Fe* I 4.52 (3), *Cr* I 4.51 (18)
4224.85. .	2	3	*Cr* II 4.85 (20), *CH* 0, 0 R 4.84
4225.24. .	1	3	*V* II 5.23 (120)
4225.45. .	4	3	*Fe* I 5.46 (6)
4225.73. .	2	2	*Fe* I 5.72 (Pr)
4225.95. .	3	3	*Fe* I 5.96 (3)
4226.45. .	5	3	*Fe* I 6.43 (3)
4226.75. .	16	3	*Ca* I 6.73 (500)
4227.40. .	7	3	*Fe* I 7.43 (30)
4227.92. .	1	3	*CH* 1, 1 R 7.93
4228.34. .	1	3	. . . .
4228.75. .	1	3	*Fe* I 8.72 (1)
4229.56. .	3	3	*Fe* I 9.52 (1)
4229.79. .	3	3	*Fe* I 9.76 (1), *CH* 0, 0 R 9.90, 9.79
4230.21. .	0	3	*Cr* I 0.29 (4)
4230.54. .	1	3	*Fe* I 0.58 (1), *Cr* I 0.48 (25)
4231.02. .	3	3	*CH* 0, 0 R 1.00, *Ni* I 1.04 (5)
4231.63. .	1	3	*Zr* II 1.64 (8), (*Fe* I 1.52 (1)), (*CH* 1, 1 R 1.59)
4231.95. .	2	3	*V* II 2.06 (80)
4232.39. .	1	2	*V* I 2.46 (15), *Nd* II 2.38 (150)
4232.74. .	1	3	*Fe* I 2.73 (1)
4233.18. .	9	3	*Fe* II 3.17 (11), *Cr* II 3.25 (10)
4233.61. .	7	3	*Fe* I 3.61 (18)
4234.18. .	1	3	*V* I 4.00 (12), *V* II 4.25 (7)
4234.54. .	1n	3	*V* II 4.55 (40), *Cr* I 4.52 (12)
4235.23. .	5	3	*Mn* I 5.29 (8), *Mn* I 5.14 (6)
4235.75. .	1b	2	*Y* II 5.73 (20), *V* I 5.76 (10), *Fe* I 5.84 (Pr), *Fe* I 5.64 (Pr)
4235.94. .	7	3	*Fe* I 5.94 (25)
4236.32. .	0	3	*Ni* I 6.37 (2), (*Cr* II 6.33 (Pr))
4236.76. .	1	3	*Fe* I 6.76 (1)
4237.16. .	3	3	*Fe* I 7.16 (2), *Fe* I 7.08 (2), *CH* 0, 0 R 7.23, 7.16
4237.64. .	1	3	*Fe* I 7.68 (Pr), *Cr* I 7.71 (12)
4238.02. .	4	3	*Fe* I 8.03 (4)
4238.35. .	0	3	*La* II 8.38 (400)
4238.81. .	5	3	*Fe* I 8.82 (10)
4239.34. .	2	3	*Fe* I 9.37 (Pr), *Cr* II 9.31 (0)
4239.80. .	5n	3	*Fe* I 9.74 (3), *Fe* I 9.85 (2), *Mn* I 9.72 (5)
4240.40. .	3	3	*Fe* I 0.37 (2), *Ca* I 0.46 (6)
4240.70. .	1	3	*Cr* I 0.70 (30)
4241.15. .	1	3	*Fe* I 1.11 (1)
4241.60. .	0	3	(*Co* I 1.52 (2))
4242.36. .	5	3	*Cr* II 2.38 (30)
4242.68. .	4	3	*Fe* I 2.73 (2), *Fe* I 2.59 (1), *CH* 0, 0 R 2.59
4243.39. .	3n	3	*Fe* I 3.37 (2), *CH* 0, 0 R 3.43, 3.32
4243.81. .	1	3	*Fe* I 3.79 (1)
4244.28. .	0	3	*Mn* II 4.26 (1)
4244.77. .	0	3	*Ni* II 4.80 (1)
4245.29. .	6	3	*Fe* I 5.26 (6), *Fe* I 5.36 (tr)
4245.76. .	0	2	. . . .
4246.07. .	4	3	*Fe* I 6.09 (3), *Fe* I 6.02 (Pr)
4246.85. .	9	3	*Sc* II 6.83 (100)
4247.42. .	7	3	*Fe* I 7.43 (12)

TABLE 1—*Continued*

λ	Int.	N	Identification
4247.77..	0	2	*CH* 1, 1 R 7.71
4248.24..	4	3	*Fe* I 8.23 (4)
4248.71..	1	3	*CH* 0, 0 R 8.73, *Cr* I 8.73 (10), *Ce* II 8.68 (200)
4248.93..	0	2	*CH* 0, 0 R 8.93
4249.57..	2n	3	*CH* 0, 0 R 9.63, 9.49
4250.13..	8	3	*Fe* I 0.12 (25)
4250.80..	9	3	*Fe* I 0.79 (25)
4251.33..	1	3	....
4251.70..	1	3	*Gd* II 1.74 (300), *Ti* I 1.77 (2)
4252.05..	1	1	*Ti* II 2.05 (Pr)
4252.14..	1	2	*Ni* I 2.11 (2)
4252.66..	3	3	*Cr* II 2.62 (10)
4252.98..	1	1	*CH* 1, 1 R 3.00, *Mn* II 3.02 (2)
4253.29..	0	2	*Ce* II 3.36 (50), *Gd* II 3.37 (150), *CH* 1, 1 R 3.20
4253.91..	1	3	*Fe* I 3.91 (Pr)
4254.35..	8	3	*Cr* I 4.35 (1000)
4254.69..	1	1	....
4254.93..	2	2	*Fe* I 4.94 (1), *CH* 0, 0 R 4.98
4255.22..	1	1	*CH* 0, 0 R 5.23
4255.52..	1	3	*Fe* I 5.50 (1), *Cr* I 5.50 (25)
4255.84..	1	2	*CH* 0, 0 R 5.80, *Ce* II 5.78 (60)
4256.20..	1	2	*Fe* I 6.21 (3)
4256.76..	1	3	*Fe* I 6.79 (1)
4257.13..	0	1	*V* II 7.02 (15), *Ce* II 7.12 (20)
4257.26..	0	1	*Cr* I 7.37 (12)
4257.65..	2	3	*Mn* I 7.66 (5)
4258.19..	4n	3	*Fe* II 8.16 (3), *Zr* II 8.05 (12)
4258.29..	2b	1	*Fe* I 8.32 (2), (*Fe* II 8.35 (Pr))
4258.63..	2	3	*Fe* I 8.62 (1)
4258.96..	1	3	*Fe* I 8.96 (1)
4259.23..	1	3	*Mn* II 9.20 (0), *Cr* I 9.15 (10), *Fe* I 9.31 (Pr), (*V* I 9.31 (8))
4259.73..	0	3	*Ce* II 9.75 (15)
4260.07..	6	3	*Fe* I 0.00 (2), *Fe* I 0.14 (1)
4260.50..	10	3	*Fe* I 0.48 (35)
4260.73..	1	3	*Fe* I 0.74 (Pr), *V* II 0.75 (9), *Ti* I 0.74 (2)
4261.29..	1n	3	*Cr* I 1.35 (25), *CH* 0, 0 R 1.22
4261.56..	1	1	*Cr* I 1.62 (12), *CH* 0, 0 R 1.52, *Ti* I 1.61 (5)
4261.93..	4	3	*Cr* II 1.92 (20)
4262.34..	1	2	*Cr* I 2.38 (8), *Cr* I 2.13 (12)
4262.67..	1	3	*Sm* II 2.68 (300)
4263.14..	2	3	*Cr* I 3.14 (35), *Ti* I 3.13 (15)
4263.37..	0	1	*Ce* II 3.43 (40)
4263.57..	1	3	*La* II 3.59 (200)
4263.89..	0	1	*Fe* II 3.90 (1)
4264.22..	3	3	*Fe* I 4.21 (2)
4264.51..	0	1	(*Ce* II 4.37 (10))
4264.73..	2	3	*Fe* I 4.74 (1)
4265.27..	3	3	*Fe* I 5.26 (2)
4265.67..	1	1	*Ti* I 5.72 (4)
4265.92..	2	3	*Mn* I 5.92 (6)
4266.25..	1	2	*Cr* II 6.23 (Pr), *Ti* I 6.23 (3)
4266.70..	0	1	*Cr* I 6.82 (8), (*Zr* II 6.72 (11), (*Nd* II 6.72 (30))
4266.97..	4	3	*Fe* I 6.97 (3)
4267.41..	1	3	*CH* 0, 0 R 7.38
4267.83..	4	3	*Fe* I 7.83 (5)
4268.15..	1	3	*CH* 0, 0 R 8.09
4268.74..	3	3	*Fe* I 8.74 (2), *Cr* I 8.79 (10)
4268.99..	1	2	*C* I 8.99 (2), (*Ti* I 8.93 (1))
4269.28..	1	3	*Cr* II 9.28 (10)
4269.77..	1n	3	*Fe* I 9.86 (Pr), *La* II 9.50 (300)
4270.18..	1	2	*Ce* II 0.19 (60), *Ti* I 0.14 (7)
4270.42..	0	1	*Fe* II 0.39 (Pr), *Fe* I 0.33 (Pr)
4270.57..	1	1	*Ce* II 0.72 (50), (*Nd* II 0.56 (25))
4271.16..	7	3	*Fe* I 1.16 (20)
4271.77..	10	3	*Fe* I 1.76 (35)
4272.15..	0	1	*Pr* II 2.27 (80)
4272.53..	1	3	....
4272.86..	1	3	*Cr* I 2.91 (12)
4273.34..	4	3	*Fe* II 3.32 (3)
4273.87..	3	3	*Fe* I 3.87 (1)
4274.20..	1	3	*CH* 0, 0 R 4.17
4274.58..	0	1	*Ti* I 4.58 (15)
4274.81..	6	3	*Cr* I 4.80 (800)
4275.33..	1	3	*CH* 0, 0 Q 5.37, 5.24
4275.57..	4	3	*Cr* II 5.57 (30)
4276.05..	1	3	*Cr* I 5.97 (15), (*Co* I 6.11 (2))
4276.69..	2	3	*Fe* I 6.68 (1)
4276.96..	0	2	*V* I 6.96 (12)
4277.47..	1nn	3	*Fe* I 7.39 (Pr), *Fe* I 7.68 (1), *Zr* II 7.37 (4), *CH* 0, 0 Q 7.54
4278.19..	4	3	*Fe* I 8.23 (1), *Fe* II 8.13 (1), *Cr* II 8.10 (1), *Ti* I 8.23 (7)
4278.80..	1n	3	*V* II 8.89 (60), (*Ce* II 8.87 (20)), (*Ti* I 8.83 (3))
4279.10..	1n	1	*Mo* II 9.02 (10), *CH* 0, 0 Q 9.05
4279.46..	1	3	*Fe* I 9.48 (1)
4279.83..	2	3	*Fe* I 9.86 (1), *CH* 0, 0 Q 9.71, 9.91, *Sc* II 9.93 (1)
4280.44..	2	3	*Cr* I 0.40 (25), *Fe* I 0.53 (1), (*Gd* II 0.50 (200))
4280.78..	1	1	*Sm* II 0.79 (400), *Fe* I 0.64 (Pr)
4281.06..	2	3	*Mn* I 1.10 (6), *CH* 0, 0 Q 1.00
4281.42..	0	1	*Ti* I 1.37 (10)
4281.96..	1	3	*CH* 0, 0 Q 1.96
4282.41..	5	3	*Fe* I 2.41 (12)
4283.02..	5	3	*Ca* I 3.01 (40)
4283.42..	1	2	*Fe* I 3.41 (Pr)
4283.74..	0	2	*Mn* II 3.77 (0)
4284.18..	4	3	*Cr* II 4.21 (20), *Mn* I 4.08 (4)

TABLE 1—*Continued*

λ	Int.	N	Identification
4284.68..	1	3	*Ni* I 4.68 (6), *Cr* I 4.72 (12)
4285.00..	1	3	*Ti* I 4.99 (8), *CH* 0, 0 Q 4.98
4285.45..	4	3	*Fe* I 5.44 (3)
4285.79..	1	1	*Co* I 5.78 (6), *Fe* I 5.83 (1)
4285.98..	2n	3	*Ti* I 6.01 (25)
4286.47..	2	3	*Fe* I 6.44 (1), *Zr* II 6.51 (5), *CH* 0, 0 Q 6.48
4286.95..	3	3	*Fe* I 6.99 (1), *La* II 6.97 (300), (*CH* 0, 0 Q 7.03, 6.88)
4287.41..	2	3	*Ti* I 7.40 (22)
4287.91..	4	3	*Ti* II 7.89 (2), *Ni* I 8.00 (15)
4288.11..	2b	3	*Fe* I 8.15 (2), *Ti* I 8.16 (3)
4288.74..	1	3	*CH* 0, 0 Q 8.73
4289.03..	2	3	*Ti* I 9.07 (25), *Fe* I 8.96 (1), *CH* 0, 0 Q 9.08, 8.95
4289.37..	3	3	*Ca* I 9.36 (40)
4289.74..	4	3	*Cr* I 9.72 (700)
4289.91..	2b	1	*Ce* II 9.94 (300), (*Ti* I 9.92 (3))
4290.25..	6	3	*Ti* II 0.22 (50), *Fe* I 0.38 (2)
4290.93..	4b	3	*Ti* I 0.93 (22), *Fe* I 0.87 (1)
4291.07..	2b	2	*CH* 0, 0 Q 1.11, *Ti* I 1.21 (5)
4291.46..	3	3	*Fe* I 1.47 (4)
4292.12..	2n	3	*Fe* I 2.14 (Pr)
4292.28..	2b	1	*Fe* I 2.29 (1), *Mn* II 2.25 (0)
4292.71..	1n	2	(*Ti* I 2.68 (1))
4293.10..	3	3	*Zr* II 3.14 (7), *CH* 0, 0 Q 3.12, 3.02
4293.49..	1	2	*Cr* I 3.56 (20)
4294.11..	7	3	*Fe* I 4.13 (15), *Ti* II 4.05, 4.10 (40)
4294.77..	3	3	*Sc* II 4.77 (8)
4295.17..	2n	3	*CH* 0, 0 Q 5.21, 5.04, *Cr* II 5.37 (Pr)
4295.79..	2	3	*Cr* I 5.76 (25), *Ti* I 5.75 (22), *Ni* I 5.89 (8)
4296.03..	0	3	*La* II 6.05 (300), *V* I 6.11 (15), *Gd* II 6.08 (150)
4296.59..	4	3	*Fe* II 6.57 (6), *Ce* II 6.68 (200)
4296.97..	1	3	*Cr* I 7.05 (15), *CH* 0, 0 Q 6.96, *Ni* I 6.99 (2)
4297.25..	1	3	*CH* 0, 0 Q 7.29, 7.20
4297.60..	0	2	(*V* I 7.68 (12))
4297.78..	0	1	*Cr* I 7.74 (30), (*Pr* II 7.76 (80))
4298.05..	3	3	*Fe* I 8.04 (2), *V* I 8.03 (12)
4298.71..	2	3	*Ti* I 8.66 (40), *CH* 0, 0 Q 8.81, *Ni* I 8.77 (2)
4298.99..	4	3	*Ca* I 8.99 (30)
4299.25..	6	3	*Fe* I 9.24 (18), *Ti* I 9.23 (15)
4299.65..	1	3	*Fe* I 9.64 (1), *Ti* I 9.64 (15), *Cr* I 9.72 (20)
4300.07..	6	3	*Ti* II 0.05 (60)
4300.56..	3	3	*Ti* I 0.57 (50), *CH* 0, 0 Q 0.58
4300.83..	1	3	*Fe* I 0.82 (1)
4301.10..	4	3	*Ti* I 1.09 (50), *CH* 0, 0 Q 1.13, *V* II 1.13 (40), *Cr* I 1.18 (25)
4301.50..	1	1	....
4301.93..	5	3	*Ti* II 1.93 (15)
4302.21..	1	3	*Fe* I 2.19 (2), *CH* 0, 0 Q 2.27
4302.55..	5	3	*Ca* I 2.53 (60), *Cr* II 2.58 (1)
4302.84..	0	2	*CH* 0, 0 Q 2.74, 2.91, *Cr* II 2.78 (2)
4303.18..	4	3	*Fe* II 3.17 (8)
4303.56..	1	3	*Nd* II 3.57 (400)
4303.88..	2	3	*CH* 0, 0 Q 3.93, 3.83
4304.25..	1	2	*CH* 0, 0 Q 4.38, *CH* 1, 1 Q 4.25
4304.56..	2	3	*Fe* I 4.55 (1), *CH* 0, 0 Q 4.57
4304.81..	1	1	*CH* 1, 1 Q 4.84
4305.15..	1	2	*Fe* I 5.20 (1), *CH* 1, 1 Q 5.09
4305.45..	4	3	*Sr* II 5.45 (40), *Fe* I 5.46 (3), *CH* 0, 0 Q 5.44, *Cr* I 5.45 (30)
4305.73..	2	2	*Sc* II 5.72 (10), (*Pr* II 5.76 (100))
4305.88..	4n	3	*Ti* I 5.91 (60)
4306.16..	0	1	*CH* 0, 0 Q 6.15
4306.68..	2n	2	*CH* 0, 0 Q 6.68, *Fe* I 6.60 (1), *Ce* II 6.72 (100)
4306.80..	3n	2	*CH* 0, 0 Q 6.84, *CH* 1, 1 Q 6.84, *Cr* II 6.95 (5)
4307.29..	1	2	*CH* 0, 0 Q 7.31, *Ni* I 7.29 (3)
4307.87..	11	3	*Ti* II 7.90 (40), *Fe* I 7.91 (35), *Ca* I 7.74 (45)
4308.54..	1n	3	*CH* 0, 0 Q 8.59, (*Ti* I 8.51 (2))
4309.04..	3	3	*Fe* I 9.04 (2)
4309.40..	3	3	*Fe* I 9.38 (4), *Fe* I 9.46 (Pr)
4309.66..	3	3	*Y* II 9.62 (50), *CH* 0, 0 Q 9.71
4310.12..	1	3	*CH* 0, 0 Q 0.10, (*Co* I 0.09 (2))
4310.45..	1	3	*CH* 0, 0 Q 0.45, *Fe* I 0.38 (Pr)
4311.10..	1n	2	*CH* 0, 0 Q 1.15, 0.98
4311.48..	1	3	*CH* 0, 0 Q 1.44, 1.51
4312.21..	1	3	*CH* 0, 0 Q 2.30, 2.17, 2.08, *Zr* II 2.23 (3)
4312.54..	0	1	*Mn* I 2.55 (3), *Cr* I 2.47 (5)
4312.88..	7	3	*Ti* II 2.86 (35)
4313.58..	1	3	*CH* 0, 0 Q 3.59, 3.66
4314.11..	6n	3	*Sc* II 4.08 (60)
4314.28..	2b	2	*Fe* II 4.29 (4)
4314.79..	0b	2	*Ti* I 4.74, 4.80 (25)
4315.06..	8	3	*Fe* I 5.09 (10), *Ti* II 4.98 (40)
4316.03..	0n	3	*Gd* II 6.06 (150), *Fe* I 5.96 (Pr)
4316.81..	3	3	*Ti* II 6.81 (1)
4317.31..	1	3	*Zr* II 7.32 (12)
4318.15..	0n	1	*Fe* II 8.22 (0)
4318.66..	5	3	*Ca* I 8.65 (45), *Ti* I 8.63 (10)
4319.46..	1n	3	*Fe* I 9.46 (Pr)

TABLE 1—*Continued*

λ	Int.	N	Identification
4319.80	0	1	*Fe* II 9.72 (1)
4320.77	6	3	*Sc* II 0.74 (50)
4320.97	4	2	*Ti* II 0.96 (1)
4321.76	1	3	*Ti* I 1.66 (8)
4323.14	1nn	3	(*Sm* II 3.28 (200))
4323.51	0n	1	*Cr* I 3.52 (30), *Fe* I 3.37 (Pr)
4323.90	1n	2	. . . .
4324.44	0	1	*Fe* II 4.36 (Pr)
4325.03	6	3	*Sc* II 5.01 (40), *Cr* I 5.08 (40), (*Fe* I 4.97 (1))
4325.35	0	1	*Ni* I 5.36 (2), (*Ti* I 5.13 (9))
4325.78	9	3	*Fe* I 5.76 (35)
4326.33	0	3	*Ti* I 6.36 (9)
4326.76	2	3	*Fe* I 6.76 (2), *Mn* II 6.76 (3)
4327.12	3	3	*Fe* I 7.10 (3)
4327.91	2	3	*Fe* I 7.92 (2)
4328.02	1	1	. . . .
4328.65	0	2	. . . .
4329.26	1	1	(*Sm* II 9.02 (400))
4329.43	0	1	(*Fe* I 9.55 (Pr))
4330.27	2	3	*Ti* II 0.26 (0)
4330.72	3	3	*Ti* II 0.71 (0), *Ni* I 0.72 (2)
4331.03	0	3	*Fe* I 0.96 (1)
4331.65	2	3	*Ni* I 1.64 (12), *Fe* II 1.53 (3)
4332.50	0	1	*Cr* I 2.57 (15)
4332.85	0	1	*Fe* II 2.88 (Pr)
4333.28	0	1	*Zr* II 3.28 (15)
4333.76	2	3	*La* II 3.76 (500)
4334.15	0	1	*Sm* II 4.15 (400)
4336.36	0	1	*Ce* II 6.26 (50)
4337.06	4	3	*Fe* I 7.05 (10)
4337.55	2	3	*Cr* I 7.57 (75)
4337.93	4	3	*Ti* II 7.92 (50)
4338.31	1	2	*Fe* I 8.26 (2)
4338.72	1	2	*Fe* II 8.70 (Pr), *Fe* I 8.84 (Pr), *Nd* II 8.70 (80), *Cr* I 8.80 (15)
4339.45	1	3	*Cr* I 9.45 (75)
4339.75	1	1	*Cr* I 9.72 (60)
4340.49	75	3	$H\gamma$ 0.47
4341.38	1	2	*Ti* II 1.37 (1)
4342.06	1	1	*Gd* II 2.19 (300), (*Nd* II 2.07 (20))
4342.36	0	1	*Fe* II 2.36 (0)
4342.80	0	1	*V* I 2.83 (6)
4343.26	2	3	*Fe* I 3.26 (2)
4343.72	1	3	*Fe* I 3.70 (2)
4344.32	4	2	*Ti* II 4.29 (2)
4344.50	3	3	*Cr* I 4.51 (100)
4344.96	0	3	*Cr* I 5.08 (15)
4346.26	0n	2	(*Mn* I 6.33 (Pr))
4346.57	2	3	*Fe* I 6.56 (2), *Fe* II 6.50 (Pr)
4346.87	1	1	*Cr* I 6.83 (30)
4347.24	1	2	*Fe* I 7.24 (1)
4347.87	1	3	*Fe* I 7.85 (1), (*Sm* II 7.80 (400))
4348.40	1	3	*CH* 0, 0 P 8.34
4349.01	2	3	*Fe* I 8.94 (1)
4349.33	0	1	*Fe* II 9.28 (Pr)
4350.20	1	1	. . . .
4350.88	2n	3	*Ti* II 0.83 (1)
4351.05	2b	2	*Cr* I 1.05 (75)
4351.58	1b	3	*Fe* I 1.55 (3)
4351.86	9	3	*Mg* I 1.90 (30), *Fe* II 1.76 (9), *Cr* I 1.77 (100)
4352.75	5	3	*Fe* I 2.74 (9)
4353.42	0	3	. . . .
4353.91	0	2	*Cr* I 3.98 (15), *Co* I 3.82 (4)
4354.21	0	1	*Fe* I 4.27 (Pr), *Fe* II 4.36 (2)
4354.61	3	3	*Sc* II 4.61 (5)
4355.11	3	3	*Ca* I 5.10 (25)
4355.91	1n	3	*Ni* I 5.91 (3), *CH* 0, 0 P 6.00, 5.69
4356.34	0	1	*CH* 0, 0 P 6.35
4356.61	1n	2	*CH* 0, 0 P 6.60, *Cr* I 6.76 (20)
4357.54	1	2	*Fe* II 7.57 (4), *Cr* I 7.52 (15), *Fe* I 7.52 (Pr)
4357.93	1	1	*Ni* I 7.85 (1)
4358.15	0	1	*Nd* II 8.17 (200)
4358.54	3	3	*Fe* I 8.50 (3)
4358.74	2	2	*Y* II 8.73 (30)
4359.65	4n	3	*Cr* I 9.63 (75), *Ni* I 9.58 (10), *Zr* II 9.74 (10)
4360.24	0	1	*CH* 0, 0 P 0.28, *Cr* I 9.99 (20), *Ce* II 0.16 (25), *Fe* II 0.03 (Pr)
4360.42	1n	2	*CH* 0, 0 P 0.47, *Ti* I 0.49 (4)
4360.80	1	3	*Fe* I 0.81 (1), *Co* I 0.83 (10)
4361.27	1	1	*Fe* II 1.25 (2)
4361.75	0	1	(*Ce* II 1.66 (18))
4362.10	1	3	*Ni* II 2.10 (1), *Sm* II 2.04 (300)
4362.50	0	3	. . . .
4363.17	1	3	*CH* 0, 0 P 3.29, 3.08, *Cr* I 3.13 (12)
4363.61	1	1	*Mo* II 3.64 (10)
4364.12	1	3	*CH* 0, 0 P 4.18, 4.03, *Cr* I 4.14 (10)
4364.68	0	2	*Ce* II 4.66 (125), *La* II 4.66 (100)
4365.04	0	1	*Fe* II 4.89 (Pr)
4365.90	1	3	*Fe* I 5.90 (1)
4366.49	0n	3	*CH* 0, 0 P 6.67, 6.49
4367.04	0n	1	*Fe* I 7.06 (Pr)
4367.63	5	3	*Ti* II 7.66 (15), *Fe* I 7.58 (5)
4367.91	3	3	*Fe* I 7.91 (2)
4368.30	1	2	*Cr* I 8.25 (20), *Fe* II 8.26 (1), *Pr* II 8.33 (150), *Ni* I 8.31 (2)
4368.69	1	2	*Fe* I 8.64 (Pr), (*Nd* II 8.63 (60))
4369.03	1	1	. . . .
4369.41	3	3	*Fe* II 9.40 (2)
4369.79	4	3	*Fe* I 9.77 (7)
4370.28	0	3	. . . .
4370.99	1	3	*CH* 0, 0 P 1.02, *Zr* II 0.96 (8)
4371.33	3	3	*C* I 1.33 (4), *Cr* I 1.28 (75)
4371.86	1	1	. . . .

TABLE 1—*Continued*

λ	Int.	N	Identification
4372.26. .	1	3	*Fe* II 2.22 (Pr), (*Ti* I 2.38 (3))
4372.82. .	1	3	*CH* 0, 0 P 2.84, 2.72, *Fe* I 2.99 (1)
4373.31. .	0	1	*Cr* I 3.25 (35)
4373.57. .	3	3	*Fe* I 3.56 (2)
4373.86. .	0	1	*Fe* I 3.90 (Pr), *Ce* II 3.82 (50)
4374.17. .	1	3	*Cr* I 4.16 (40), *CH* 0, 0 P 4.21
4374.49. .	4	3	*Sc* II 4.46 (40), *Fe* I 4.49 (1)
4374.92. .	5	3	*Y* II 4.94 (300), *Ti* II 4.82 (1)
4375.32. .	1	2	*Cr* I 5.33 (30), *Ti* II 5.35 (Pr)
4375.58. .	0	1	*CH* 0, 0 P 5.57, 5.65, *Fe* I 5.49 (Pr), (*Co* I 5.54 (2))
4375.95. .	4	3	*Fe* I 5.93 (9)
4376.40. .	1	2	....
4376.80. .	2	3	*Fe* I 6.78 (1), *Cr* I 6.80 (25)
4377.25. .	1	3	*Fe* I 7.33 (1), *CH* 0, 0 P 7.23
4377.79. .	1	2	*Fe* I 7.79 (1), *Mo* II 7.76 (10)
4378.28. .	1	3	*CH* 0, 0 P 8.28, 8.22
4379 26. .	3	3	*V* I 9.24 (150)
4379.78. .	1	3	*Zr* II 9.78 (9), *Cr* I 9.78 (20)
4380.10. .	1	2	*CH* 0, 0 P 0.06, *Co* I 0.07 (5), *Ce* II 0.06 (30)
4380.51. .	1	3	*Cr* I 0.55 (10), *Mg* I 0.38 (5)
4380.75. .	1	1	*CH* 0, 0 P 0.71
4381.14. .	0	1	*Cr* I 1.11 (35)
4381.71. .	0	1	*Fe* II 1.79 (Pr)
4382.13. .	0	3	*Ce* II 2.17 (200), *Fe* I 2.00 (Pr)
4382.78. .	3	3	*Fe* I 2.77 (2), *Cr* I 2.85 (20)
4383.57. .	9	3	*Fe* I 3.55 (45)
4384.34. .	2	3	*Fe* II 4.33 (Pr)
4384.76. .	3b	3	*V* I 4.72 (125), *Sc* II 4.81 (6), *Fe* I 4.68 (1), (*Ni* II 4.6 (Pr))
4384.99. .	1b	3	*Cr* I 4.98 (75)
4385.40. .	4	3	*Fe* II 5.38 (7), *Fe* I 5.26 (1)
4386 19. .	0	3	....
4386.86. .	3	3	*Ti* II 6.86 (10)
4387.49. .	1	3	*Cr* I 7.50 (30)
4387.90. .	3	3	*Fe* I 7.90 (3)
4388.42. .	3	3	*Fe* I 8.41 (4)
4388.78. .	1	3	(*CH* 0, 0 P 8.85)
4389.24. .	2	3	*Fe* I 9.24 (2)
4389.69. .	0	1	*CH* 0, 0 P 9.76, 9.64
4389.98. .	2	3	*V* I 9.97 (100)
4390.54. .	1	3	*Mg* II 0.58 (10), *Fe* I 0.46 (1)
4391.00. .	4	3	*Fe* I 0.95 (4), *Ti* II 0.98 (tr)
4391.71. .	1nn	3	*Cr* I 1.75 (40), *Ce* II 1.66 (250), *Fe* I 1.88 (Pr)
4392.11. .	0	1	*V* I 2.07 (5), *Cr* I 2.26 (10), *Fe* I 2.31 (Pr), (*CH* 0, 0 P 2.07)
4392.58. .	1	3	*Fe* I 2.58 (1)
4393.37. .	0nn	3	*Cr* I 3.53 (12), (*CH* 0, 0 P 3.42)
4394.07. .	4	3	*Ti* II 4.06 (2)
4395.06. .	7	3	*Ti* II 5.03 (60)
4395.50. .	1	2	*Fe* I 5.51 (1), *Fe* I 5.29 (2), *Cr* I 5.42 (18), *CH* 0, 0 P 5.47
4395.85. .	4	3	*Ti* II 5.85 (2)
4396.38. .	0n	2	....
4396.81. .	0	1	(*Ni* I 6.89 (1))
4397.13. .	0n	3	*Cr* I 7.25 (30), *CH* 0, 0 P 7.14, 6.94, *Fe* II 7.27 (Pr)
4398.03. .	2	3	*Y* II 8.02 (50)
4398.33. .	1	3	*Ti* II 8.31 (1), *CH* 0, 0 P 8.50
4398.74. .	0	2	*Ce* II 8.79 (20), *Ni* I 8.62 (3), *CH* 0, 0 P 8.70
4399.14. .	0	1	*Ce* II 9.20 (60)
4399.77. .	5	3	*Ti* II 9.77 (35), *Cr* I 9.82 (30)
4400.40. .	4	3	*Sc* II 0.36 (30), *V* I 0.58 (60)
4400.95. .	1	3	*Ni* I 0.87 (3)
4401.32. .	2	3	*Fe* I 1.29 (5), *Zr* II 1.35 (2)
4401.54. .	4	3	*Ni* I 1.55 (30), *Fe* I 1.45 (2)
4402.84. .	0	1	*Fe* II 2.88 (2), (*CH* 0, 0 P 2.83)
4403.17. .	2n	3	(*CH* 0, 0 P 3.09)
4403.39. .	1n	1	*Cr* I 3.37 (35), *Cr* I 3.50 (40), *Zr* II 3.35 (6)
4404.26. .	1	1	*Ti* I 4.28 (10), *Fe* I 4.10 (Pr)
4404.76. .	9	3	*Fe* I 4.75 (30)
4405.36. .	0	1	*Fe* I 5.42 (Pr), (*Fe* I 5.04 (Pr))
4405.69. .	1n	2	*Pr* II 5.85 (80), (*Ti* I 5.69 (2)), (*CH* 0, 0 P 5.75)
4405.95. .	1	2	....
4406.12. .	1n	1	*V* I 6.15 (6)
4406.65. .	1	3	*V* I 6.64 (80)
4407.18. .	1n	2	(*Ce* II 7.28 (40))
4407.70. .	5	3	*Fe* I 7.71 (5), *Ti* II 7.68 (1), *Cr* I 7.72 (40), *V* I 7.64 (70)
4408.19. .	1	2	*V* I 8.20 (70)
4408.46. .	5	3	*Fe* I 8.42 (6), *V* I 8.51 (90)
4408.89. .	0	1	*Pr* II 8.84 (200), *V* II 8.92 (40)
4409.23. .	3	3	*Ti* II 9.22 (tr), *Fe* I 9.12 (2)
4409.51. .	3	3	*Ti* II 9.52 (tr)
4410.02. .	1	2	....
4410.20. .	0	1	*Cr* I 0.30 (40)
4410.57. .	1	3	*Ni* I 0.52 (4), (*Ce* II 0.64 (30))
4411.04. .	3	3	*Ti* II 1.08 (15), *Cr* I 1.09 (40), *Cr* I 0.97 (25)
4411.47. .	1	2	....
4411.92. .	3	3	*Ti* II 1.94 (1), *Mn* I 1.88 (3)
4412.28. .	1	2	*Cr* I 2.25 (40)
4412.83. .	0	2	....
4413.60. .	2	3	*Fe* II 3.60 (0)
4414.25. .	0	1	*Fe* I 4.23 (Pr), (*Ni* I 4.20 (Pr))
4414.49. .	0	3	*Zr* II 4.54 (5), *Fe* I 4.46 (1)

TABLE 1—*Continued*

λ	Int.	N	Identification
4414.92..	2b	2	*Mn* I 4.88 (10)
4415.15..	6	3	*Fe* I 5.12 (20)
4415.57..	4	3	*Sc* II 5.56 (20)
4416.37..	0n	2	*V* I 6.47 (20), *Ti* I 6.54 (4)
4416.83..	5	3	*Fe* II 6.82 (7)
4417.27..	1	3	*Ti* I 7.27 (15)
4417.74..	5	3	*Ti* II 7.72 (40)
4418.36..	4	3	*Ti* II 8.34 (1), *Fe* I 8.43 (1)
4418.80..	0	3	*Ce* II 8.78 (200)
4419.06..	0	1	*Cr* I 9.10 (10), *Gd* II 9.04 (150)
4420.30..	1n	2	....
4420.66..	0n	2	*Sc* II 0.66 (2), *Fe* II 0.75 (Pr), (*Sm* II 0.53 (200))
4421.96..	3	3	*Ti* II 1.95 (1)
4422.58..	4	3	*Fe* I 2.57 (6), *Y* II 2.59 (40)
4423.17..	2n	3	*Ti* II 3.22 (Pr), *Fe* I 3.14 (1), (*V* I 3.22 (8))
4423.82..	1	3	*Fe* I 3.86 (2)
4424.13..	1n	1	*Cr* I 4.08 (10), *Fe* I 4.19 (1)
4424.32..	1	2	*Cr* I 4.28 (40), *Sm* II 4.34 (600)
4425.44..	5	3	*Ca* I 5.44 (50)
4425.73..	0	1	*Fe* I 5.66 (1), *Fe* I 5.77 (Pr)
4426.00..	0	1	*V* I 6.00 (20), *Ti* I 6.05 (4)
4427.08..	1	1	*Ti* I 7.10 (40)
4427.31..	4	3	*Fe* I 7.31 (10)
4427.96..	1n	3	*Ti* II 7.90 (Pr), *Mg* II 8.00 (7)
4428.57..	1	3	*Cr* I 8.50 (35), *Fe* I 8.55 (Pr), *V* I 8.52 (15)
4429.30..	0n	3	*Fe* I 9.32 (1), *Fe* I 9.21 (Pr), *Ce* II 9.27 (100), *Pr* II 9.24 (100), *Zr* II 9.34 (2)
4429.90..	1n	1	*La* II 9.90 (400), *V* I 9.80 (15)
4430.13..	2n	3	*Fe* I 0.20 (2), *Ti* I 0.02 (3)
4430.64..	4	3	*Fe* I 0.62 (6)
4431.35..	1	3	*Sc* II 1.37 (3), *Ti* I 1.28 (4)
4431.83..	1	2	(*Fe* II 1.63 (1)), (*Mn* I 1.92 (1))
4432.09..	1	3	*Ti* II 2.09 (tr), *Cr* I 2.18 (40)
4432.58..	1	3	*Fe* I 2.57 (3)
4433.22..	3	3	*Fe* I 3.22 (3)
4433.80..	2	3	*Fe* I 3.79 (3)
4434.00..	1	1	*Ti* I 4.00 (15), *Mg* II 3.99 (8), *Cr* I 3.97 (20)
4434.32..	1	1	*Sm* II 4.32 (400)
4434.44..	1	2	....
4434.98..	5	3	*Ca* I 4.96 (60)
4435.10..	2b	1	*Fe* I 5.15 (2)
4435.67..	4	3	*Ca* I 5.69 (40)
4436.08..	1	1	*V* I 6.14 (15), (*Mn* I 6.02 (2))
4436.32..	1	3	*Mn* I 6.35 (8), (*Zr* II 6.36 (2))
4436.96..	2	3	*Fe* I 6.93 (2), *Ni* I 6.98 (5)
4437.53..	0	3	*Ni* I 7.57 (2)
4437.87..	0	1	*V* I 7.84 (20)
4438.33..	1	3	*Fe* I 8.35 (2)
4439.14..	1	2	*Fe* II 9.13 (Pr)
4439.87..	1	2	*Fe* I 9.88 (2)
4440.44..	1	3	*Zr* II 0.45 (10), *Fe* I 0.48 (1), *Ti* I 0.34 (10)
4441.01..	1n	3	*Fe* I 0.97 (2), *Fe* I 0.84 (1)
4441.43..	0	1	*Ni* I 1.45 (2), *Fe* I 1.56 (Pr)
4441.73..	3	3	*Ti* II 1.73 (Pr), *V* I 1.68 (25)
4442.34..	5	3	*Fe* I 2.34 (12)
4442.88..	2	3	*Fe* I 2.84 (2), *Zr* II 2.99 (25)
4443.20..	3	3	*Fe* I 3.20 (7)
4443.82..	5	3	*Ti* II 3.80 (50)
4444.22..	0	1	*V* I 4.21 (20)
4444.57..	3	3	*Ti* II 4.56 (1), *Fe* II 4.56 (1)
4445.40..	0	3	*Fe* I 5.48 (1), *Fe* II 5.26 (Pr)
4445.83..	0	1	*Zr* II 5.88 (1), *Co* I 5.71 (4)
4446.30..	0n	2	*Fe* II 6.25 (1), *Nd* II 6.39 (200)
4446.85..	2	3	*Fe* I 6.84 (2)
4447.15..	2	3	*Fe* I 7.13 (2)
4447.73..	5	3	*Fe* I 7.72 (9)
4448.54..	0n	2	....
4449.15..	2	3	*Ti* I 9.14 (30), *Ce* II 9.34 (200)
4449.54..	0	1	*Fe* II 9.66 (1), (*V* I 9.57 (5))
4449.93..	0	1	*Pr* II 9.87 (150)
4450.35..	1b	1	*Fe* I 0.32 (2), *Ni* I 0.30 (2)
4450.47..	4n	3	*Ti* II 0.49 (10)
4450.87..	1	3	*Ti* I 0.90 (25), *Fe* I 0.76 (Pr)
4451.54..	4	3	*Fe* II 1.54 (4), *Mn* I 1.59 (15), *Nd* II 1.56 (400)
4452.04..	1	1	*V* I 2.01 (20)
4452.57..	1	2	*Fe* I 2.62 (Pr)
4453.01..	1	3	*Mn* I 3.00 (6)
4453.32..	2	3	*Ti* I 3.31 (30), *V* II 3.35 (30)
4453.68..	1	2	*Ti* I 3.71 (20)
4454.07..	1	2	....
4454.37..	4	2	*Fe* I 4.38 (5)
4454.78..	6	3	*Ca* I 4.78 (80), *Zr* II 4.80 (10)
4455.05..	1	2	*Fe* I 5.03 (2), *Mn* I 5.01 (5)
4455.30..	3	3	*Fe* II 5.26 (3), *Ti* I 5.32 (30), *Mn* I 5.32 (6)
4455.88..	4	3	*Ca* I 5.89 (40), *Mn* I 5.82 (6)
4456.32..	1	3	*Fe* I 6.33 (1)
4456.63..	2	3	*Ca* I 6.61 (10), *Ti* II 6.65 (tr)
4457.04..	0	3	*Mn* I 7.04 (5)
4457.49..	3	3	*Ti* I 7.43 (40), *Mn* I 7.55 (8), *V* I 7.48 (15), *Zr* II 7.42 (8)
4458.20..	3	3	*Fe* I 8.10 (3), *Mn* I 8.26 (12)
4458.50..	1	2	*Cr* I 8.54 (45)
4459.09..	5	3	*Fe* I 9.12 (10), *Ni* I 9.04 (20)
4459.32..	1	3	*Cr* I 9.34 (18)
4459.76..	1	3	*V* I 9.76 (30), *Cr* I 9.74 (25)
4460.27..	2	3	*V* I 0.29 (50), *Ce* II 0.21 (400)
4460.71..	0	1	*Cr* I 0.77 (18)
4461.14..	2	3	*Fe* I 1.20 (2), *Zr* II 1.22 (10), *Mn* I 1.08 (8), *Ce* II 1.14 (50)
4461.37..	1	1	*Fe* I 1.37 (1), *Fe* II 1.43 (Pr)
4461.66..	3	3	*Fe* I 1.65 (8)

TABLE 1—*Continued*

λ	Int.	N	Identification
4462.00	3	3	*Mn* I 2.02 (20), *Fe* I 1.99 (4)
4462.44	2	3	*Ni* I 2.46 (10)
4462.73	0	1	*Cr* I 2.77 (30)
4462.98	1	2	*Nd* II 2.98 (250)
4463.40	1n	2	*Ti* I 3.39 (8), *Ni* I 3.43 (3), *Ce* II 3.41 (60)
4463.84	1	1	....
4464.49	4	3	*Ti* II 4.46 (1), *V* II 4.32(40)
4464.72	3	2	*Fe* I 4.77 (2), *Mn* I 4.68 (8)
4465.28	1nn	2	*Cr* I 5.36 (35), *Fe* I 5.33 (1), *Cr* I 5.15 (20)
4465.81	1	3	*Ti* I 5.81 (20)
4466.55	4	3	*Fe* I 6.55 (12)
4466.92	1	3	*Fe* I 6.94 (2), *Co* I 6.88 (10)
4467.30	1	1	*Sm* II 7.34 (500)
4467.48	0	1	*Fe* I 7.45 (1), *Cr* I 7.56 (30)
4467.86	0	1	*Ni* I 7.94 (2)
4468.50	7	3	*Ti* II 8.49 (50)
4469.17	2	3	*Ti* II 9.16 (tr)
4469.39	3	3	*Fe* I 9.38 (5)
4469.73	0	1	*V* I 9.71 (15)
4470.15	1	3	*Mn* I 0.14 (6)
4470.50	2	3	*Ni* I 0.48 (15)
4470.85	3	3	*Ti* II 0.86 (tr)
4471.24	0	3	*Ti* I 1.24 (20), *Ce* II 1.24 (200)
4471.77	0	1	*Fe* I 1.81 (1), *Fe* I 1.68 (1)
4472.15	0	1	*Ca* II 2.09 (0)
4472.67	1	1	*Fe* I 2.72 (2)
4472.89	3	3	*Fe* II 2.92 (2), *Mn* I 2.79 (5)
4473.69	1	2	*Cr* I 3.78 (40)
4474.79	0	3	*Ti* I 4.85 (8), *V* I 4.71 (12)
4476.04	7	4	*Fe* I 6.02 (10), *Fe* I 6.08 (4)
4476.62	0	2	....
4476.92	0	1	*Cr* I 7.02 (35)
4477.50	0n	3	(*Y* I 7.44 (25))
4478.62	0nn	4	(*Sm* II 8.66 (125))
4479.57	2	4	*Fe* I 9.61 (3)
4480.12	2n	4	*Fe* I 0.14 (3), *Fe* I 9.97 (Pr)
4480.57	1	3	*Ni* I 0.57 (3), *Fe* II 0.69 (1), *Ti* I 0.60 (5)
4481.21	11	4	*Mg* II 1.13, 1.33 (100), *Ti* I 1.26 (30)
4481.60	1	2	*Fe* I 1.62 (2), *Cr* I 1.44 (18), *Cr* II 1.49 (1)
4482.20	7	4	*Fe* I 2.26 (6), *Fe* I 2.17 (4)
4482.74	2	4	*Fe* I 2.75 (2), *Ti* I 2.69 (10)
4483.28	0	2	....
4483.75	0	3	*Fe* I 3.78 (Pr)
4484.22	4	4	*Fe* I 4.23 (4)
4484.85	0	2	*Fe* II 4.93 (Pr)
4485.13	0	1	*Eu* II 5.15 (100)
4485.69	3	4	*Fe* I 5.68 (2)
4486.92	1	1	*Ce* II 6.91 (150)
4487.19	0	3	*Fe* I 7.01 (Pr)
4487.71	0	2	*Fe* I 7.75 (Pr)
4488.28	4	4	*Ti* II 8.32 (15), *Fe* I 8.14 (2)
4488.88	1b	2	*Fe* I 8.92 (2), *V* I 8.90 (20)
4489.16	4n	4	*Fe* II 9.18 (4), *Ti* I 9.09 (20)
4489.75	2	4	*Fe* I 9.74 (3)

λ	Int.	N	Identification
4490.09	3	4	*Fe* I 0.08 (2), *Mn* I 0.08 (5)
4490.75	3	4	*Fe* I 0.77 (2)
4491.41	4	4	*Fe* II 1.40 (5)
4492.23	0	3	*Cr* I 2.31 (40)
4492.64	0	4	*Fe* I 2.69 (1), *Ti* I 2.54 (3)
4493.53	2	4	*Ti* II 3.53 (1), *Fe* II 3.58 (1)
4494.06	1	4	*Fe* I 4.06 (Pr)
4494.57	6	4	*Fe* I 4.57 (12)
4495.03	0	1	*Ti* I 5.01 (4), *Cr* I 5.04 (4)
4495.47	1n	4	*Ti* II 5.46 (Pr), *Fe* I 5.39 (1), *Fe* I 5.57 (1), *Zr* II 5.44 (3), *Fe* II 5.52 (Pr)
4496.01	1	4	*Fe* I 5.97 (1), (*V* I 6.06 (8))
4496.88	4	4	*Cr* I 6.86 (100), *Zr* II 6.96 (15)
4497.71	1n	4	*Ti* I 7.71 (3), *Na* I 7.66 (2)
4498.30	0	2	....
4498.93	1nn	4	*Mn* I 8.90 (7), *Cr* I 8.73 (35)
4499.14	1	1	....
4499.74	0	1	*Fe* II 9.71 (0)
4500.34	2	4	*Ti* II 0.32 (Pr), *Cr* I 0.30 (40)
4501.26	8	4	*Ti* II 1.27 (40), (*Cr* I 1.11 (35))
4501.81	1	1	*Cr* I 1.79 (25), *Nd* II 1.81 (50)
4502.22	1	4	*Mn* I 2.22 (7)
4502.69	0	2	*Fe* I 2.59 (1)
4503.19	0	1	*Cr* I 3.05 (12)
4503.77	0	2	*Ti* I 3.76 (4)
4504.34	0	1	(*Fe* I 4.21 (Pr))
4504.78	1	4	*Fe* I 4.84 (2)
4506.76	1	2	*Ti* II 6.74 (Pr), *Cr* I 6.85 (30)
4508.29	7	4	*Fe* II 8.28 (8)
4509.18	0	3	(*Fe* I 9.13 (Pr))
4509.35	0	1	*Fe* I 9.31 (1), *Ca* I 9.45 (3)
4509.75	1	4	....
4510.83	0	1	*Fe* I 0.84 (Pr)
4511.00	1	2	*Fe* I 1.07 (Pr)
4511.91	1	4	*Cr* I 1.90 (60)
4512.72	1	4	*Ti* I 2.73 (40), *V* II 2.72 (60)
4513.44	0	4	....
4514.30	2n	4	*Fe* I 4.19 (2), *Cr* I 4.37 (20)
4515.34	6	4	*Fe* I 5.34 (7)
4516.30	1n	2	*Fe* I 6.27 (Pr)
4516.44	0	1	(*Fe* I 6.46 (Pr))
4516.68	1	2	....
4517.12	0	4	*Co* I 7.09 (4)
4517.54	2	4	*Fe* I 7.53 (2), *Fe* I 7.60 (Pr)
4518.01	1	3	*Ti* I 8.02 (50)
4518.34	2	4	*Ti* II 8.30 (Pr), *Fe* I 8.45 (1)
4519.26	0	2	(*Co* I 9.28 (1))
4519.67	0	1	*Sm* II 9.63 (200)
4520.23	6	4	*Fe* II 0.22 (7), *Fe* I 0.24 (1)
4521.19	1	1	*Cr* I 1.14 (25)
4522.66	8	4	*Fe* II 2.63 (9), *Ti* I 2.80 (40)
4523.43	1	3	*Fe* I 3.40 (2)
4524.04	0	2	....
4524.71	1	3	*Ti* II 4.73 (1)

TABLE 1—*Continued*

λ	Int.	N	Identification
4525.15	4	4	*Fe* I 5.14 (5), *Ti* II 5.21 (Pr)
4526.03	0	1	*Cr* I 6.11 (40), *La* II 6.12 (200), *Fe* II 5.75 (Pr), (*Fe* I 5.87 (1))
4526.44	4	4	*Cr* I 6.47 (75), *Fe* I 6.56 (2) *Fe* I 6.41 (Pr)
4526.94	2	4	*Ca* I 6.94 (30)
4527.36	2	4	*Cr* I 7.34 (40), *Ti* I 7.30 (35), *Ce* II 7.35 (200)
4527.77	0	2	*Fe* I 7.78 (1)
4528.09	0	1	*Fe* I 7.90 (1), *V* I 7.99 (5)
4528.62	8	5	*Fe* I 8.62 (18), *V* II 8.51 (300)
4529.54	5	6	*Ti* II 9.46 (1), *Fe* I 9.56 (1)
4530.04	0	2	*Mn* II 0.03 (5)
4530.24	0	3	....
4530.78	2	6	*Cr* I 0.69, 0.76 (100), *Cu* I 0.78 (110)
4531.16	4	6	*Fe* I 1.15 (8)
4531.63	1	5	*Fe* I 1.63 (2)
4532.04	1	2	....
4532.22	0	2	*V* II 2.19 (40)
4532.41	1	3	....
4533.01	2b	5	(*Fe* I 3.14 (1))
4533.25	3b	5	*Ti* I 3.24 (80)
4534.02	7	5	*Ti* II 3.97 (30)
4534.18	1b	1	*Fe* II 4.17 (2)
4534.80	3	5	*Ti* I 4.78 (60)
4535.16	1	2	*Cr* I 5.15 (35)
4535.64	3	5	*Ti* I 5.57 (50), *Cr* I 5.72 (60)
4535.98	3	5	*Ti* I 5.92 (40), *Ti* I 6.05 (40)
4536.48	0	1	*Fe* I 6.51 (1)
4537.23	0n	2	....
4537.48	0	1	*Fe* I 7.68 (1)
4538.76	1	4	*Fe* I 8.76 (1), *Fe* I 8.84 (2)
4539.67	1	4	*Cr* I 9.79 (30), *Cr* II 9.62 (2), *Ce* II 9.76 (200)
4540.58	2n	4	*Cr* I 0.50 (50)
4540.81	2	1	*Cr* I 0.72 (50)
4541.08	1	1	*Cr* I 1.07 (30)
4541.54	4	4	*Fe* II 1.52 (4), *Cr* I 1.51 (25)
4542.43	1n	4	*Fe* I 2.42 (2)
4542.74	1n	2	*Fe* I 2.72 (1), *Cr* I 2.62 (35), (*Cr* II 2.77 (Pr))
4543.26	0	2	*Fe* I 3.24 (Pr)
4543.44	1	2	....
4543.70	0	1	*Cr* I 3.74 (20), *Co* I 3.81 (6)
4544.02	2	4	*Ti* II 4.01 (tr)
4544.66	3	4	*Ti* I 4.69 (30), *Cr* I 4.62 (50)
4545.16	3	4	*Ti* II 5.14 (tr)
4545.39	0	1	*V* I 5.39 (25), *Cr* I 5.34 (25)
4545.96	3	4	*Cr* I 5.96 (50)
4546.34	0	1	*Fe* I 6.48 (Pr)
4546.97	2	4	*Fe* I 7.02 (2), *Ni* I 6.93 (5)
4547.20	0	3	*Ni* I 7.23 (3)
4547.84	2	4	*Fe* I 7.85 (4), *Ti* I 7.85 (2)
4548.15	0	1	*Ti* I 8.09 (2)
4548.62	1	1	....
4548.77	2	3	*Ti* I 8.76 (35)
4549.60	14	4	*Ti* II 9.62 (60), *Fe* II 9.47 (10)
4550.76	2	4	....
4551.25	1	3	*Ni* I 1.24 (3)
4551.66	1	4	*Fe* I 1.67 (1)
4552.44	2n	4	*Ti* I 2.45 (35), *Fe* I 2.54 (3), *Ti* II 2.25 (Pr)
4553.04	0	1	*V* I 3.06 (7)
4553.30	0	3	(*Co* I 3.34 (2))
4554.05	7	4	*Ba* II 4.03 (1000)
4554.54	1	2	*Fe* I 4.46 (1), (*Ru* I 4.51 (1000))
4555.00	2	4	*Cr* II 5.02 (20)
4555.49	1	3	*Ti* I 5.49 (30)
4555.91	3b	3	*Fe* II 5.89 (8)
4556.08	3b	4	*Fe* I 6.13 (4), *Cr* I 6.17 (40)
4556.76	1n	1	(*Fe* I 5.74 (Pr))
4556.97	0	1	*Fe* I 6.94 (1)
4557.29	1n	4	....
4558.14	0	1	*Fe* I 8.11 (1), *Ti* I 8.09 (2)
4558.66	4	4	*Cr* II 8.66 (100)
4559.32	0	1	*La* II 9.28 (100)
4560.08	1	4	*Fe* I 0.10 (2)
4560.79	0	1	*V* I 0.71 (20)
4560.95	0	1	*Ce* II 0.96 (60)
4561.43	1n	2	*Cr* I 1.54 (10), (*Cr* I 1.20 (2))
4562.36	1	3	*Ce* II 2.36 (400)
4563.17	0	3	*Cr* I 3.24 (15)
4563.77	6	4	*Ti* II 3.76 (30)
4564.64	1n	4	*V* II 4.59 (200), *Fe* I 4.71 (1), *Fe* I 4.83 (1)
4565.47	2b	2	*Cr* I 5.51 (50), *Co* I 5.58 (15), *Fe* I 5.32 (2), *Zr* II 5.43 (3)
4565.66	2b	4	*Fe* I 5.67 (2), *Cr* II 5.78 (10)
4566.30	0	1	(*Sm* II 6.21 (200))
4566.51	1	3	*Fe* I 6.52 (2), *Cr* I 6.60 (7)
4566.87	1	4	(*Fe* I 6.99 (1))
4567.36	1	1	*Ni* I 7.42 (1)
4568.32	1	4	*Ti* II 8.31 (1)
4568.82	1	4	*Fe* I 8.79 (1), *Fe* I 8.84 (1)
4569.40	0	1	*Cr* I 9.53 (20)
4569.60	0	4	*Cr* I 9.64 (30), *Cr* I 9.53 (20)
4571.08	2	4	*Mg* I 1.10 (5), *Cr* I 1.10 (25)
4571.98	7	4	*Ti* II 1.97 (50)
4572.80	0	3	*Fe* I 2.86 (1)
4573.62	0	1	(*Cr* II 3.63 (Pr))
4574.18	1	3	*Fe* I 4.24 (1), (*Ni* I 4.03 (1))
4574.45	0n	1	*Zr* II 4.49 (6), *Cr* I 4.45 (6)
4574.76	1	3	*Fe* I 4.72 (2), *La* II 4.87 (200)
4575.61	0	3	(*Fe* I 5.80 (1))
4576.34	3	4	*Fe* II 6.33 (4)
4577.16	0	2	*V* I 7.17 (40)
4577.74	0	1	*Fe* II 7.78 (Pr), (*Sm* II 7.69 (250))
4578.55	3	4	*Ca* I 8.56 (30)
4579.30	1	1	*Fe* I 9.34 (1), *Fe* I 9.06 (Pr)
4579.44	0n	2	*Fe* II 9.52 (1)
4580.04	2	4	*Cr* I 0.06 (40), *Fe* II 0.06 (1), *La* II 0.05 (150)
4580.47	1	1	*Ti* II 0.46 (1), *V* I 0.39 (40)
4580.54	2	2	*Fe* I 0.60 (2) *Ni* I 0.62 (3)
4580.93	1	1	*Cr* I 1.06 (10)

TABLE 1—*Continued*

λ	Int.	N	Identification
4581.44	5	4	*Ca* I 1.40 (40), *Fe* I 1.52 (2), *Co* I 1.60 (20)
4582.37	0	3	(*Fe* II 2.12 (Pr))
4582.85	3	4	*Fe* II 2.84 (3), *Fe* I 2.94 (1)
4583.42	1	3	*Ti* II 3.44 (1)
4583.86	6	4	*Fe* II 3.83 (11)
4584.28	0	1	*Cr* I 4.10 (20), *Fe* II 3.99 (Pr), (*Ru* I 4.44 (150))
4584.78	2	4	*Fe* I 4.82 (2), *Fe* I 4.72 (1), *Cr* I 4.75 (12)
4585.20	0	1	*Cr* I 5.09 (18), *Cr* I 4.93 (15)
4585.37	0n	2	....
4585.90	3	4	*Ca* I 5.87 (50), *Ca* I 5.92 (2)
4586.26	1	4	*V* I 6.36 (50), *Cr* I 6.14 (20)
4587.12	1	4	*Fe* I 7.13 (2)
4587.58	0	1	*Fe* I 7.73 (Pr)
4588.21	3	4	*Cr* II 8.22 (75)
4588.72	0	1	(*Co* I 8.73 (1))
4589.15	0	2	....
4589.40	0	1	(*Dy* I 9.37 (150))
4589.95	4	4	*Ti* II 9.96 (2), *Cr* II 9.89 (3)
4590.75	1n	4	*Cr* I 0.69 (8)
4591.46	2	4	*Cr* I 1.39 (60)
4592.06	2	4	*Cr* II 2.09 (20)
4592.60	4	4	*Fe* I 2.66 (5), *Ni* I 2.53 (10)
4593.34	1	4	*Fe* I 3.54 (1), (*Cs* I 3.18 (1000))
4593.86	1n	2	*Ce* II 3.93 (200), *Cr* I 3.84 (8)
4594.04	1n	2	*V* I 4.10 (60), (*Eu* I 4.03 (10000))
4594.88	1	4	*Ni* I 4.91 (5), (*Fe* I 4.96 (2))
4595.39	2	4	*Fe* I 5.36 (2), *Cr* I 5.59 (45)
4596.01	2	4	*Fe* I 6.06 (2), *Ni* I 5.95 (4)
4596.46	0	3	*Fe* I 6.43 (1)
4597.07	0	1	*Co* I 6.90 (5), (*Fe* I 7.04 (Pr))
4597.32	1	3	....
4597.82	1	4	....
4598.12	2	4	*Fe* I 8.12 (2)
4598.82	0	2	*Cr* I 9.00 (8), *Fe* I 8.74 (Pr)
4599.86	1	3	....
4600.01	1n	1	*Cr* I 0.10 (40), *V* II 0.19 (150)
4600.31	2	3	*Ti* II 0.28 (Pr), *Ni* I 0.37 (6)
4600.79	2	4	*Cr* I 0.75 (75)
4601.38	1	4	*Fe* II 1.34 (Pr), (*Sc* II 1.45 (Pr))
4602.02	2	4	*Fe* I 2.01 (2)
4602.95	4	4	*Fe* I 2.94 (9)
4603.40	0	1	*Fe* I 3.35 (Pr)
4603.81	0	3	....
4603.95	1	1	*Fe* I 3.96 (1)
4604.55	1	4	(*Cr* I 4.58 (5))
4605.02	2	4	*Ni* I 4.99 (12)
4605.44	1	1	*Mn* I 5.36 (4)
4605.58	1	3	(*La* II 5.78 (100))
4606.23	1	3	*Ni* I 6.23 (3)
4607.34	1	2	*Sr* I 7.33 (1000)
4607.64	2	4	*Fe* I 7.66 (3)
4609.28	1	2	*Ti* II 9.26 (Pr)
4611.28	4	4	*Fe* I 1.28 (5)
4612.02	0	2	*Cr* I 1.97 (15)
4612.50	0	1	(*Dy* I 2.27 (100))
4612.62	0	1	*Fe* I 2.62 (Pr)
4613.28	4	4	*Fe* I 3.21 (2), *Cr* I 3.37 (60)
4613.95	1n	4	*Zr* II 3.95 (5)
4614.56	0	1	*Cr* I 4.52 (12), (*Ni* I 4.58 (1))
4614.74	0	1	*Cr* I 4.73 (10)
4615.60	1	4	(*Sm* II 5.69 (300))
4616.14	2	4	*Cr* I 6.14 (75)
4616.65	2	4	*Cr* II 6.64 (18)
4617.28	2	4	*Ti* I 7.27 (30)
4617.93	0	2	(*Ni* I 7.94 (1))
4618.04	0	2	....
4618.38	1	1	....
4618.81	4	4	*Cr* II 8.83 (35), *Fe* I 8.76 (2)
4619.30	2	4	*Fe* I 9.29 (3)
4619.53	1	1	*Cr* I 9.55 (40), *Ti* I 9.52 (3)
4619.99	1n	1	*La* II 9.87 (300)
4620.54	3	4	*Fe* II 0.51 (3)
4621.15	0	1	(*Cr* I 1.00 (4))
4621.43	0	1	(*Cr* II 1.41 (Pr))
4621.58	0	1	*Fe* I 1.62 (Pr)
4621.95	1	3	*Cr* I 1.89, 1.96 (45)
4622.50	1	3	*Cr* I 2.49 (35), *Fe* II 2.40 (Pr)
4623.10	1	3	*Ti* I 3.10 (25)
4623.81	0	3	....
4625.05	3	4	*Fe* I 5.05 (3)
4625.81	0	1	*Fe* II 5.91 (1), (*Co* I 5.77 (2))
4626.16	3	4	*Cr* I 6.19 (65)
4627.36	0n	4	(*Eu* I 7.22 (8000))
4628.19	0	4	*Ce* II 8.16 (500)
4629.35	4	4	*Fe* II 9.34 (7), *Ti* I 9.34 (15), *Co* I 9.36 (15)
4630.13	2	4	*Fe* I 0.12 (2)
4630.56	0	1	*Fe* I 0.78 (1)
4631.51	0	1	*Fe* I 1.50 (1)
4632.07	1	1	*Cr* I 2.18 (35), *Fe* I 2.15 (Pr), *Fe* II 1.90 (0)
4632.90	3	4	*Fe* I 2.92 (2), *Fe* I 2.82 (Pr)
4633.70	1	1	*Fe* I 3.76 (1)
4634.09	4	4	*Cr* II 4.11 (25)
4634.59	0	1	*Fe* II 4.60 (Pr), *Cr* I 4.59 (5)
4634.71	1	3	....
4635.32	1	4	*Fe* II 5.33 (5)
4635.89	1	4	*Fe* I 5.85 (1)
4636.38	1	4	*Ti* II 6.34 (1), *La* II 6.42 (80)
4636.84	0	1	(*Fe* I 6.68 (Pr))
4637.12	0	1	*Cr* I 7.18 (40)
4637.51	2	4	*Fe* I 7.51 (3)
4638.03	3	4	*Fe* I 8.02 (3)
4639.38	1n	3	*Ti* I 9.37 (18)
4639.55	1	1	*Cr* I 9.54 (35)
4639.64	0	1	*Ti* I 9.67 (15)
4640.28	1	2	....
4641.17	1	3	*Fe* I 1.22 (Pr)
4641.95	0	1	*Cr* I 2.01 (10)

TABLE 1—*Continued*

λ	Int.	N	Identification
4642.23..	0	1	*Sm* II 2.24 (500)
4642.73..	0	1	*Fe* I 2.59 (Pr)
4643.47..	2	4	*Fe* I 3.47 (2)
4644.22..	0	1	*Fe* II 4.09 (Pr)
4645.20..	0	1	*Ti* I 5.19 (12)
4646.18..	3	4	*Cr* I 6.17 (100)
4646.66..	1	2	(*Cr* I 6.81 (20)), (*Sm* II 6.68 (200))
4647.42..	3	4	*Fe* I 7.44 (6)
4647.95..	1	4	....
4648.70..	2	4	*Ni* I 8.66 (15)
4648.95..	1	1	*Cr* I 8.87 (35), *Fe* II 8.93 (0)
4649.55..	1nn	2	*Cr* I 9.46 (45)
4649.63..	1n	2	....
4650.12..	1nn	1	*Ti* I 0.02 (10), *Fe* II 0.04 (Pr)
4650.47..	0	3	....
4651.27..	2	4	*Cr* I 1.28 (75)
4652.16..	3	4	*Cr* I 2.16 (100)
4652.63..	0	1	*Fe* II 2.28 (tr)
4652.90..	0	1	(*Mn* II 2.82 (1))
4653.34..	0	1	(*Ni* I 3.31 (1))
4653.47..	0	1	*Fe* I 3.50 (Pr)
4654.05..	0	1	*Ce* II 4.29 (30)
4654.58..	5	4	*Fe* I 4.63 (5), *Fe* I 4.50 (5)
4655.65..	1n	4	*La* II 5.49 (400), *Ti* II 5.75 (—), *Ni* I 5.66 (2), *Ti* I 5.71 (3)
4656.44..	1	3	*Ti* I 6.47 (25)
4656.66..	1	1	....
4657.13..	3n	4	*Ti* II 7.21 (tr), *Fe* II 6.97 (1)
4658.13..	0	1	*Fe* I 8.29 (1), *Fe* II 8.03 (Pr)
4660.45..	0n	2	....
4660.89..	0	1	*Fe* II 0.93 (Pr)
4661.50..	1	3	*Fe* I 1.54 (2)
4662.00..	1	3	*Fe* I 1.98 (2)
4662.80..	0	1	*Ti* II 2.74 (Pr)
4663.29..	1	4	*Cr* I 3.33 (40), *Fe* I 3.18 (1), *Co* I 3.40 (12)
4663.78..	2	4	*La* II 3.76 (300), *Cr* I 3.83 (55), *Fe* II 3.70 (0)
4664.76..	2	4	*Cr* I 4.80 (60), *Na* I 4.81 (3), (*Fe* II 4.79 (Pr))
4665.89..	1n	4	*Cr* I 5.90 (35), *Fe* II 5.80 (Pr)
4666.74..	3	4	*Fe* II 6.75 (2)
4667.48..	4	4	*Fe* I 7.46 (6), *Ti* I 7.58 (25)
4668.12..	3	4	*Fe* I 8.14 (6), *Fe* I 8.07 (Pr)
4668.60..	1	4	*Na* I 8.56 (4)
4669.20..	2	4	*Fe* I 9.17 (4), *Cr* I 9.34 (50)
4669.74..	1	1	*Cr* I 9.67 (10), (*Sm* II 9.65 (500))
4670.32..	4n	4	*Sc* II 0.40 (15), *V* I 0.48 (25), *Fe* II 0.17 (0)
4671.38..	1n	4	*Cr* II 1.36 (Pr)
4672.33..	2	4	....
4672.78..	0	1	*Fe* I 2.84 (Pr)
4673.19..	2	4	*Fe* I 3.17 (4), *Fe* I 3.28 (Pr)
4674.07..	1	4	....
4674.70..	0	1	*Fe* I 4.66 (Pr), *Ni* I 4.73 (2), *Sm* II 4.60 (600)
4675.01..	0	1	*Ti* I 5.12 (10), (*Y* I 4.85 (125))
4675.59..	0	1	*Ni* I 5.64 (2)
4677.61..	0	1	*Fe* I 7.60 (Pr)
4678.17..	2	4	....
4678.84..	3	4	*Fe* I 8.85 (7)
4679.22..	1	3	*Fe* I 9.23 (1)
4679.74..	0	1	(*Cr* II 9.87 (Pr))
4680.14..	2b	2	*Zn* I 0.14 (45)
4680.31..	2n	1	*Fe* I 0.30 (2)
4680.51..	1b	2	*Cr* I 0.49 (50), *Fe* I 0.48 (1)
4680.75..	1	1	*Cr* I 0.87 (35), (*Nd* II 0.73 (30))
4682.04..	2nn	3	*Ti* I 1.91 (30)
4682.36..	0n	1	*Y* II 2.32 (20), *Co* I 2.36 (9)
4683.53..	1	3	*Fe* I 3.56 (2)
4684.54..	0	2	*Cr* I 4.60 (12)
4685.29..	1	3	*Ca* I 5.26 (12)
4686.23..	2	3	*Ni* I 6.22 (5)
4686.78..	0	1	*V* I 6.93 (6)
4687.32..	1n	2	*Fe* I 7.39 (1), *Fe* I 7.31 (Pr)
4688.22..	1	3	(*Fe* I 8.38 (Pr))
4688.71..	0	1	(*La* II 8.65 (40))
4689.44..	1	3	*Cr* I 9.37 (65), *Ti* II 9.46 (Pr)
4690.14..	1	3	*Fe* I 0.15 (3)
4691.42..	4	3	*Fe* I 1.41 (6), *Ti* I 1.34 (20)
4691.58..	1	1	*Fe* II 1.55 (Pr)
4691.86..	1	1	....
4692.53..	0n	1	*La* II 2.50 (200)
4692.71..	1	1	....
4693.26..	1	1	*Co* I 3.19 (6)
4694.07..	1	3	*S* I 4.13 (10), *Cr* I 3.95 (45)
4694.81..	1	2	....
4694.96..	0	1	*Cr* I 5.15 (30)
4695.48..	1	3	*S* I 5.45 (8)
4696.26..	0	2	*S* I 6.25 (6)
4696.95..	1	1	*Cr* I 7.06 (40), *Ti* I 6.92 (4)
4697.63..	0	1	*Cr* II 7.62 (2)
4698.54..	2n	3	*Cr* I 8.46 (60), *Cr* I 8.62 (50), *Ti* II 8.67 (Pr), *Ti* I 8.77 (20), *Sc* II 8.28 (2)
4699.35..	2	3	....
4700.17..	2	3	*Fe* I 0.17 (2)
4701.06..	1	3	*Fe* I 1.05 (1), *Mn* I 1.16 (3)
4701.52..	1	3	*Ni* I 1.54 (3)
4702.34..	0	2	....
4703.00..	8	3	*Mg* I 2.99 (40)
4703.80..	1	3	*Ni* I 3.81 (4)
4704.50..	0	3	(*Sm* II 4.40 (500))
4704.95..	1	3	*Fe* I 4.96 (5)
4705.47..	1	2	*Fe* I 5.46 (1)
4706.52..	1	3	*V* I 6.57 (12), *Nd* II 6.54 (100)
4707.31..	4	3	*Fe* I 7.28 (8), *Fe* I 7.49 (2)
4708.01..	1	3	*Cr* I 8.04 (60)
4708.67..	2	3	*Ti* II 8.66 (tr)
4709.08..	2	3	*Fe* I 9.09 (3), *Fe* I 8.97 (1), *Fe* II 8.97 (3)
4709.70..	1	3	*Mn* I 9.72 (10)
4710.27..	3	3	*Fe* I 0.29 (5), *Cr* I 0.24 (6), *Ti* I 0.19 (18)

TABLE 1—*Continued*

λ	Int.	N	Identification
4711.49..	0	3	....
4712.11..	0	3	*Fe* I 2.10 (1), *Ni* I 2.07 (2)
4714.42..	5	3	*Ni* I 4.42 (25)
4715.10..	1	1	*Cr* II 5.12 (1)
4715.77..	3	3	*Ni* I 5.78 (8)
4716.62..	0	2	*Fe* I 6.84 (Pr), *La* II 6.44 (80)
4717.61..	1	1	*V* I 7.69 (10), *Cr* I 7.69 (10), (*La* II 7.58 (50))
4718.42..	2	3	*Cr* I 8.43 (75)
4719.52..	1	3	*Ti* II 9.52 (1)
4721.01..	1	3	*Fe* I 1.00 (1)
4721.54..	0	1	*V* I 1.52 (6)
4722.15..	3	3	*Zn* I 2.16 (75)
4722.77..	0	1	*Cr* I 2.74 (10), *Ti* I 2.60 (10), *V* I 2.88 (8)
4723.15..	1	3	*Ti* I 3.17 (10), *Cr* I 3.06 (15)
4724.46..	1	3	*Cr* I 4.42 (35)
4725.03..	0	1	*Ce* II 5.09 (20)
4725.40..	1	1	....
4726.23..	1	1	*Fe* I 6.16 (1)
4727.40..	4	3	*Fe* I 7.40 (3)
4727.94..	0	1	*Co* I 7.94 (3)
4728.53..	2	3	*Fe* I 8.56 (3)
4729.08..	1	3	*Fe* I 9.03 (1)
4729.58..	0	2	*Fe* I 9.70 (1), *V* I 9.54 (6)
4730.01..	1	3	*Mg* I 0.03 (2)
4730.72..	1	3	*Cr* I 0.71 (50)
4731.47..	4	3	*Fe* II 1.44 (3)
4731.83..	0	1	*Ni* I 1.81 (3)
4732.48..	1	3	*Ni* I 2.46 (3)
4732.97..	0	1	*Ti* II 2.96 (Pr)
4733.23..	0	1	*Ti* I 3.43 (6), (*Ni* I 3.22 (1))
4733.58..	2	3	*Fe* I 3.60 (4)
4734.12..	1	3	*Fe* I 4.10 (1)
4734.74..	0	2	....
4735.84..	2	3	*Fe* I 5.85 (2)
4736.79..	4	3	*Fe* I 6.78 (12)
4737.35..	1	3	*Cr* I 7.35 (75), *Ce* II 7.28 (60)
4737.67..	0	1	*Fe* I 7.63 (1), *Sc* I 7.64 (20)
4738.32..	0	3	*Mn* II 8.29 (−), (*Fe* II 8.52 (Pr))
4739.13..	1	3	*Mn* I 9.11 (8)
4739.57..	0	1	*Mg* II 9.59 (5), (*Ce* II 9.49 (25))
4740.37..	1	3	*Fe* I 0.34 (1), (*Ti* II 0.52 (Pr))
4741.00..	1	3	*Fe* I 1.08 (1), *Sc* I 1.02 (30)
4741.55..	2	3	*Fe* I 1.53 (3)
4742.17..	1	1	*Ti* I 2.13 (3), (*Ti* I 2.32 (Pr))
4742.64..	0n	1	*V* I 2.63 (5)
4742.82..	1	1	*Ti* I 2.79 (20), *La* II 3.08 (250)
4743.82..	0	1	*Sc* I 3.81 (40)
4744.39..	2	3	(*Fe* I 4.64 (Pr)), (*Fe* I 4.13 (Pr))
4745.08..	0	1	*Fe* I 5.13 (1)
4745.29..	0	1	*Cr* I 5.31 (30)
4745.81..	2	3	*Fe* I 5.81 (3)

λ	Int.	N	Identification
4747.12..	0	1	*Ce* II 7.14 (20), *Cr* I 7.00 (4)
4748.11..	3	3	(*Sc* II 8.12 (Pr))
4749.89..	1n	3	*Fe* I 9.93 (1)
4751.06..	0n	2	*V* I 0.99 (8), *Cr* I 1.04 (5)
4751.20..	1	1	....
4752.07..	0	1	*Cr* I 2.08 (50), *Ni* I 2.12 (3)
4752.28..	1nn	2	....
4752.47..	1	1	*Ni* I 2.43 (4)
4753.09..	0	1	*Sc* I 3.15 (15)
4754.03..	4	3	*Mn* I 4.04 (50)
4754.76..	1	3	*Cr* I 4.74 (20), *Ni* I 4.77 (3)
4755.74..	1b	2	*Mn* II 5.73 (0)
4756.12..	1	3	*Cr* I 6.11 (100)
4756.54..	2	3	*Ni* I 6.52 (10)
4756.98..	0	1	*Cr* I 7.33 (15)
4757.58..	1	3	*Fe* I 7.58 (2), *Cr* I 7.59 (18)
4758.17..	1	3	*Ti* I 8.12 (25)
4759.29..	1	3	*Ti* I 9.27 (25)
4760.01..	0	1	*Fe* I 0.08 (Pr), *Cr* I 9.91 (10)
4761.51..	2	3	*Mn* I 1.53 (10), (*Cr* II 1.42 (1))
4762.39..	4b	3	*Mn* I 2.38 (30), *C* I 2.41 (4)
4762.67..	2b	3	*Ti* II 2.77 (1), *Ni* I 2.63 (3)
4763.37..	1	1	....
4763.90..	4	3	*Ni* I 3.95 (4), *Ti* II 3.84 (Pr), *Fe* II 3.79 (Pr)
4764.32..	1	1	*Cr* I 4.29 (50)
4764.53..	2	3	*Ti* II 4.54 (1), *Cr* I 4.64 (20)
4765.45..	1	3	*Fe* I 5.48 (1)
4765.84..	2	3	*Mn* I 5.86 (10)
4766.44..	3	3	*Mn* I 6.43 (20)
4766.78..	1	2	*Fe* I 6.88 (Pr), *Cr* I 6.63 (35), *V* I 6.64 (10), *C* I 6.62 (2)
4767.36..	0	1	*Ti* II 7.30 (Pr), *Cr* I 7.28 (25)
4768.34..	3	3	*Fe* I 8.33 (1), *Fe* I 8.40 (3)
4768.78..	1	1	(*Ti* II 8.83 (Pr))
4769.96..	1n	3	*C* I 0.00 (2)
4770.92..	0	1	*Cr* I 0.67 (12)
4771.59..	3nn	3	*Cr* I 1.57 (10), *C* I 1.72 (4), *Fe* I 1.70 (1)
4772.75..	3	3	*Fe* I 2.82 (3), *Fe* II 2.77 (Pr)
4773.83..	0	1	*Ce* II 3.94 (50)
4775.90..	1	3	*C* I 5.87 (3)
4776.37..	1	2	*Fe* I 6.34 (1), *Co* I 6.31 (6), *V* I 6.36 (10)
4776.64..	1	1	*V* I 6.52 (5)
4779.41..	0	1	*Fe* I 9.44 (1), *Sc* I 9.35 (20)
4780.00..	6	3	*Ti* II 9.99 (1)
4780.92..	0	1	*Fe* I 0.82 (Pr)
4782.08..	0	1	(*Ti* II 2.03 (Pr))
4782.91..	0	1	*Fe* I 2.81 (Pr)
4783.42..	5	3	*Mn* I 3.42 (50)
4784.00..	1	1	....
4785.15..	0	1	(*Co* I 5.07 (1))
4785.80..	0	1	(*Fe* I 5.96 (1))
4786.58..	3b	3	*Ni* I 6.54 (15), *Y* II 6.58 (20), *V* I 6.52 (20)
4786.80..	3b	2	*Fe* I 6.81 (5)
4787.79..	1	3	*Fe* I 7.84 (1), *Cr* I 7.74 (5)
4788.76..	2	3	*Fe* I 8.76 (4)

TABLE 1—*Continued*

λ	Int.	N	Identification
4789.32	1	2	*Cr* I 9.35 (75)
4789.65	3	3	*Fe* I 9.65 (7)
4790.25	0	1	*Cr* I 0.34 (20)
4790.51	0	1	*Fe* I 0.57 (Pr)
4791.20	1n	3	*Fe* I 1.25 (1)
4792.42	2	3	*Ti* II 2.39 (Pr), *Cr* I 2.51 (75), *Ti* I 2.48 (10)
4793.42	0	1	(*Ni* I 3.47 (Pr))
4794.35	0	1	*Fe* I 4.36 (Pr)
4796.26	0	1	*Cr* I 6.17 (40), *Ti* I 6.21 (6)
4797.37	0	1	(*Nd* II 7.16 (20))
4798.46	2	3	*Ti* II 8.54 (2), *Fe* I 8.27 (1), *Fe* I 8.74 (1)
4799.29	1	2	*Fe* I 9.41 (1)
4799.86	1	3	*Ti* I 9.80 (12)
4800.66	1	3	*Fe* I 0.65 (2)
4801.04	1	2	*Cr* I 1.03 (75)
4801.69	0	1	*Fe* I 1.62 (Pr)
4802.90	2	3	*Fe* I 2.88 (3)
4803.69	0	1	(*Ti* II 3.63 (Pr))
4804.53	0	1	*Fe* I 4.53 (1), *Fe* I 4.59 (1), *Cr* I 4.64 (15), *Cr* II 4.57 (1)
4805.09	6	3	*Ti* II 5.10 (2)
4805.53	0	1	*Ti* I 5.42 (12)
4806.22	1	1	*Cr* I 6.26 (25)
4806.98	1	3	*Ni* I 7.00 (4)
4807.68	1	3	*Fe* I 7.72 (2), *V* I 7.54 (25)
4808.69	1	3	*Ni* I 8.86 (2)
4809.17	0	1	*Fe* I 9.15 (1), *Fe* I 9.26 (1)
4810.54	3	3	*Zn* I 0.53 (65)
4812.33	2	3	*Cr* II 2.35 (25)
4812.93	0	1	*Fe* I 3.12 (1)
4813.53	1	1	*Co* I 3.48 (20), (*V* II 3.95 (50))
4815.90	0	1	*Sm* II 5.81 (400), (*Ni* I 5.92 (1))
4817.83	1	3	*Ni* I 7.85 (2), *Fe* I 7.77 (1)
4820.28	1	1	(*Nd* II 0.34 (30))
4820.46	1	1	*Ti* I 0.41 (20)
4821.27	0	1	*Ni* I 1.14 (2), (*Ti* II 1.01 (Pr))
4822.70	0	1	*Fe* I 2.68 (Pr)
4823.49	6	3	*Mn* I 3.52 (50)
4824.14	5	3	*Cr* II 4.13 (75), *Fe* I 4.16 (1)
4824.97	0	1	*Cr* II 4.97 (Pr)
4825.42	0	3	*Ti* I 5.44 (3), *Nd* II 5.48 (150)
4827.62	0	2	*Ti* I 7.60 (2)
4829.03	3	3	*Ni* I 9.03 (15)
4829.38	1	3	*Cr* I 9.38 (100)
4831.15	2	3	*Ni* I 1.18 (10), *Fe* II 1.11 (Pr)
4832.73	1	3	*Fe* I 2.73 (2), *Ni* I 2.70 (2)
4833.31	0	1	*Fe* II 3.21 (Pr)
4834.66	0	1	*Fe* I 4.51 (1)
4835.86	1	2	*Fe* I 5.86 (3)
4836.23	2	3	*Cr* II 6.22 (25), *Ti* I 6.12 (6)
4836.94	0	1	*Cr* I 6.86 (40)
4837.58	0	2	*Fe* I 7.67 (Pr)
4838.57	2	3	*Fe* I 8.52 (2), *Ni* I 8.65 (2)
4839.55	2	3	*Fe* I 9.55 (3)
4840.29	2	3	*Co* I 0.25 (25), *Fe* I 0.33 (1)
4840.89	1	3	*Ti* I 0.87 (25)
4841.60	1	1	*Fe* I 1.68 (Pr)
4841.76	1	2	*Fe* I 1.80 (1), *Cr* I 1.73 (12)
4842.78	1	2	*Fe* I 2.79 (1)
4843.17	1	3	*Fe* I 3.16 (3), *Ni* I 3.16 (2)
4844.05	1	3	*Fe* I 4.00 (2)
4845.65	0	3	*Fe* I 5.66 (2)
4847.27	1n	2	*Ca* I 7.30 (2)
4848.25	4	3	*Cr* II 8.24 (60)
4849.10	1n	3	*Ti* II 9.18 (Pr), *Fe* I 8.88 (1)
4852.54	0	2	*Ni* I 2.56 (2)
4852.70	0	1	(*Y* I 2.69 (50))
4853.86	0	1	(*Ni* I 3.74 (1))
4854.87	3	3	*Y* II 4.87 (150), *Fe* I 4.89 (1)
4855.48	3	3	*Ni* I 5.41 (15), *Fe* II 5.54 (Pr)
4855.83	1	1	*Fe* I 5.68 (3)
4856.10	2	1	*Ti* II 5.95 (Pr), *Ti* I 6.01 (20)
4856.18	1	2	*Cr* II 6.19 (20)
4857.39	0	3	*Cr* I 7.34 (18), *Ni* I 7.38 (2)
4858.99	0	1	*Fe* I 9.14 (1), *Nd* II 9.03 (100)
4859.74	4	4	*Fe* I 9.75 (15)
4860.40	1	1	*Cr* II 0.20 (20)
4861.34	45	4	*Hβ* 1.33
4862.61	0nn	1	*Fe* I 2.55 (Pr)
4863.65	1	1	*Fe* I 3.65 (2)
4863.77	1	2	(*Fe* I 3.78 (Pr))
4864.32	3	3	*Cr* II 4.32 (50), *Ni* I 4.28 (2)
4865.13	0	2	....
4865.61	2	3	*Ti* II 5.62 (tr)
4866.26	3	3	*Ni* I 6.27 (10)
4867.81	1	2	*Co* I 7.87 (25), *Fe* II 7.73 (Pr)
4868.42	1	2	*Fe* I 8.38 (Pr), *Ti* I 8.26 (18)
4868.76	0	1	*Fe* II 8.82 (Pr)
4869.40	0	2	*Fe* I 9.47 (Pr)
4870.07	1n	3	*Ti* I 0.13 (20), *Fe* I 0.05 (Pr)
4870.83	1	3	*Cr* I 0.80 (100), *Ni* I 0.84 (2)
4871.31	6	3	*Fe* I 1.32 (25)
4872.12	6	3	*Fe* I 2.14 (20)
4872.80	1	3	(*Fe* I 2.70 (Pr))
4873.42	2	3	*Ni* I 3.44 (4)
4873.99	2	3	*Ti* II 4.02 (tr), *Fe* I 4.36 (Pr), *Fe* I 3.75 (Pr)
4874.81	1	1	*Ni* I 4.81 (2), *Cr* I 4.65 (20)
4874.97	1n	2	....
4875.85	1	3	*Fe* I 5.90 (1), *Fe* I 5.74 (Pr)
4876.44	4	3	*Cr* II 6.41 (50), *Cr* II 6.48 (Pr)
4877.25	0	1	(*Fe* I 7.59 (Pr))
4878.18	7	3	*Fe* I 8.22 (12), *Ca* I 8.13 (50)
4879.08	0	1	(*Pr* II 9.12 (30))
4879.29	1nn	1	....
4880.39	0	1	(*Cr* I 0.06 (25))

TABLE 1—*Continued*

λ	Int.	N	Identification
4881.67..	1	3	*Fe* I 1.73 (2)
4882.18..	1	3	*Fe* I 2.15 (2)
4882.90..	0	2	....
4883.69..	3	3	*Y* II 3.69 (200)
4884.60..	1	3	*Cr* II 4.57 (10)
4885.08..	1	3	*Ti* I 5.08 (20), *Cr* I 4.95 (25)
4885.45..	2	3	*Fe* I 5.44 (2)
4885.82..	0	1	*Cr* I 5.78 (75), *Cr* I 5.96 (50)
4886.32..	3	3	*Fe* I 6.34 (1)
4887.09..	3n	3	*Cr* I 7.01 (150), *Fe* I 7.19 (—), *Ni* I 6.99 (3)
4887.69..	0	1	*Cr* I 7.73 (25)
4887.90..	1	2	....
4888.63..	3	3	*Fe* I 8.65 (1), *Cr* I 8.53 (40)
4889.05..	3	3	*Fe* I 9.11 (2), *Fe* I 9.01 (1)
4889.88..	0	1	*Cr* I 9.73 (20)
4890.20..	1n	1	....
4890.77..	6	3	*Fe* I 0.76 (25)
4891.50..	6	3	*Fe* I 1.50 (50)
4892.11..	1	1	*Cr* I 1.97 (18)
4892.86..	1	3	*Fe* I 2.87 (1)
4893.80..	1n	3	*Fe* II 3.78 (0), *Fe* I 3.71 (Pr)
4896.47..	1	2	*Fe* I 6.44 (1)
4900.06..	5	3	*Y* II 0.13 (150), *Ti* I 9.91 (20)
4900.82..	0	3	*Cr* I 0.82 (Pr), *Ti* I 0.62 (7)
4903.31..	4	3	*Fe* I 3.32 (12), *Cr* I 3.24 (70)
4904.42..	3	3	*Ni* I 4.41 (10)
4905.11..	1	3	*Fe* I 5.15 (1)
4906.86..	1n	2	*Fe* I 6.78 (Pr)
4907.78..	1n	3	*Fe* I 7.74 (1)
4908.18..	1n	2	(*Fe* II 8.21 (0))
4908.78..	1	2	*Fe* I 8.61 (Pr)
4909.40..	2	3	*Fe* I 9.39 (1)
4910.05..	3b	3	*Fe* I 0.03 (2)
4910.30..	3b	3	*Fe* I 0.33 (1)
4910.56..	3b	1	*Fe* I 0.57 (1)
4911.21..	3	3	*Ti* II 1.20 (0)
4911.90..	1n	3	*Fe* I 1.79 (1), *Ni* I 2.03 (2)
4912.56..	0	2	*Cr* II 2.49 (12)
4913.16..	0	2	*Fe* II 3.37 (1)
4913.62..	1	1	*Ti* I 3.62 (20)
4913.93..	1n	3	*Ni* I 3.97 (3)
4915.38..	0	1	*Ti* I 5.24 (5)
4916.57..	0	1	*Fe* I 6.68 (Pr)
4917.23..	2	3	*Fe* I 7.24 (1)
4918.02..	1	3	*Fe* I 8.02 (1)
4918.40..	1	3	*Ni* I 8.36 (4)
4918.99..	6	3	*Fe* I 9.00 (30)
4919.69..	1	2	*Fe* I 9.75 (Pr)
4919.86..	0	1	*Ti* I 9.87 (10)
4920.52..	7	3	*Fe* I 0.51 (60)
4920.89..	1n	2	*Cr* I 0.94 (50), *La* II 0.98 (300)
4921.80..	1	3	*La* II 1.80 (300), *Ti* I 1.77 (12)
4922.26..	2	3	*Cr* I 2.27 (300)
4923.28..	0n	2	....
4923.94..	8	3	*Fe* II 3.92 (12)
4924.80..	3	3	*Fe* I 4.78 (3)
4925.57..	2	3	*Ni* I 5.58 (2)
4926.67..	0	1	*Fe* I 6.85 (Pr)
4927.39..	1	3	*Fe* I 7.45 (1)
4927.87..	1	3	....
4930.29..	2	3	*Fe* I 0.33 (2)
4931.32..	0	2	....
4932.05..	2	3	*C* I 2.00 (5), *V* I 2.03 (4)
4933.29..	3	3	*Fe* I 3.35 (2), *Fe* I 3.19 (Pr)
4934.07..	8	3	*Ba* II 4.09 (700), *Fe* I 4.02 (2)
4934.85..	0	2	*La* II 4.83 (100), *Cr* I 4.89 (15)
4935.81..	1	3	*Ni* I 5.83 (4)
4936.38..	1	3	*Cr* I 6.33 (150)
4937.31..	3	3	*Ni* I 7.34 (4)
4938.19..	2	3	*Fe* I 8.18 (2)
4938.81..	3	3	*Fe* I 8.82 (10)
4939.27..	2	3	*Fe* I 9.24 (2)
4939.71..	2	3	*Fe* I 9.69 (4)
4940.45..	1	1	....
4941.62..	2	1	....
4941.90..	0	1	*Ni* I 1.92 (2)
4942.52..	2	2	*Cr* I 2.50 (200), *Fe* I 2.60 (Pr)
4944.28..	0	2	....
4945.51..	1	3	*Fe* I 5.63 (1), *Ni* I 5.46 (2)
4946.40..	3	3	*Fe* I 6.39 (4)
4947.54..	1	2	*V* II 7.58 (40)
4948.34..	0	2	....
4948.59..	0	1	*Fe* II 8.85 (1), *Ti* I 8.49 (8)
4950.10..	2	3	*Fe* I 0.11 (2)
4952.36..	0	1	(*Ni* I 2.33 (1))
4952.67..	2	3	*Fe* I 2.65 (1), *Cr* II 2.78 (10)
4953.25..	1	3	*Ni* I 3.20 (3)
4953.96..	0	3	*Fe* II 3.98 (0)
4954.72..	1	3	*Cr* I 4.81 (80)
4955.89..	0	2	....
4957.34..	6b	2	*Fe* I 7.30 (20), (*Dy* II 7.36 (1500))
4957.56..	10b	3	*Fe* I 7.60 (60)
4959.09..	1n	3	*Nd* II 9.13 (60)
4961.82..	1	1	*Fe* I 1.91 (1)
4962.62..	1	3	*Fe* I 2.56 (1)
4964.97..	1	2	*Cr* I 4.93 (100)
4965.23..	0	1	*V* II 5.40 (40), *Ni* I 5.14 (1)
4966.10..	4	3	*Fe* I 6.10 (8)
4966.94..	0	1	*Cr* I 6.80 (25)
4967.53..	0	1	*Ni* I 7.55 (1)
4967.91..	2	3	*Fe* I 7.90 (3)
4968.67..	1	3	*Fe* I 8.70 (1)
4969.94..	2	3	*Fe* I 9.93 (3)
4970.57..	1	3	*Fe* I 0.49 (2), *Fe* I 0.65 (Pr)
4971.37..	1	3	*Ni* I 1.35 (2)
4972.18..	0	3	(*Fe* I 2.40 (1))
4973.10..	3	3	*Fe* I 3.11 (3)
4974.30..	1n	2	....
4975.21..	1	1	....
4975.35..	1	1	*Ti* I 5.34 (10), *Fe* I 5.42 (1)
4976.17..	0	3	*Ni* I 6.16 (1), *Ni* I 6.34 (2)
4977.59..	1	2	*Fe* I 7.65 (1)
4978.06..	0	1	*Ti* I 8.19 (10), *Fe* I 8.12 (Pr)

TABLE 1—*Continued*

λ	Int.	N	Identification
4978.61..	3	3	*Fe* I 8.61 (2), *Na* I 8.54 (5)
4979.44..	1	1	*Fe* I 9.59 (1)
4980.20..	3	3	*Ni* I 0.16 (12)
4981.17..	0	1	*Ti* II 1.38 (Pr)
4981.75..	3	3	*Ti* I 1.73 (60)
4982.15..	0	1	*Y* II 2.13 (15)
4982.51..	4	3	*Fe* I 2.51 (8)
4982.78..	1b	3	*Na* I 2.81 (6)
4983.27..	3	3	*Fe* I 3.26 (5)
4983.94..	4n	3	*Fe* I 3.86 (6)
4984.11..	2b	1	*Ni* I 4.13 (10)
4984.69..	0	2	....
4985.24..	2b	1	*Fe* I 5.26 (7)
4985.44..	3nn	3	*Fe* I 5.55 (7), (*Cr* II 5.46 (Pr))
4986.21..	1	3	*Fe* I 6.22 (1)
4986.91..	0	1	*Fe* I 6.92 (Pr), (*La* II 6.82 (100))
4988.04..	0	2	(*Co* I 8.04 (2))
4988.96..	3	3	*Fe* I 8.96 (6)
4990.38..	1	2	....
4991.17..	5	3	*Ti* I 1.07 (50), *Fe* I 1.28 (3), *Fe* II 1.11 (Pr)
4991.88..	0	1	*Fe* I 1.86 (Pr)
4992.78..	0	2	(*Fe* I 2.79 (Pr))
4993.13..	1	1	....
4993.37..	2	2	*Fe* II 3.36 (1)
4993.64..	1	1	*Fe* I 3.69 (1)
4994.15..	3	3	*Fe* I 4.13 (8)
4994.98..	1	1	(*Ti* I 5.06 (0))
4995.45..	1n	3	*Fe* I 5.41 (Pr)
4995.72..	1	1	*Ti* II 5.89 (Pr)
4996.22..	1n	2	....
4996.86..	1	3	*Ni* I 6.85 (2), (*La* II 6.82 (50))
4997.73..	0	2	....
4998.19..	2	3	*Ni* I 8.23 (2)
4999.52..	3	3	*Ti* I 9.50 (45), *La* II 9.46 (200)
5000.33..	2	3	*Ni* I 0.34 (4)
5000.88..	0	3	*Zr* II 0.91 (3), *Ti* I 0.99 (10), *Fe* II 0.73 (Pr)
5001.48..	1	3	*Ca* II 1.49 (1)
5001.89..	3	3	*Fe* I 1.87 (12)
5002.79..	2	2	*Fe* I 2.80 (6)
5003.99..	1n	3	*Fe* I 4.03 (1), *Ni* I 3.75 (2)
5004.27..	1b	1	*Fe* II 4.26 (3), *Cr* I 4.38 (35)
5005.15..	1	3	*Ti* II 5.18 (Pr)
5005.72..	3	3	*Fe* I 5.72 (10)
5006.13..	5	3	*Fe* I 6.13 (20)
5007.27..	5	3	*Fe* I 7.29 (3), *Ti* I 7.21 (40)
5010.17..	1n	3	*Ti* II 0.20 (tr), *Ni* I 0.04 (2)
5010.95..	1	3	*Ni* I 0.96 (3)
5012.10..	5	3	*Fe* I 2.07 (12), *Fe* I 2.16 (Pr)
5012.48..	2b	2	*Ni* I 2.46 (2)
5012.73..	1b	1	*Fe* I 2.70 (Pr)
5013.28..	1	3	*Cr* I 3.32 (100), *Ti* I 3.28 (18)
5013.71..	1	3	*Ti* II 3.71 (tr)
5014.27..	3	3	*Ti* I 4.28 (25), *Ti* I 4.18 (25)
5014.96..	4	3	*Fe* I 4.95 (10)
5015.72..	0	1	(*Fe* II 5.77 (0))
5016.13..	1	3	*Ti* I 6.16 (20)
5016.40..	0	1	*Fe* I 6.48 (Pr)
5016.90..	1	3	....
5017.59..	3	3	*Ni* I 7.59 (10)
5018.44..	10	3	*Fe* II 8.43 (12)
5019.42..	1	3	*Fe* II 9.48 (0)
5019.98..	2n	2	*Ti* I 0.03 (25), *Ca* II 9.98 (2), *V* II 9.86 (100)
5020.88..	0n	2	*Fe* I 0.82 (1)
5021.63..	1	3	*Fe* I 1.60 (Pr), (*Fe* I 1.69 (Pr))
5022.24..	3	3	*Fe* I 2.24 (6)
5022.89..	1	3	*Ti* I 2.87 (25), *Fe* II 2.87 (1)
5023.37..	1	2	*Fe* I 3.48 (−), *Fe* I 3.23 (−)
5024.80..	1	3	*Ti* I 4.84 (20)
5025.55..	1	3	*Ti* I 5.57 (18)
5026.30..	0	2	....
5027.17..	3	3	*Fe* I 7.14 (5), *Fe* I 7.21 (1)
5027.76..	1	3	*Fe* I 7.78 (−)
5028.15..	2	3	*Fe* I 8.13 (4)
5029.63..	2n	2	*Fe* I 9.62 (1)
5029.77..	1n	1	(*Mn* I 9.81 (1))
5031.03..	5	3	*Sc* II 1.02 (40), *Fe* I 0.78 (5), *Fe* I 1.03 (2)
5031.88..	0n	2	*Fe* I 1.90 (8)
5032.70..	0n	3	*Fe* II 2.79 (1), *Ni* I 2.75 (1)
5034.68..	0	1	(*Fe* I 5.02 (3))
5035.36..	3	3	*Ni* I 5.37 (12)
5035.97..	2	3	*Ti* I 5.91 (25), *Ni* I 5.96 (3)
5036.44..	1	3	*Ti* I 6.47 (25), *Fe* I 6.29 (6)
5036.97..	1	1	*Fe* I 6.93 (2), *Fe* II 6.92 (2)
5037.87..	0	1	*Ti* II 7.81 (Pr)
5038.47..	2	3	*Ti* I 8.40 (25), *Ni* I 8.60 (4)
5039.22..	2	3	*Fe* I 9.26 (2), *Ni* I 9.26 (2)
5040.01..	2	3	*Ti* I 9.96 (22)
5041.00..	5	3	*Fe* I 1.07 (7), *Fe* I 0.90 (2)
5041.70..	6	3	*Fe* I 1.76 (10), *Ca* I 1.62 (40)
5042.19..	1	3	*Ni* I 2.20 (4)
5043.62..	0	2	*Ti* I 3.58 (7)
5044.21..	1	3	*Fe* I 4.22 (2)
5048.05..	1b	3	*Ni* I 8.08 (1)
5048.46..	2b	3	*Fe* I 8.46 (2)
5048.82..	2b	3	*Ni* I 8.85 (4), *Cr* I 8.75 (25)
5049.82..	5	3	*Fe* I 9.82 (15)
5050.58..	1	2	....
5050.93..	1	2	....
5051.64..	3	3	*Fe* I 1.64 (10)
5052.18..	3	3	*C* I 2.12 (6)
5053.00..	1	2	*Fe* I 2.99 (Pr), *Ti* I 2.88 (8)
5053.52..	1	2	....
5053.93..	0	1	(*Ti* I 4.07 (3))
5054.72..	1	2	*Fe* I 4.65 (1)
5056.03..	1	3	*Si* II 6.02 (10), *Fe* I 6.02 (1)
5057.98..	1	2	*Fe* I 8.00 (1), *Ni* I 8.03 (2)
5060.09..	1n	2	*Fe* I 0.08 (1)
5061.72..	0	1	*Fe* II 1.79 (1)
5064.68..	1	3	*Ti* I 4.65 (25)

TABLE 1—*Continued*

λ	Int.	N	Identification
5065.08	5	3	*Fe* I 5.02 (6), *Fe* I 5.21 (2), *Fe* I 4.98 (Pr)
5065.83	0	3	*Cr* I 5.91 (50), *Ti* I 5.98 (7)
5066.24	1	2	(*Fe* I 6.28 (Pr))
5067.19	2	3	*Fe* I 7.16 (1)
5067.79	0	1	*Cr* I 7.71 (75), (*Ni* I 7.82 (1))
5068.80	4	3	*Fe* I 8.77 (10)
5069.90	0n	2	(*Sc* I 0.25 (40))
5070.96	0	1	*Fe* II 0.96 (2)
5071.25	0	1	(*Ti* I 1.48 (7))
5072.16	3	3	*Ti* II 2.30 (2), *Fe* I 2.08 (1)
5072.71	2	3	*Fe* I 2.69 (1), *Cr* I 2.92 (60)
5073.60	0	2	(*Fe* II 3.56 (tr))
5073.83	0	1	*Fe* II 4.06 (1)
5074.77	4	3	*Fe* I 4.76 (10)
5075.44	0	2	*Fe* I 5.17 (Pr), *Ce* II 5.30 (20)
5076.30	2	3	*Fe* I 6.29 (2), *Ni* I 6.32 (2)
5077.32	0	2	(*Ti* II 7.33 (Pr))
5079.13	4n	3	*Fe* I 9.23 (6), *Fe* I 8.98 (1)
5079.78	3	3	*Fe* I 9.74 (4), *Ce* II 9.68 (75)
5080.54	3	3	*Ni* I 0.52 (30)
5081.15	3	3	*Ni* I 1.11 (25)
5081.62	0	1	*Sc* I 1.55 (125)
5081.76	0	1	*Fe* I 1.84 (Pr), *Fe* II 1.92 (tr)
5082.36	2	3	*Ni* I 2.35 (4)
5083.37	3	3	*Fe* I 3.34 (7)
5084.12	3	3	*Ni* I 4.08 (15)
5085.51	1n	3	*Sc* I 5.55 (40), *Ni* I 5.48 (2)
5086.42	0	2	(*Fe* II 6.36 (tr))
5087.45	2	3	*Y* II 7.42 (100)
5088.21	0	1	*Fe* I 8.16 (1)
5088.43	0	1	*Ni* I 8.53 (2)
5089.02	1	1	*Ni* I 8.96 (2), *Fe* II 9.28 (0)
5090.81	3	3	*Fe* I 0.79 (6)
5091.69	1	2	*Fe* I 1.73 (Pr)
5092.77	0	1	(*Ti* II 2.82 (Pr)), *Nd* II 2.80 (30))
5093.63	0	1	*Fe* II 3.65 (1), *Fe* II 3.47 (1)
5094.42	0	3	*Ni* I 4.42 (2)
5097.00	3	3	*Fe* I 7.00 (6)
5097.43	1	3	*Fe* II 7.38 (1), *Cr* II 7.29 (7)
5098.69	4	3	*Fe* I 8.70 (8), *Fe* I 8.59 (3)
5099.27	2n	3	*Ni* I 9.32 (5), *Fe* I 9.09 (1), (*Sc* I 9.23 (40))
5099.98	3	3	*Ni* I 9.95 (10)
5100.70	1	3	*Fe* II 0.84 (4), *Fe* II 0.70 (2), *Fe* II 0.66 (Pr)
5101.32	0	1	*Fe* II 1.48 (2)
5102.97	1	3	*Ni* I 2.97 (4)
5104.11	1	2	*Fe* I 4.04 (−)
5104.24	1n	1	*Fe* I 4.21 (1)
5104.42	1n	1	*Fe* I 4.44 (1)
5105.53	2	3	*Cu* I 5.54 (300)
5106.46	0n	2	....
5107.58	7	3	*Fe* I 7.64 (8), *Fe* I 7.45 (6)
5108.47	0	3	....
5108.93	0	1	*Co* I 8.90 (10), *Cr* I 8.93 (12)
5109.68	3	3	*Fe* I 9.65 (2)
5110.43	4	3	*Fe* I 0.41 (10)
5111.02	0	1	*Cr* I 0.75 (40)
5112.28	1n	1	*Zr* II 2.28 (7)
5112.45	1	1	*Cr* I 2.49 (25)
5113.28	0	1	*Cr* I 3.13 (45), *Co* I 3.23 (6)
5114.52	0	3	*La* II 4.55 (200), *Fe* I 4.52 (Pr)
5115.43	2	3	*Ni* I 5.40 (8)
5116.02	0	2	*Cr* II 6.06 (2), (*Fe* I 5.79 (1))
5116.76	0	1	(*Fe* II 7.11 (0))
5119.13	1	2	*Y* II 9.12 (20)
5120.41	1	3	*Ti* I 0.42 (12), *Fe* II 0.34 (Pr)
5121.63	3	3	*Fe* I 1.63 (2)
5122.13	0	1	*Cr* I 2.12 (30)
5122.66	1n	1	*Co* I 2.77 (8)
5123.10	1	2	*Y* II 3.21 (50), *La* II 2.99 (200)
5123.75	3	3	*Fe* I 3.72 (6)
5124.65	0	1	*Fe* I 4.62 (Pr)
5125.16	5	3	*Fe* I 5.13 (6), *Ni* I 5.21 (4)
5126.20	2	3	*Co* I 6.20 (10), *Fe* I 6.22 (1), *Fe* II 6.19 (Pr)
5126.65	0	1	*Fe* I 6.60 (1)
5127.37	3	3	*Fe* I 7.36 (5)
5127.84	1	2	*Fe* II 7.87 (1), *Fe* I 7.69 (Pr)
5128.44	0	1	*V* I 8.53 (7)
5129.22	4n	3	*Ti* II 9.14 (1)
5129.44	0	1	*Ni* I 9.38 (5)
5129.63	0	1	*Fe* I 9.66 (1)
5130.44	1	2	*Ni* I 0.39 (2)
5131.50	2	3	*Fe* I 1.48 (2)
5132.65	1	3	*Fe* II 2.67 (Pr)
5133.68	4	3	*Fe* I 3.69 (20)
5134.62	1	2	....
5136.08	0	1	*Fe* I 6.09 (1)
5137.00	1b	1	*Ni* I 7.08 (8), *Cr* II 7.09 (7)
5137.32	3nn	3	*Fe* I 7.39 (6)
5138.43	1n	1	*V* I 8.43 (5)
5139.38	9	3	*Fe* I 9.47 (20), *Fe* I 9.26 (10)
5140.81	0	3	....
5141.75	2	3	*Fe* I 1.75 (2)
5142.58	3b	3	*Fe* I 2.54 (3)
5142.83	3b	3	*Fe* I 2.93 (6), *Ni* I 2.77 (10)
5143.73	1n	2	*Fe* I 3.73 (Pr)
5145.05	1n	2	*Fe* I 5.10 (−), *Cr* I 4.67 (50)
5145.32	0	1	*Ti* I 5.46 (12)
5145.91	1	1	(*Fe* II 5.87 (0))
5146.13	0	1	*Fe* II 6.12 (Pr)
5146.44	3	3	*Ni* I 6.48 (12)
5148.15	3	3	*Fe* I 8.23 (3), *Fe* I 8.06 (3)
5149.48	0	1	*Fe* II 9.54 (3)
5150.29	0	1	*Fe* I 0.20 (Pr)
5150.86	3	3	*Fe* I 0.84 (6), (*Fe* II 0.93 (Pr))
5151.91	2	3	*Fe* I 1.92 (4)
5152.34	0	1	*Ti* I 2.18 (10)
5152.54	1	1	....

TABLE 1—*Continued*

λ	Int.	N	Identification
5152.95..	1	1	....
5153.36..	2n	3	*Cr* II 3.49 (15), *Na* I 3.40 (6), *Cu* I 3.24 (100)
5154.09..	3	3	*Ti* II 4.06 (0), *Co* I 4.07 (8)
5154.39..	1	2	*Fe* II 4.40 (Pr)
5155.15..	1	3	*Ni* I 5.14 (4)
5155.79..	2	3	*Ni* I 5.76 (9)
5156.46..	0	1	*Co* I 6.37 (10)
5156.62..	0	2	....
5157.24..	1	2	(*La* II 7.43 (150))
5159.06..	1	3	*Fe* I 9.07 (2)
5160.09..	0	1	*Fe* II 9.93 (4), *Fe* I 9.97 (Pr)
5161.04..	1	1	*Fe* II 0.82 (1), *Fe* II 1.18 (Pr)
5162.28..	4	3	*Fe* I 2.29 (10)
5162.92..	0	1	(*Ni* I 2.93 (Pr))
5164.46..	1n	2	....
5164.65..	1n	1	*Fe* I 4.56 (1), *Fe* I 4.69 (Pr), *Fe* II 4.69 (Pr)
5165.42..	2	3	*Fe* I 5.42 (4)
5166.27..	3	3	*Fe* I 6.29 (4), *Cr* I 6.23 (150)
5167.39..	10	3	*Mg* I 7.32 (40), *Fe* I 7.49 (40)
5168.75..	1	1	*Fe* I 8.90 (4), *Ni* I 8.66 (6)
5169.00..	9	3	*Fe* II 9.03 (12)
5169.97..	0	1	*Fe* II 9.73 (1), *Fe* I 0.08 (Pr)
5170.18..	1	1	....
5170.58..	1n	1	....
5170.86..	1n	2	....
5171.62..	4	3	*Fe* I 1.60 (20), *Fe* II 1.62 (Pr)
5172.71..	10	3	*Mg* I 2.68 (80)
5173.73..	1	3	*Ti* I 3.74 (30)
5176.51..	1	3	*Ni* I 6.56 (5)
5177.36..	1	1	*Cr* I 7.43 (75), *Fe* I 7.23 (−)
5177.73..	0	1	*Cr* I 7.83 (10)
5178.85..	0	1	*Fe* I 8.80 (1), *Fe* II 8.71 (Pr)
5178.98..	1n	1	*Fe* II 8.95 (Pr)
5180.02..	1	3	*Fe* I 0.06 (−)
5183.62..	12	3	*Mg* I 3.60 (125)
5184.34..	0	3	*Fe* I 4.29 (3), *Cr* I 4.59 (100), *Ni* I 4.58 (4)
5185.91..	3	3	*Ti* II 5.90 (2)
5187.32..	0	1	*Ce* II 7.45 (60)
5187.94..	1	3	*Fe* I 7.92 (2)
5188.76..	6	3	*Ca* I 8.85 (50), *Ti* II 8.70 (6)
5189.76..	0	1	(*Ti* I 9.61 (Pr))
5190.46..	0	2	....
5191.47..	4	3	*Fe* I 1.46 (20)
5192.35..	4	3	*Fe* I 2.35 (30)
5192.93..	1	3	*Ti* I 2.97 (35), *Dy* II 2.87 (800)
5193.74..	0	2	(*Cr* I 3.49 (35))
5194.22..	0	1	*Ti* I 4.04 (4)
5194.96..	3	3	*Fe* I 4.94 (10)
5195.49..	2	3	*Fe* I 5.47 (8)
5196.08..	2	3	*Fe* I 6.10 (2)
5196.47..	1	3	*Cr* I 6.44, 6.57 (100)
5197.57..	3	3	*Fe* II 7.57 (6)
5197.93..	1	2	*Fe* I 7.94 (Pr)
5198.71..	3	3	*Fe* I 8.71 (4)
5199.42..	0	2	....
5200.39..	2n	3	*Y* II 0.42 (60)
5201.01..	0	1	*Ti* I 1.10 (4)
5202.30..	4	3	*Fe* I 2.34 (8), *Fe* I 2.27 (1)
5203.52..	1n	2	....
5204.56..	5	3	*Cr* I 4.52 (200), *Fe* I 4.58 (2)
5205.75..	1	2	*Y* II 5.73 (80)
5206.04..	5	3	*Cr* I 6.04 (300)
5207.22..	0	2	....
5208.49..	6	3	*Cr* I 8.44 (500), *Fe* I 8.60 (7)
5209.84..	0	1	*Fe* I 9.90 (Pr)
5210.40..	1	3	*Ti* I 0.39 (40)
5211.55..	1	3	*Ti* II 1.54 (0)
5212.78..	0	2	*Co* I 2.70 (25)
5215.19..	3	3	*Fe* I 5.18 (6)
5216.31..	4	3	*Fe* I 6.28 (10)
5217.41..	3	3	*Fe* I 7.40 (5)
5218.16..	1	3	*Cu* I 8.20 (80)
5218.86..	0	1	....
5220.26..	1	2	*Ni* I 0.31 (2)
5220.98..	0	1	*Cr* I 0.91 (40)
5222.62..	1	1	*Cr* I 2.68 (30), *Ti* I 2.68 (6)
5223.38..	1	1	*Fe* I 3.19 (1)
5224.38..	1	1	*Ti* I 4.30 (15)
5224.95..	1	3	*Cr* I 4.94 (150), *Ti* I 4.93 (8)
5225.56..	1	3	*Fe* I 5.53 (1)
5226.66..	4b	3	*Fe* I 6.87 (15), *Ti* II 6.53 (5)
5227.10..	5b	3	*Fe* I 7.19 (40)
5227.98..	0	1	*Cr* I 8.08 (50), *Ti* II 7.87 (Pr)
5228.39..	1	3	*Fe* I 8.39 (1)
5229.86..	3	3	*Fe* I 9.86 (5)
5231.03..	0	1	(*Ti* I 0.97 (0))
5232.40..	0	1	*Cr* II 2.50 (15)
5232.96..	4	3	*Fe* I 2.95 (40)
5234.09..	0	1	*V* I 4.09 (8)
5234.65..	3	3	*Fe* II 4.62 (7)
5235.43..	2	3	*Fe* I 5.39 (2), *Ni* I 5.49 (2)
5236.08..	1n	2	*Fe* I 6.20 (1)
5236.45..	1	1	*Fe* I 6.39 (Pr)
5237.35..	3	3	*Cr* II 7.34 (75)
5239.00..	0n	1	*Cr* I 8.97 (65)
5239.82..	2	3	*Sc* II 9.82 (15)
5241.02..	0	2	*V* II 1.19 (100), *V* I 0.88 (9)
5241.95..	0	1	*Fe* I 1.93 (1)
5242.50..	2	3	*Fe* I 2.50 (4)
5243.36..	1	2	*Cr* I 3.40 (75)
5243.77..	1	3	*Fe* I 3.79 (1)
5245.47..	1n	1	....
5245.81..	0	1	*Fe* I 5.74 (Pr)
5245.97..	0	1	(*Fe* I 6.01 (Pr))
5246.87..	1n	2	*Cr* II 6.75 (15), *Fe* I 7.06 (1)
5247.57..	1	3	*Cr* I 7.56 (150)
5249.08..	1	2	*Fe* I 9.10 (1)
5249.44..	1	1	*Cr* II 9.40 (10)
5249.63..	1	1	*Nd* II 9.58 (100)
5250.15..	0	2	*Fe* I 0.21 (1), *Co* I 0.00 (7)
5250.67..	2	3	*Fe* I 0.65 (6)
5252.02..	1	3	*Ti* II 2.04 (Pr), *Ti* I 2.10 (8)
5252.76..	0	2	....
5253.45..	2	3	*Fe* I 3.48 (2)
5254.96..	3	3	*Cr* I 4.92 (100), *Fe* I 4.96 (1), *Fe* II 4.92 (Pr)

TABLE 1—*Continued*

λ	Int.	N	Identification
5255.25	1	1	*Cr* I 5.13 (125), *Mn* I 5.32 (4)
5255.71	1	1	*Fe* I 5.75 (Pr), *Ti* I 5.81 (5), *Fe* I 5.67 (Pr)
5256.90	1	2	*Fe* II 6.89 (1)
5257.09	1	1	*Cr* I 7.07 (10)
5257.63	0	1	*Co* I 7.62 (10), *Fe* I 7.65 (Pr)
5260.36	0	3	*Ca* I 0.38 (2)
5260.88	0	1	*Mn* I 0.77 (3)
5261.74	2	3	*Ca* I 1.71 (20), *Cr* I 1.75 (50)
5262.23	3	3	*Ca* I 2.24 (25)
5263.33	3	3	*Fe* I 3.31 (8)
5263.88	1	1	*Fe* I 3.87 (1)
5264.20	3	3	*Ca* I 4.24 (20), *Cr* I 4.15 (200)
5264.83	2	3	*Fe* II 4.80 (2)
5265.61	4	3	*Ca* I 5.56 (40), *Cr* I 5.72 (100)
5266.57	4	3	*Fe* I 6.56 (30)
5267.27	0	3	*Fe* I 7.28 (Pr), *Cr* II 7.10 (4)
5268.52	1nn	3	*Co* I 8.50 (10), *Ti* II 8.62 (1)
5269.56	5	3	*Fe* I 9.54 (60)
5270.33	5	3	*Fe* I 0.36 (30), *Ca* I 0.27 (60)
5271.16	0n	3	....
5272.34	1n	3	*Fe* II 2.41 (2), (*Cr* I 2.01 (50))
5273.28	4	3	*Fe* I 3.18 (5), *Fe* I 3.38 (4)
5274.18	1	2	*Ce* II 4.24 (75)
5275.08	3	3	*Cr* II 4.99 (20), *Fe* I 5.02 (1), *Cr* I 5.17 (75)
5276.00	4	3	*Fe* II 5.99 (7), *Cr* I 6.03 (75)
5277.00	0	3	....
5278.04	0	2	(*Fe* II 8.26 (0))
5278.91	1	3	*Fe* II 8.96 (0), (*S* I 8.99 (3))
5280.01	1n	3	*Cr* II 9.92 (15), *Cr* II 0.08 (10)
5280.39	1	1	*Fe* I 0.36 (1), *Cr* I 0.29 (30)
5281.79	3	3	*Fe* I 1.80 (10)
5282.47	0	2	(*Ti* I 2.38 (3))
5283.64	3	3	*Fe* I 3.63 (18)
5284.14	2	3	*Fe* II 4.09 (3)
5285.22	1	3	*Fe* I 5.13 (Pr), *Cr* I 5.38 (7)
5288.56	1	3	*Fe* I 8.54 (2)
5292.57	1	2	....
5292.72	0	1	*Cr* I 2.86 (10)
5295.35	1	2	*Fe* I 5.32 (1), *Mn* II 5.29 (15), (*Sc* II 5.30 (Pr))
5296.69	2	3	*Cr* I 6.69 (100)
5297.40	1	3	*Cr* I 7.36 (60)
5298.24	3nn	3	*Cr* I 8.27 (100)
5298.64	1n	1	*Fe* I 8.78 (1)
5299.36	1	1	*Mn* II 9.28 (25)
5299.93	0n	3	....
5300.81	1	3	*Cr* I 0.75 (75), (*Co* I 1.04 (15))
5302.32	3	3	*Fe* I 2.31 (10), *Mn* II 2.32 (30)
5303.33	1	2	*Fe* II 3.42 (0), *V* II 3.26 (40)
5304.37	0	1	*Cr* I 4.21 (40), (*Fe* II 4.26 (Pr))
5305.90	1	3	*Cr* II 5.85 (25)
5307.34	3	3	*Fe* I 7.36 (2)
5308.39	1	3	*Cr* II 8.44 (20), *Fe* I 8.69 (Pr)
5309.37	0	1	*Cr* I 9.47 (8)
5310.69	1	3	*Cr* II 0.70 (12)
5311.67	0	1	*Zr* II 1.78 (2)
5313.61	1	3	*Cr* II 3.59 (25), *Fe* I 3.84 (−), *Ti* II 3.76 (Pr)
5315.03	1	3	*Fe* I 5.08 (1)
5316.69	9	3	*Fe* II 6.61 (8), *Fe* II 6.78 (4)
5318.30	0	3	*Sc* II 8.34 (3), *Fe* II 8.03 (1), *Fe* II 8.27 (0), *Cr* II 8.41 (4)
5320.04	1	2	*Fe* I 0.05 (1)
5321.09	1	1	*Fe* I 1.11 (1)
5322.07	1	3	*Fe* I 2.05 (2)
5322.65	0	1	*Cr* II 2.78 (Pr), (*Pr* II 2.78 (60))
5324.16	4	3	*Fe* I 4.18 (30)
5325.54	2	3	*Fe* II 5.56 (2)
5326.10	0	1	*Fe* I 6.15 (1)
5327.01	0n	2	*Fe* I 7.27 (Pr), (*Fe* I 6.79 (−))
5328.10	5	3	*Fe* I 8.04 (50)
5328.45	4	3	*Fe* I 8.53 (15), *Cr* I 8.34 (200)
5329.16	1	3	*Cr* I 9.12 (65)
5329.97	1	3	*Fe* I 9.99 (2)
5330.84	0	3	....
5332.83	2	3	*Fe* I 2.90 (4), *Fe* I 2.68 (1)
5333.68	1n	1	*Co* I 3.65 (5)
5334.83	2n	3	*Cr* II 4.88 (40)
5336.80	3	3	*Ti* II 6.81 (4)
5337.78	1	3	*Cr* II 7.79 (12), *Fe* II 7.71 (0)
5338.48	0	1	(*Ti* I 8.33 (1))
5339.23	0	2	*Ca* II 9.29 (−)
5339.93	3	3	*Fe* I 9.94 (12)
5341.05	3	3	*Fe* I 1.03 (20), *Mn* I 1.06 (20)
5342.26	1	1	....
5342.66	1	3	*Co* I 2.70 (50)
5343.44	1	3	*Co* I 3.38 (20)
5344.47	0	2	....
5344.77	1n	1	(*Cr* I 4.76 (20))
5345.83	3	3	*Cr* I 5.81 (500)
5346.57	0	2	*Fe* II 6.56 (1), *Cr* II 6.54 (5)
5348.34	2	3	*Cr* I 8.32 (350)
5349.52	3n	3	*Ca* I 9.47 (25), *Fe* I 9.74 (3)
5350.62	0n	1	*Zr* II 0.36 (5), (*Tl* I 0.46 (5000))
5352.18	0n	2	(*Co* I 2.05 (20))
5353.42	2	3	*Fe* I 3.39 (2), *Ni* I 3.42 (3), *Co* I 3.50 (25)
5354.79	0	1	(*Cr* II 4.66 (Pr))
5361.58	1	3	*Fe* I 1.64 (1), *Nd* II 1.47 (60)
5362.85	4	3	*Fe* II 2.86 (5)
5364.41	0	1	*Mn* I 4.48 (2)
5364.89	3	3	*Fe* I 4.87 (15)
5365.41	2	3	*Fe* I 5.40 (3)

TABLE 1—*Continued*

λ	Int.	N	Identification
5367.47	3	3	*Fe* I 7.47 (20)
5368.40	0	1	*Cr* I 8.55 (35)
5369.22	0	1	*Cr* II 9.25 (Pr)
5369.54	0	1	*Co* I 9.59 (20)
5369.96	4	3	*Fe* I 9.96 (25)
5371.50	5	3	*Fe* I 1.49 (50), *Cr* I 1.48 (50)
5373.68	1	3	*Fe* I 3.70 (1), *Cr* I 3.72 (30)
5375.70	0	1	(*Sc* I 5.35 (20))
5377.66	1	2	*Mn* I 7.63 (6)
5379.58	1	3	*Fe* I 9.58 (2)
5380.30	1	3	*C* I 0.24 (8)
5381.03	2	3	*Ti* II 1.02 (1)
5382.07	0n	1	*La* II 1.91 (100)
5382.62	0	1	*Fe* I 2.75 (−), *Fe* II 2.52 (Pr)
5383.39	4	3	*Fe* I 3.37 (35)
5389.50	2	3	*Fe* I 9.46 (5)
5390.47	1	3	*Cr* I 0.39 (40)
5391.48	2	3	*Fe* I 1.47 (1)
5392.35	1	2	*Ni* I 2.37 (2), *Sc* I 2.08 (30)
5393.16	3	3	*Fe* I 3.17 (10)
5394.62	1	2	*Mn* I 4.67 (10), *Fe* I 4.68 (−)
5395.18	0	3	*Fe* I 5.25 (1)
5396.23	1	2	*Ti* II 6.3 (1)
5397.15	4	3	*Fe* I 7.13 (40)
5398.31	2	3	*Fe* I 8.28 (1)
5399.12	1	1	....
5399.44	1	2	*Mn* I 9.49 (4)
5399.72	1	1	(*Co* I 9.76 (10))
5400.53	3	3	*Fe* I 0.51 (5)
5401.39	0	3	(*Fe* I 1.27 (Pr))
5402.10	0	1	*Fe* II 2.11 (2), *V* I 1.94 (8)
5402.85	1	2	*Y* II 2.78 (50)
5403.00	1	1	....
5404.09	4	3	*Fe* I 4.14 (30)
5405.80	4	3	*Fe* I 5.78 (40)
5406.76	1	2	*Fe* I 6.78 (Pr)
5407.60	1	3	*Cr* II 7.62 (10), *Co* I 7.52 (5)
5408.43	0	1	(*Fe* II 8.84 (1))
5409.08	1	3	*Fe* I 9.12 (1)
5409.82	3	3	*Cr* I 9.79 (500)
5410.94	3	3	*Fe* I 0.91 (15)
5412.52	0	1	*Fe* I 2.58 (Pr)
5413.01	1n	2	....
5414.03	1	3	*Fe* II 4.09 (2)
5415.21	3	3	*Fe* I 5.20 (35), *V* I 5.28 (10)
5416.25	0	2	....
5417.02	1	3	*Fe* I 7.04 (1)
5418.20	0	2	....
5418.80	2	3	*Ti* II 8.80 (0)
5420.29	0	3	*Mn* I 0.36 (10)
5421.07	1	3	(*Cr* II 0.90 (10))
5422.17	0	1	*Fe* I 2.17 (Pr)
5422.71	0	1	*Ti* II 2.47 (1)
5424.08	4	3	*Fe* I 4.07 (45)
5424.67	0	1	*Ni* I 4.65 (4)
5425.28	2	3	*Fe* II 5.27 (2)
5427.90	0	1	*Fe* II 7.82 (3)
5429.70	6	3	*Fe* I 9.70 (40)
5431.26	0n	2	....
5432.94	1	3	*Fe* I 2.95 (2), *Fe* II 2.98 (Pr)
5433.67	0	1	(*Sc* II 3.73 (Pr))
5434.55	3	3	*Fe* I 4.53 (30)
5435.14	1	1	(*Fe* I 5.18 (Pr))
5435.80	1b	2	*Ni* I 5.87 (5), *Fe* II 5.79 (Pr)
5436.32	0n	2	*Fe* I 6.30 (1)
5436.68	1b	1	*Fe* I 6.59 (2)
5438.18	1	1	*Fe* I 8.06 (Pr)
5439.56	1	1	....
5441.28	1n	2	*Fe* I 1.32 (1)
5442.51	1n	1	*Cr* I 2.41 (35)
5443.58	0	2	(*Fe* I 3.42 (Pr))
5445.05	3	3	*Fe* I 5.04 (15)
5445.72	0	1	*Fe* II 5.97 (Pr)
5446.61	1b	1	*Ti* II 6.46 (Pr), *Cr* II 6.57 (Pr), (*Ti* I 6.59 (2))
5446.88	5	3	*Fe* I 6.92 (40)
5448.05	1	2	....
5453.51	0n	1	(*Ti* I 3.65 (3))
5455.57	9	3	*Fe* I 5.61 (40), *Fe* I 5.43 (5)
5456.56	0	2	*Fe* I 6.47 (1)
5457.63	1	1	....
5463.16	4n	3	*Fe* I 3.28 (10), *Fe* I 2.97 (2)
5464.25	1n	2	*Fe* I 4.29 (1)
5465.49	0	1	(*Ag* I 5.49 (1000)), (*Fe* I 5.04 (1))
5466.01	0	1	*Fe* II 6.02 (2)
5466.42	2	3	*Fe* I 6.40 (3), (*Y* I 6.47 (300))
5467.07	0	1	*Fe* I 6.99 (1), *Fe* II 6.95 (3)
5468.17	0	2	(*Ni* I 8.10 (2)), (*Ti* II 8.44 (Pr))
5469.41	0	1	*Fe* I 9.28 (Pr), (*Co* I 9.30 (4))
5470.04	0	1	*Fe* I 0.17 (1)
5470.62	0	1	*Mn* I 0.64 (8), (*Fe* II 0.81 (Pr))
5472.60	1	1	*Cr* II 2.63 (3)
5472.73	1	1	*Fe* I 2.73 (1)
5473.92	2	3	*Fe* I 3.91 (3)
5476.54	3n	3	*Fe* I 6.57 (10), *Fe* I 6.30 (2)
5476.86	4b	2	*Ni* I 6.91 (50)
5477.78	1	1	*Ti* I 7.70 (8), *Zr* II 7.82 (2), *Fe* II 7.67 (Pr)
5478.41	2	3	*Cr* II 8.35 (15), *Fe* I 8.46 (1)
5479.77	0	1	*Fe* I 9.98 (Pr)
5480.82	1	3	*Fe* I 0.87 (2), (*Y* II 0.75 (15))
5481.34	2	3	*Fe* I 1.45 (3), *Fe* I 1.26 (2), *Ti* I 1.43 (6), *Mn* I 1.40 (4)
5481.94	0	1	*Sc* I 1.99 (100), *Ti* I 1.86 (5)
5483.18	1	3	*Fe* I 3.12 (1), *Co* I 3.35 (40)
5487.07	1	1	*Fe* I 7.14 (1)
5487.76	3	3	*Fe* I 7.75 (8)
5488.90	1	3	*Cr* II 8.97 (Pr)
5490.01	0n	2	*Ti* I 0.15 (12), *Fe* I 9.87 (Pr)
5490.72	1	3	*Ti* II 0.65 (Pr), (*Sc* II 1.00 (Pr))
5491.91	0	1	*Fe* I 1.84 (2)
5493.00	1n	2	*Ti* II 2.82 (Pr), (*Fe* I 3.36 (Pr))
5494.48	0	1	*Fe* I 4.46 (1)
5497.52	4	3	*Fe* I 7.52 (15)
5500.89	0	1	(*Cr* II 0.61 (Pr))

TABLE 1—*Continued*

λ	Int.	N	Identification
5501.47. .	3	3	*Fe* I 1.47 (12)
5502.14. .	1	3	*Cr* II 2.05 (12)
5503.15. .	2	3	*Cr* II 3.18 (8), (*Fe* II 3.40 (1))
5504.30. .	0	1	*Mn* I 4.21 (2)
5505.80. .	1	3	*Fe* I 5.89 (−)
5506.81. .	3	3	*Fe* I 6.78 (18)
5507.68. .	1	1	*V* I 7.75 (8)
5508.39. .	1n	1	. . . .
5508.59. .	1	2	*Cr* II 8.60 (8)
5509.93. .	1	3	*Y* II 9.91 (30), *Ni* I 0.00 (4)
5510.72. .	1	3	*Cr* II 0.68 (7)
5512.29. .	1	3	*Fe* I 2.28 (1), *Fe* I 2.41 (Pr)
5513.01. .	2	3	*Ca* I 2.98 (20)
5514.40. .	1	3	*Ti* I 4.35 (20), *Ti* I 4.54 (25)
5519.56. .	1	2	*Fe* II 9.72 (Pr)
5520.90. .	0	1	*Fe* I 1.14 (1), *Sc* I 0.50 (100)
5522.43. .	1	3	*Fe* I 2.46 (2)
5523.30. .	0	1	*Co* I 3.31 (8)
5524.01. .	0	1	*Fe* I 4.27 (1)
5524.92. .	1	1	*Co* I 4.99 (4)
5525.53. .	1n	3	*Fe* I 5.55 (3)
5526.82. .	3	3	*Sc* II 6.81 (75)
5527.76. .	0	1	(*V* I 7.54 (250))
5528.45. .	5	3	*Mg* I 8.41 (10)
5532.14. .	0	1	*Fe* I 1.95 (1)
5532.84. .	0	3	*Fe* I 2.74 (1), (*Mo* I 3.01 (30))
5534.87. .	3	3	*Fe* II 4.86 (4)
5535.53. .	2	3	*Fe* I 5.42 (2), (*Ba* I 5.48 (1000))
5536.34. .	0	2	*Fe* I 6.60 (Pr)
5537.23. .	0	1	(*Ni* I 7.11 (1))
5538.53. .	1	2	*Fe* I 8.54 (1)
5539.33. .	0	1	*Fe* I 9.27 (1)
5543.18. .	1	3	*Fe* I 3.18 (2)
5544.00. .	1	3	*Fe* I 3.93 (2)
5545.16. .	0	2	*Fe* II 5.26 (Pr)
5546.41. .	1n	3	*Fe* I 6.49 (1)
5549.86. .	0	1	*Fe* I 9.94 (2)
5553.56. .	1	3	*Fe* I 3.59 (1)
5554.91. .	2	3	*Fe* I 4.90 (4)
5556.06. .	0	1	*Cr* I 6.19 (10)
5557.96. .	1	3	*Fe* I 7.96 (1), *Fe* I 7.92 (Pr)
5560.14. .	1	3	*Fe* I 0.23 (1)
5561.46. .	0	3	. . . .
5562.69. .	2	3	*Fe* I 2.71 (2)
5563.60. .	2	3	*Fe* I 3.60 (3)
5565.69. .	3	3	*Fe* I 5.71 (4)
5567.32. .	1n	3	*Fe* I 7.40 (2)
5569.60. .	3	3	*Fe* I 9.62 (20)
5570.63. .	1	1	(*Mo* I 0.46 (25))
5572.87. .	4	3	*Fe* I 2.85 (30)
5574.36. .	1	2	*Cr* I 4.41 (12)
5576.09. .	2	3	*Fe* I 6.10 (10)
5577.00. .	0	2	(*Fe* I 7.04 (Pr))
5578.70. .	1	3	*Ni* I 8.73 (5)
5581.98. .	2	3	*Ca* I 1.97 (25)
5584.75. .	0	3	*Fe* I 4.77 (1)
5586.76. .	4	3	*Fe* I 6.76 (40)
5587.73. .	1	3	*Fe* I 7.58 (1), *Ni* I 7.86 (5)
5588.77. .	4	3	*Ca* I 8.76 (80)
5590.10. .	3	3	*Ca* I 0.12 (20)
5591.23. .	0	1	*Fe* II 1.38 (Pr)
5592.05. .	1n	1	*Ni* I 2.15 (1)
5592.26. .	1	2	*Ni* I 2.28 (8)
5593.75. .	0	3	*Ni* I 3.74 (4)
5594.53. .	5	3	*Ca* I 4.47 (60), *Fe* I 4.67 (2)
5595.83. .	1	1	. . . .
5596.89. .	0	3	. . . .
5598.45. .	5	3	*Ca* I 8.49 (50), *Fe* I 8.30 (4)
5600.16. .	1	3	*Fe* I 0.24 (1), *Ni* I 0.04 (4)
5601.32. .	3	3	*Ca* I 1.28 (30)
5601.94. .	1	1	. . . .
5602.91. .	4	3	*Fe* I 2.96 (10), *Ca* I 2.85 (25), *Fe* I 2.77 (2)
5604.75. .	0	1	*V* I 4.94 (8)
5608.62. .	0	1	(*Fe* I 8.98 (Pr))
5614.86. .	1	1	*Ni* I 4.79 (5)
5615.63. .	5	3	*Fe* I 5.65 (50)
5617.22. .	1	3	*Fe* I 7.22 (1), *Fe* I 7.15 (Pr)
5618.67. .	1	3	*Fe* I 8.63 (1)
5619.65. .	0	2	*Fe* I 9.60 (1)
5620.51. .	1	3	*Fe* I 0.53 (1), *Cr* II 0.63 (12), *Nd* II 0.62 (500)
5622.34. .	0	1	(*Si* I 2.23 (3))
5623.22. .	1	1	. . . .
5624.06. .	1	2	*Fe* I 4.06 (1)
5624.57. .	3	3	*Fe* I 4.55 (10), *V* I 4.60 (20)
5625.46. .	1	3	*Ni* I 5.33 (4)
5627.49. .	0	3	*Fe* II 7.49 (Pr), *V* I 7.63 (30)
5628.57. .	0	3	*Cr* I 8.64 (50)
5631.78. .	1	1	*Fe* I 1.72 (2)
5633.90. .	2	3	*Fe* I 3.97 (2)
5634.87. .	1	1	. . . .
5635.90. .	1	3	*Fe* I 5.84 (1), (*Fe* I 6.00 (Pr))
5637.35. .	1	3	. . . .
5638.27. .	2	3	*Fe* I 8.27 (3)
5640.06. .	0n	2	(*Fe* I 0.46 (1))
5641.07. .	1n	3	*Sc* II 0.97 (15), *Ni* I 1.11 (1)
5641.51. .	1b	2	*Fe* I 1.45 (2)
5642.89. .	1	1	*Fe* I 2.76 (Pr)
5643.14. .	0	1	*Ni* I 3.10 (2)
5644.02. .	1	3	*Ti* I 4.14 (18), *Fe* I 3.94 (Pr)
5645.61. .	1	3	*Si* I 5.66 (25)
5647.20. .	0	1	*Co* I 7.23 (12)
5647.82. .	0	2	. . . .
5649.64. .	0	3	*Fe* I 9.66 (1), *Ni* I 9.70 (2)
5650.61. .	1	2	*Fe* I 0.72 (1)
5652.27. .	0n	2	*Fe* I 2.32 (1)
5653.88. .	1n	2	*Fe* I 3.89 (1)
5654.04. .	1	1	. . . .
5654.91. .	1	1	. . . .
5655.37. .	2n	3	*Fe* I 5.51 (4), *Fe* I 5.18 (2)
5656.48. .	1n	1	. . . .
5656.94. .	0	2	. . . .
5657.91. .	2	3	*Sc* II 7.87 (25), *Fe* II 7.92 (Pr)
5658.73. .	4	3	*Fe* I 8.83 (10), *Fe* I 8.54 (1)
5659.74. .	0n	2	. . . .
5660.74. .	1n	2	*Fe* I 0.79 (1)
5660.94. .	1nn	1	*Fe* I 1.03 (Pr)

TABLE 1—*Continued*

λ	Int.	N	Identification
5661.58	1	2	*Fe* I 1.36 (1)
5662.50	2b	3	*Fe* I 2.52 (6)
5662.90	1b	3	*V* II 2.95 (200), *Fe* I 2.94 (1), *Ti* I 2.89 (4)
5664.06	1	3	*Cr* I 4.04 (40), *Ni* I 4.02 (3)
5665.56	1n	3	*Si* I 5.60 (25)
5667.25	2n	3	*Sc* II 7.16 (10)
5668.46	0	1	*V* I 8.37 (12)
5669.05	2	3	*Sc* II 9.03 (12)
5669.80	0	3	*Na* I 9.80 (3), *Ni* I 9.94 (3)
5671.70	0	1	*Sc* I 1.80 (200), (*Cr* II 1.62 (Pr))
5675.46	2	3	*Ti* I 5.41 (9), (*Na* I 5.34 (3))
5677.67	0n	1	*Fe* I 7.70 (Pr)
5678.35	1	1	*Cr* II 8.42 (10), *Fe* I 8.41 (Pr), (*Fe* I 8.62 (Pr))
5679.04	2	2	*Fe* I 9.02 (2)
5679.46	1	1	....
5679.78	1	1	(*Fe* I 0.26 (1)), (*Ti* I 9.91 (2))
5680.98	0	1	*Cr* I 1.20 (60)
5682.61	3	3	*Ni* I 2.63 (8)
5684.39	3	3	*Si* I 4.52 (50), *Sc* II 4.19 (15)
5686.48	3	3	*Fe* I 6.53 (3)
5687.17	0	1	*Sc* I 6.83 (150)
5688.25	3	3	*Na* I 8.20 (10)
5689.40	0	1	*Ti* I 9.46 (10)
5690.40	1	3	*Si* I 0.47 (40)
5691.55	1	3	*Fe* I 1.51 (1), *Fe* II 1.38 (Pr), *Fe* I 1.71 (Pr), (*Ni* I 1.52 (1))
5693.61	1	3	....
5694.97	1	3	*Ni* I 5.00 (6)
5696.49	0n	2	(*Fe* I 6.11 (Pr)), (*Fe* II 6.11 (Pr)), (*S* I 6.63 (2))
5698.35	1	2	*Fe* I 8.37 (2), *Cr* I 8.33 (100), *V* I 8.51 (60)
5700.31	1	3	*Cr* I 0.51 (40), *Sc* I 0.23 (100), *S* I 0.24 (4), *Cu* I 0.24 (30)
5701.17	1	1	*Si* I 1.14 (25)
5701.57	2	3	*Fe* I 1.55 (7)
5706.06	2	3	*Fe* I 5.99 (2), *S* I 6.11 (6), *Fe* I 6.12 (Pr)
5707.06	1	3	*Fe* I 7.06 (1), *V* I 6.97 (30), (*Fe* I 7.25 (Pr))
5708.40	2	3	*Si* I 8.44 (75)
5709.50	3	3	*Fe* I 9.38 (10), *Ni* I 9.56 (12)
5710.30	0	2	*Fe* I 9.93 (Pr)
5711.15	2	3	*Mg* I 1.09 (6)
5712.07	3	3	*Fe* I 2.14 (2), *Fe* I 1.87 (2)
5712.75	0	1	*Cr* I 2.78 (100)
5713.97	0n	2	*Ti* I 3.90 (3)
5714.48	1	1	....
5715.21	2	3	*Fe* I 5.11 (1), *Ni* I 5.09 (6), *Ti* I 5.12 (9)
5717.86	1	3	*Fe* I 7.84 (3)
5722.00	0	1	*Fe* I 1.72 (Pr)
5731.89	1	3	*Fe* I 1.77 (3)
5732.80	0	1	*Fe* I 2.89 (Pr), *Fe* II 2.72 (Pr)
5741.93	0	3	*Fe* I 1.86 (2)
5743.30	0	1	*Fe* I 2.97 (1), (*Ca* I 3.28 (3))
5747.85	1	3	*Fe* I 7.96 (1), *Fe* I 7.87 (Pr), (*Fe* II 7.88 (Pr))
5749.90	0	1	*Fe* I 9.64 (Pr)
5752.09	1	3	*Fe* I 2.04 (2)
5753.16	2	3	*Fe* I 3.14 (5), (*Fe* I 3.40 (Pr))
5753.83	1	3	*Cr* I 3.69 (25), *Fe* I 3.98 (Pr)
5754.79	1	3	*Ni* I 4.68 (10), *Fe* I 4.92 (Pr)
5757.08	1n	2	....
5759.08	0	1	*Fe* I 9.27 (1)
5760.84	1n	2	*Ni* I 0.85 (4)
5763.12	3	3	*Fe* I 2.99 (10)
5766.43	0	1	*Ti* I 6.33 (4)
5769.52	0	1	*Fe* I 9.34 (1)
5772.37	1	3	*Si* I 2.26 (50), *V* I 2.40 (6)
5774.20	0	1	*Ti* I 4.04 (5)
5775.21	1	3	*Fe* I 5.09 (5)
5778.29	1	1	....
5778.62	0n	2	*Fe* I 8.47 (1), *Fe* I 8.81 (Pr)
5780.70	1	3	*Fe* I 0.62 (2), *Fe* I 0.83 (1)
5782.25	1	2	*Cu* I 2.13 (200)
5784.11	0	3	*Cr* I 3.93 (50), (*Ba* II 4.18 (8))
5785.53	1nn	3	*Fe* II 5.0 (5), *Cr* I 5.82 (50)
5786.90	0	1	(*Cr* I 7.04 (20)), (*Fe* I 7.02 (Pr))
5787.92	1	1	*Cr* I 7.99 (75)
5788.24	1n	2	....
5789.75	0	2	....
5791.01	2	3	*Fe* I 1.04 (2), *Cr* I 1.00 (100)
5792.47	1	1	....
5793.29	1	3	*Si* I 3.13 (30), *C* I 3.51 (3), *Fe* II 3.16 (Pr)
5794.33	0	2	*Fe* I 3.93 (2)
5798.08	1	3	*Fe* I 8.19 (2), *Si* I 7.91 (40), *Cr* I 8.46 (25), *Fe* II 7.81 (Pr)
5799.82	0	1	*Fe* II 0.02 (Pr)
5804.43	1	1	*Fe* I 4.48 (1)
5805.23	0	2	*Ni* I 5.23 (5), *Fe* II 4.91 (Pr)
5805.60	0	1	*Fe* I 5.77 (1)
5806.77	1	2	*Fe* I 6.73 (2)
5807.14	1	1	*V* I 7.14 (3)
5808.17	1	1	....
5809.24	1	2	*Fe* I 9.24 (2)
5809.55	1	1	....
5811.74	0	1	*Fe* I 1.94 (1), *Fe* II 1.93 (Pr)
5813.74	0	1	*Fe* II 3.67 (3)
5814.91	0	1	*Fe* I 4.82 (1), *Fe* I 5.16 (1)
5816.47	1	3	*Fe* I 6.36 (3)
5817.76	0	2	*V* I 7.53 (5)
5823.30	0	1	*Fe* II 3.17 (3)
5833.95	0	1	*Fe* II 4.06 (Pr)
5835.46	1	1	*Fe* II 5.61 (3), *Fe* I 5.59 (Pr), *Fe* II 5.43 (Pr), *Fe* II 5.50 (Pr), (*Fe* I 5.43 (Pr))
5837.72	0	1	*Fe* I 7.70 (1)
5843.95	0	1	(*Ti* II 3.77 (Pr))
5846.42	0	1	*V* I 6.31 (8)

TABLE 1—*Continued*

λ	Int.	N	Identification
5848.12	0	3	*Fe* I 8.09 (2)
5850.20	1	1	....
5852.34	1	3	*Fe* I 2.19 (2)
5853.77	2	3	*Ba* II 3.68 (200), *Cr* I 4.27 (75)
5855.03	1	2	*Fe* I 5.13 (1)
5856.06	1	1	*Fe* I 6.08 (2)
5857.61	3	3	*Ca* I 7.45 (100), *Ni* I 7.76 (9)
5858.54	0	1	*Fe* I 8.28 (Pr)
5859.70	2	3	*Fe* I 9.61 (5)
5860.50	0	2	(*Ti* II 0.92 (Pr))
5862.43	2	3	*Fe* I 2.36 (8)
5863.78	0	1	*La* II 3.70 (80)
5864.07	0	1	*Fe* I 4.25 (1)
5866.59	0	1	*Ti* I 6.45 (35)
5867.35	0	1	*Ca* I 7.57 (1), (*Fe* I 7.01 (Pr))
5873.11	0n	1	*Fe* I 3.22 (2), *Eu* II 2.98 (500)
5877.71	0	1	*Fe* I 7.77 (1)
5883.80	1	3	*Fe* I 3.84 (4), Atm. 3.91 (5)
5885.92	1nn	2	*Fe* II 5.73 (6), Atm. 5.98 (5)
5887.58	0nn	1	Atm. 7.66 (3), Atm. 7.23 (5), *Fe* I 7.48 (Pr)
5889.99	10	3	*Na* I 9.95 (10)
5891.28	1	3	*Fe* II 1.36 (8), *Fe* I 1.12 (1), *Fe* I 1.19 (Pr)
5892.43	1	1	Atm. 2.40 (3)
5892.86	1n	2	*Ni* I 2.88 (12), *Fe* I 2.71 (2)
5893.00	1	1	....
5895.99	4	3	*Na* I 5.92 (9)
5897.57	1n	1	*V* II 7.54 (50)
5898.21	1n	2	Atm. 8.17 (4), *Fe* I 8.21 (1)
5899.50	1n	1	*Ti* I 9.30 (25)
5900.10	2	2	Atm. 0.05 (4)
5901.54	1	2	Atm. 1.47 (6)
5902.74	1	1	*Fe* I 2.53 (1)
5903.42	1	1	*Ti* I 3.32 (5)
5904.01	1	1	....
5904.93	1	2	....
5905.71	1	3	*Fe* I 5.67 (3)
5908.30	0	1	*Fe* I 8.25 (2)
5909.15	0	1	Atm. 9.00 (3)
5909.70	0n	1	*Fe* I 9.99 (3), *Fe* II 9.38 (Pr)
5910.18	1n	2	(*Ti* II 0.07 (Pr))
5912.55	1	1	....
5914.21	3	3	*Fe* I 4.16 (8)
5915.50	1	1	*Co* I 5.55 (10)
5916.29	1	3	*Fe* I 6.25 (3), (*V* II 6.36 (15))
5918.30	1n	1	*Ti* I 8.55 (10)
5919.16	1	1	Atm. 9.06 (5)
5919.55	1n	1	Atm. 9.65 (7)
5920.56	1	2	*Fe* I 0.52 (2)
5922.25	1	1	*Ti* I 2.11 (18)
5922.45	0	2	....
5923.62	1n	2	....
5927.96	2	2	*Fe* I 7.80 (2)
5929.54	0	1	*Fe* I 9.70 (1)
5930.17	2	3	*Fe* I 0.17 (8)
5932.21	1	2	Atm. 2.10 (5), (*Fe* I 1.90 (Pr)), (*Fe* II 2.05 (Pr))
5932.79	0	1	Atm. 2.79 (2)
5934.67	1	2	*Fe* I 4.66 (5)
5934.82	1	1	....
5937.80	0	1	*Ti* I 7.81 (6)
5941.12	1n	2	Atm. 1.08 (5), *Fe* I 0.97 (2), (*Fe* II 1.36 (Pr))
5942.50	1	2	Atm. 2.58 (3)
5944.10	0n	2	(*Fe* I 3.60 (Pr))
5945.90	1n	2	Atm. 6.01 (3)
5946.58	1	1	(*Co* I 6.48 (5))
5946.98	1	1	Atm. 7.07 (2)
5948.59	2	3	*Si* I 8.58 (100)
5949.51	1	3	*Fe* I 9.35 (2)
5951.26	0n	2	....
5952.69	1	3	*Fe* I 2.75 (3), *Fe* II 2.55 (Pr)
5954.60	1n	2	....
5956.16	1n	2	*Fe* II 6.5 (4), *Fe* I 6.70 (3), (*Fe* I 5.68 (1))
5957.43	1n	1	*Si* II 7.61 (5)
5958.16	1n	1	*Fe* I 8.25 (2), *Fe* I 8.35 (Pr)
5965.95	1	1	*Ti* I 5.83 (30), *Eu* II 6.07 (1200)
5967.64	1	1	....
5973.75	0	1	(*La* II 3.52 (120))
5974.63	1	1	(*Fe* I 4.60 (Pr))
5975.36	1	3	*Fe* I 5.36 (4)
5976.72	2	3	*Fe* I 6.80 (5)
5978.62	0	3	*Ti* I 8.54 (25); *Si* II 8.97 (7)
5981.44	0	1	(*Fe* I 1.40 (Pr))
5983.65	2	3	*Fe* I 3.70 (6)
5984.84	2	3	*Fe* I 4.80 (8)
5987.08	1	3	*Fe* I 7.06 (6)
5988.46	0	3	(*Ti* I 8.56 (2))
5989.89	1	1	....
5991.33	1	3	*Fe* II 1.38 (10)
5996.76	0	1	*Ni* I 6.74 (3)
5997.75	1	3	*Fe* I 7.80 (1), *Ni* I 7.61 (2)
5999.46	1	1	*Ti* I 9.67 (8)
6001.15	0	2	....
6003.08	2	3	*Fe* I 3.03 (8)
6004.48	1	1	....
6006.13	0	3	(*Co* I 6.36 (5))
6007.97	1	3	*Fe* I 7.96 (3)
6008.56	2	3	*Fe* I 8.58 (9)
6013.44	1	3	*Mn* I 3.50 (30)
6015.02	1	2	(*Fe* I 5.26 (Pr))
6016.66	2	3	*Mn* I 6.64 (40), *Fe* I 6.66 (2)
6018.32	0	1	*Fe* I 8.31 (Pr)
6020.19	3	3	*Fe* I 0.17 (10)
6021.87	1	3	*Mn* I 1.80 (50), *Fe* I 1.82 (2)
6023.22	1	1	....
6024.11	3	3	*Fe* I 4.07 (15)
6025.54	0	2	....
6027.08	1	3	*Fe* I 7.06 (4)
6029.16	1n	1	*Cr* I 9.28 (18)
6032.23	0n	1	(*Fe* I 2.67 (1))
6042.08	1	3	*Fe* I 2.09 (2), (*S* I 1.93 (3))
6043.38	0	1	*Ce* II 3.39 (60)
6045.74	1n	1	*Fe* II 5.50 (6)
6046.07	1n	2	*S* I 6.04 (5)
6052.75	1	2	*S* I 2.66 (10), (*Mn* II 2.89 (0))
6056.10	2	3	*Fe* I 5.99 (4)
6061.05	0	1	*Fe* II 1.04 (3)

TABLE 1—*Continued*

λ	Int.	N	Identification
6062.80..	0	1	*Fe* I 2.89 (1), *Cr* I 2.75 (50)
6065.53..	3	3	*Fe* I 5.49 (15)
6067.47..	1n	1	....
6078.58..	2n	3	*Fe* I 8.50 (4)
6081.15..	1	1	(*V* I 1.42 (25))
6082.82..	0	1	*Fe* I 2.71 (1)
6084.13..	1	3	*Fe* II 4.11 (5)
6086.39..	1	2	*Ni* I 6.29 (5)
6089.71..	1	1	*Cr* II 9.69 (15)
6091.85..	0	3	(*Fe* I 1.74 (Pr))
6093.61..	0	1	*Fe* I 3.66 (1)
6094.90..	0n	1	(*Fe* I 4.42 (1))
6096.72..	1	2	*Fe* I 6.69 (1)
6098.29..	0	1	*Fe* I 8.26 (Pr)
6098.98..	1n	1	(*Ti* I 8.66 (7))
6100.28..	1	1	(*Fe* I 0.28 (1))
6102.34..	2b	2	*Fe* I 2.18 (5)
6102.84..	3b	3	*Ca* I 2.72 (80)
6103.38..	1b	1	*Fe* I 3.19 (3), *Fe* II 3.54 (1)
6104.95..	0	3	*Fe* I 5.14 (Pr), (*Mn* II 5.38 (5))
6106.57..	1	3	(*Zr* II 6.47 (2)), (*Fe* I 6.86 (Pr))
6108.09..	1	3	*Ni* I 8.12 (8)
6111.08..	1	1	*Ni* I 1.06 (2)
6113.05..	1n	2	....
6113.48..	1	1	*Fe* II 3.33 (2)
6114.84..	0	1	*Zr* II 4.78 (2)
6116.13..	1	3	*Ni* I 6.18 (6), (*Fe* II 6.04 (Pr))
6118.24..	0n	1	(*Ni* I 8.06 (1))
6119.86..	0	1	*Ni* I 9.78 (2)
6120.92..	0	1	(*Ti* I 1.01 (3))
6122.25..	4	3	*Ca* I 2.22 (100)
6125.08..	1	3	*Si* I 5.03 (4)
6126.30..	0	2	*Ti* I 6.22 (20), (*Mn* II 6.21 (10))
6127.92..	1	3	*Fe* I 7.91 (2)
6130.16..	0	1	*Fe* I 0.36 (1), *Ni* I 0.17 (3)
6131.73..	1	2	*Si* I 1.86 (5), *Si* I 1.54 (4)
6136.73..	3	3	*Fe* I 6.62 (20), *Fe* I 7.00 (2)
6137.81..	3	3	*Fe* I 7.70 (18)
6138.56..	1	1	....
6139.64..	0	2	*Fe* I 9.66 (Pr)
6141.78..	4	3	*Ba* II 1.72 (600), *Fe* I 1.73 (4)
6142.71..	0	3	*Si* I 2.53 (5)
6144.97..	1n	3	*Si* I 5.08 (10)
6146.60..	0	2	....
6147.83..	2	3	*Fe* II 7.74 (2), *Fe* I 7.85 (–)
6149.27..	2	3	*Fe* II 9.24 (2)
6150.21..	0	2	(*Fe* II 0.10 (Pr))
6151.68..	1	2	*Fe* I 1.62 (2)
6155.16..	1	3	*Si* I 5.22 (20), *Fe* II 5.24 (Pr)
6157.85..	1	3	*Fe* I 7.73 (4)
6160.81..	1	3	*Na* I 0.75 (8), (*Fe* II 0.75 (Pr))
6161.30..	0	2	*Ca* I 1.29 (10), (*Ti* II 1.54 (Pr))
6162.21..	4	3	*Ca* I 2.17 (150)
6163.65..	1	3	*Ca* I 3.76 (10), *Fe* I 3.54 (1)
6165.19..	1	3	*Fe* I 5.37 (2)
6166.46..	1	3	*Ca* I 6.44 (15)
6169.32..	3n	3	*Ca* I 9.56 (40), *Ca* I 9.06 (25)
6170.57..	1	3	*Fe* I 0.49 (4), *Ni* I 0.57 (3)
6172.08..	0	2	....
6173.37..	1n	3	*Fe* I 3.34 (3)
6175.34..	1	3	*Ni* I 5.42 (8), *Fe* II 5.16 (1)
6176.77..	1	3	*Ni* I 6.81 (12)
6180.18..	0	3	*Fe* I 0.21 (2)
6187.98..	0	2	*Fe* I 8.04 (2)
6191.48..	4	3	*Fe* I 1.56 (20), *Ni* I 1.19 (12)
6200.23..	1	3	*Fe* I 0.32 (4)
6204.68..	0n	1	*Ni* I 4.64 (2)
6213.40..	2	3	*Fe* I 3.44 (5)
6215.20..	1	3	*Ti* I 5.21 (20), *Fe* I 5.15 (2)
6216.51..	1	2	*V* I 6.37 (30)
6219.28..	2	3	*Fe* I 9.29 (6)
6220.55..	1	1	*Ti* I 0.46 (12), *Fe* I 0.77 (1)
6221.30..	0	1	*Fe* I 1.40 (1), *Ti* I 1.41 (8)
6224.05..	0	1	*Ni* I 3.99 (3)
6229.20..	0n	2	*Fe* I 9.23 (1)
6230.77..	3	3	*Fe* I 0.73 (25)
6232.66..	1	3	*Fe* I 2.66 (5)
6233.78..	0	1	*Fe* II 3.52 (3)
6237.35..	1	3	*Si* I 7.34 (5)
6238.40..	2	3	*Fe* II 8.38 (1)
6240.14..	0n	3	*Fe* II 9.95 (2), *Fe* I 0.27 (1)
6240.94..	0	1	*Fe* I 0.66 (2)
6241.93..	0	1	(*Ti* II 1.80 (Pr))
6243.74..	1	1	*Si* I 3.86 (10)
6244.30..	1nn	1	*Si* I 4.56 (10)
6245.63..	0	1	*Sc* II 5.63 (20)
6246.18..	1	1	....
6246.43..	2	2	*Fe* I 6.33 (15)
6247.59..	3	3	*Fe* II 7.56 (80)
6249.17..	1n	2	....
6250.89..	0	2	....
6251.04..	1	1	(*Fe* I 1.29 (Pr))
6252.61..	3	3	*Fe* I 2.56 (20)
6254.22..	3	3	*Fe* I 4.26 (6), *Si* I 4.25 (25)
6255.28..	1	1	....
6256.39..	1	3	*Fe* I 6.37 (4), *Ni* I 6.36 (15)
6258.39..	0	1	*Ti* I 8.10 (40)
6258.68..	1n	2	*Ti* I 8.71 (50), *Ni* I 8.59 (2)
6260.90..	0	1	*Ti* I 1.10 (35)
6265.15..	1	3	*Fe* I 5.14 (6)
6267.48..	0	1	*Fe* I 7.84 (1)
6270.12..	1	2	*Fe* I 0.24 (2)
6271.53..	0	1	(*Fe* I 1.29 (1)), (*Cr* II 1.83 (5))
6273.12..	0	1	(*Co* I 3.03 (70))
6276.47..	1	1	....
6276.96..	1nn	1	....
6277.82..	2nn	2	Atm. 8.10 (4)
6279.15..	1	1	Atm. 9.11 (3)
6279.78..	1	2	*Sc* II 9.76 (15)
6280.72..	1	1	*Fe* I 0.62 (2)
6282.68..	1nn	1	*Co* I 2.64 (40)
6284.11..	0	1	(*Fe* I 4.01 (Pr))
6285.62..	1	1	....
6287.44..	1	1	....
6289.28..	1	1	....

TABLE 1—*Continued*

λ	Int.	N	Identification
6291.01	1	3	*Fe* I 0.97 (3)
6292.96	1	3	Atm. 2.97 (3)
6295.20	1	1	Atm. 5.19 (3)
6295.93	1	1	*La* II 6.08 (300)
6298.01	1	1	*Fe* I 7.80 (5)
6299.06	1	1	....
6299.50	1	2	....
6301.54	3	3	*Fe* I 1.52 (15)
6302.56	2	3	*Fe* I 2.51 (6)
6304.04	1	2	(*Ti* I 3.75 (10))
6305.31	1n	1	*Fe* II 5.32 (1)
6309.98	1nn	2	*Sc* II 9.90 (15), (*Fe* I 0.54 (1))
6314.63	1	2	*Ni* I 4.68 (15)
6314.99	1nn	1	*Fe* I 5.32 (3)
6315.81	0	1	*Fe* I 5.81 (2)
6318.08	2	3	*Fe* I 8.02 (10)
6319.57	1	2	....
6320.89	1	2	*Sc* II 0.85 (7)
6322.69	1	3	*Fe* I 2.69 (5)
6324.40	0	2	....
6335.38	2	3	*Fe* I 5.34 (10)
6336.88	3	3	*Fe* I 6.84 (12)
6339.00	1	3	*Fe* I 8.90 (1), *Ni* I 9.15 (7), *V* I 9.09 (5)
6340.60	1	2	....
6344.13	1	3	*Fe* I 4.15 (2)
6347.10	3	3	*Si* II 7.09 (10)
6349.26	0	1	(*V* I 9.48 (5))
6353.12	0	2	....
6355.13	1	3	*Fe* I 5.04 (4)
6357.08	0	1	*V* I 7.30 (4)
6358.72	1	3	*Fe* I 8.69 (3)
6360.62	0	2	*Ni* I 0.80 (5)
6362.44	0	2	*Zn* I 2.35 (100)
6364.20	0	2	*Fe* I 4.38 (1)
6364.70	0	1	*Fe* I 4.72 (1)
6366.54	1	2	*Ni* I 6.48 (4), *Ti* I 6.35 (8)
6368.85	1n	1	....
6369.42	1	1	*Fe* II 9.45 (4)
6369.75	1	1	....
6371.41	2	3	*Si* II 1.36 (8)
6375.23	0n	1	(*Ni* I 5.32 (1))
6380.23	0	1	*V* II 0.11 (40)
6380.96	1n	2	*Fe* I 0.75 (3)
6385.11	0n	1	*Fe* II 5.47 (5), *Nd* II 5.20 (150)
6393.61	3	3	*Fe* I 3.60 (400)
6400.03	4	3	*Fe* I 0.01 (800), *Fe* I 0.32 (50)
6402.26	0	1	(*Fe* I 2.43 (1))
6407.06	1b	1	*Fe* II 7.30 (1), (*Fe* I 6.42 (1))
6408.01	2n	3	*Fe* I 8.03 (60)
6409.81	1	1	....
6411.69	3	3	*Fe* I 1.66 (400)
6413.10	0	2	....
6414.96	1	3	(*Ni* I 4.60 (5))
6416.92	1	3	*Fe* II 6.90 (1)
6418.72	0	1	*Cr* II 8.87 (7)
6419.98	1	3	*Fe* I 9.98 (30)
6421.41	2	3	*Fe* I 1.36 (200)
6430.84	2	3	*Fe* I 0.85 (300)

λ	Int.	N	Identification
6432.58	1	3	*Fe* II 2.65 (8)
6434.00	0	2	*Fe* II 3.85 (3)
6434.58	0	1	(*Y* I 5.00 (500))
6436.81	0n	1	*Fe* I 6.43 (1)
6439.10	4	3	*Ca* I 9.07 (150)
6440.79	0	1	*Mn* I 0.97 (8)
6443.08	0	1	*Fe* II 2.97 (6)
6446.78	0n	1	*Fe* II 6.43 (0), (*Mn* II 6.28 (15)), (*La* II 6.62 (200))
6449.84	2	3	*Ca* I 9.81 (50), *Co* I 0.23 (80)
6451.87	0	1	*Fe* I 1.59 (2)
6452.17	1	1	*V* I 2.35 (10)
6456.47	3n	3	*Fe* II 6.38 (3)
6462.62	3	3	*Ca* I 2.57 (125), *Fe* I 2.73 (30)
6464.68	0	2	(*Ti* II 4.66 (Pr))
6466.54	1n	1	....
6469.15	1n	3	*Fe* I 9.21 (15)
6471.72	1	3	*Ca* I 1.66 (40), (*Sc* II 1.55 (Pr))
6472.68	0	1	(*Sm* II 2.34 (300))
6475.17	1	3	*Fe* I 5.63 (12), *Fe* I 4.61 (1)
6479.37	1nn	1	....
6481.78	0	2	*Fe* I 1.88 (20)
6482.79	1nn	1	*Ni* I 2.81 (5)
6490.37	1	1	....
6491.24	1n	2	*Fe* II 1.28 (4)
6491.65	0	1	*Mn* I 1.71 (15), *Ti* II 1.61 (2)
6493.81	2	3	*Ca* I 3.78 (80)
6495.08	3	3	*Fe* I 4.98 (1000)
6495.79	0	1	*Fe* I 5.78 (3)
6496.84	3	3	*Ba* II 6.90 (600)
6499.52	1n	3	*Ca* I 9.65 (30), *Fe* I 8.95 (5)
6501.77	0	3	*Fe* I 1.68 (4)
6502.79	1	1	....
6507.08	1nn	1	....
6508.48	0n	1	(*Ca* I 8.74 (1))
6514.28	1	1	....
6516.15	3	3	*Fe* II 6.05 (20), (*Sc* II 6.17 (Pr))
6518.28	1	1	*Fe* I 8.38 (20)
6518.70	1nn	2	....
6526.49	0n	1	(*Mn* I 6.54 (2))
6526.87	1nn	1	(*La* II 6.99 (200))
6527.28	0	1	*Si* I 7.20 (Pr), *Si* I 7.49 (3)
6531.98	1	1	....
6533.76	0n	1	*Fe* I 3.97 (8), Atm. 3.95 (2)
6544.04	1	1	Atm. 3.91 (2)
6546.19	2	3	*Fe* I 6.24 (200), *Ti* I 6.28 (20)
6551.94	1	1	....
6555.69	0	1	(*Fe* I 5.86 (Pr))
6562.84	40	3	$H\alpha$ 2.82
6569.29	1	3	*Fe* I 9.23 (50), *Sm* II 9.31 (1000)
6571.63	1	1	(*Fe* II 1.41 (tr))
6574.55	1	1	*Fe* I 4.24 (3)
6575.17	0	1	*Fe* I 5.02 (30)
6580.86	0	1	(*Cr* I 0.96 (8))
6586.42	1	1	*Ni* I 6.33 (6)
6587.70	1	3	*C* I 7.75 (4)

TABLE 1—*Continued*

λ	Int.	N	Identification	λ	Int.	N	Identification
6582.95..	2	3	*Fe* I 2.92 (300)	6678.00..	2	2	*Fe* I 7.99 (600)
6593.83..	1	3	*Fe* I 3.88 (60)	6717.62..	2	2	*Ca* I 7.68 (500), *Fe* I 7.56 (3)
6597.45..	0	2	*Fe* I 7.61 (15), *Cr* I 7.56 (40)				
6604.54..	1n	2	*Sc* II 4.60 (10), *Fe* I 4.67 (1)	6722.04..	1	1	*Si* I 1.97 (4)
6609.04..	1	1	*Fe* I 9.12 (30)	6726.31..	1	1	*Fe* I 6.67 (20)
6609.80..	1	1	(*Fe* I 9.56 (1)), (*Fe* I 9.69 (Pr))	6748.68..	1	1	*S* I 8.79 (8)
				6750.15..	1	1	*Fe* I 0.15 (100)
6615.26..	0	1	(*Fe* I 5.01 (Pr))	6752.28..	0	1	*Fe* I 2.72 (10)
6633.72..	1	2	*Fe* I 3.76 (50)	6756.76..	1n	1	....
6639.83..	1	1	*Fe* I 9.72 (4), *Fe* I 9.90 (2)	6757.23..	1	1	*S* I 7.16 (10)
6643.56..	2	2	*Ni* I 3.64 (20)	6767.80..	1	1	*Ni* I 7.78 (20)
6663.34..	2	2	*Fe* I 3.45 (80), *Fe* I 3.26 (1)				

TABLE 2

THE VARIATION OF INTENSITY WITH SPECTRAL TYPE FOR THE STRONGEST UNIDENTIFIED LINES IN THE SPECTRUM OF PROCYON

λ	INTENSITY*					NOTE
	α Sco cM1	α Boo gK0	⊙ Disk dG2	α CMi dF4	α Car cF0	
3943.11..............	—		2	3	—	1
4015.58..............	1		3	4n	0	2
4025.83..............	—		2	3	1	
4159.19..............	2s	6n	4	4	2	3
4184.00..............	2h	5	4	4	2	
4188.72..............	7	6	4	4	2	2, 4

* A blank space indicates that the line lies outside the range of the table consulted, whereas a dash (—) means the line is not recorded. Of course, the intensities are not strictly comparable.

Intensity sources:

α Sco: Dorothy N. Davis, *Ap. J.*, **89**, 41, 1939.
α Boo: Sidney Guy Hacker, *Contr. Princeton Obs.*, No. 16, 1935.
⊙ Disk: St. John, Moore, Ware, Adams, and Babcock, *Revised Rowland, Pub. Carnegie Inst. Washington*, No. 396, 1928.
α CMi: Table 1.
α Car: Jesse L. Greenstein, *Ap. J.*, **95**, 161, 1942.

NOTES TO TABLE 2

1. Masked in α Sco. Probably also in α Car.
2. Blended in α Sco.
3. The contribution of the unclassified line *Fe* I 4159.3(1) may play a very minor role.
4. The contribution of the line *Ti* I 4188.69(5) is rather negligible in α CMi and α Car.

is shown in Table 2. For the most part, these lines are unidentified throughout the sequence of spectral types in this table. Their presence in the spectrum of α Carinae does not favor the hypothesis that they are of molecular origin; moreover, they are all listed in the sun-spot spectrum as atomic lines.[12]

[12] Charlotte E. Moore, *Atomic Lines in the Sun-Spot Spectrum* (Princeton, 1933).

ATOMIC IDENTIFICATIONS

The atoms and ions identified in the spectrum of Procyon, listed according to atomic number, are the following: *H*, *C* I, *Na* I, *Mg* I, *Mg* II, *Al* I, *Si* I, *Si* II, *S* I, *Ca* I, *Ca* II, *Sc* I, *Sc* II, *Ti* I, *Ti* II, *V* I, *V* II, *Cr* I, *Cr* II, *Mn* I, *Mn* II, *Fe* I, *Fe* II, *Co* I, *Ni* I, *Ni* II, *Cu* I, *Zn* I, *Sr* I, *Sr* II, *Y* II, *Zr* II, *Mo* I, *Mo* II, *Ba* I, *Ba* II, *La* II, *Ce* II, *Pr* II, *Nd* II, *Sm* II, *Eu* II, *Gd* II, *Dy* II, *Yb* I. The element *Mo* II is most probably present, but the evidence rests principally with the weak line at λ 4364. The *Sr* I identification is based only on the line *Sr* I 4607. But this is an ultimate line, and there seems to be little question of the presence of this atom. In addition, *K* I, *V* I, *Ru* I, and *Ba* I entries appear in Table 1, but the atoms corresponding to these lines have not been identified with the same certainty as those listed above. The well-known lines *Ag* I 5465, *Cs* I 4593, *Tl* I 5350, and *Pb* I 4058 have been entered; but, for reasons of ionization and expected abundances, these "identifications" must be considered as doubtful.

Of the singly ionized rare earths identified, *Ce* II and *Eu* II are the most obviously present, their strongest lines reaching an intensity approaching 2. The main lines of *Pr* II, *Nd* II, and *Gd* II appear with intensities of about 1, and those of *Sm* II and *Dy* II are just faintly visible. The element *La*, which is sometimes considered as a rare earth, is well represented by *La* II lines, the strongest of which appear with an intensity of 3 in Procyon. The rare earths have low first and second ionization potentials. They are, therefore, to a great extent ionized in a source like the sun; in Procyon this must be true to an even greater extent. However, owing to the fact that the spectral energy of some of the neutral rare earths is concentrated in but a few strong lines, there is a possibility of detecting their presence. To wit, *Yb* I has been identified in the solar disk and is probably also present in Procyon owing to a rather close coincidence with *Yb* I 3988. A. S. King[13] has called attention to the faint presence of *Eu* I lines in the sun-spot spectrum. The apparent absence of these lines in the solar-disk spectrum seems to preclude their observability in Procyon. For this reason the entries *Eu* I 4594 and *Eu* I 4627 should be regarded with suspicion. A note by King[14] concerning the possible presence of neutral dysprosium in the sun is of interest in the present connection because many of the prominent *Dy* I lines listed by him coincide with faint Procyon lines. Unfortunately, the leading line is masked on our spectra; but the lines *Dy* I 4046, 4195, 4215, 4221, 4589, 4612, have been entered—in parentheses, however, since these coincidences must be taken only as a tentative identification of *Dy* I; indeed, if they are not chance coincidences, they should perhaps be taken rather as evidence against the presence of *Dy* I in the sun and Procyon, since the latter star is of earlier spectral type.

All the atoms and ions which have been identified by Dunham[15] in α Persei are included in the above list; however, the relative intensities in these stars indicates a somewhat lower level of ionization for the present star, obviously a result of its dwarf character.

Needless to say, the presence of some elements—e.g., *B* I, *As* I—or stages of ionization —e.g., *Co* II, *Yb* II—cannot be discussed, since the ultimate lines and sometimes all or all but the very faintest lines lie outside the astronomical region or even the wave-length limits of Table 1. No additional atoms or ions are identified in Roach's[16] table extending to λ 7593. Paul W. Merrill's[17] photographic reconnaissance of stellar spectra in the region λλ 7000–9000 has revealed that oxygen is represented in Procyon by *O* I 8446.

MOLECULAR IDENTIFICATIONS

From a study of Yerkes Observatory spectra having a linear dispersion of 30 A/mm at λ 4500, Swings and Struve[18] have placed the upper limit of visibility of the bands

[13] *Ap. J.*, **89**, 377, 1939.
[14] *Pub. A.S.P.*, **54**, 201, 1942.
[15] *Loc. cit.*
[16] *Ap. J.*, **80**, 233, 1934.
[17] *Ap. J.*, **79**, 192, 1934.
[18] *Phys. Rev.*, **39**, 142, 1932.

CN 4216, 4197, and *CH* 4315 at spectral type *F*8. For *CN* 3883, however, this limit must occur slightly earlier in the spectral sequence, since this band is substantially stronger than the bands in the sequence starting with *CN* 4216. Accordingly, in a type dF4 star such as the present one, having an effective temperature of about 6500° K, the molecular lines must be very weak or absent; only those molecules, such as *CN*, *CH*, *OH*, and *NH*, which show moderately strong lines in the spectrum of the solar spots as well as in that of the disk, are to be expected. We proceed with a discussion of *CN* and *CH*, both of which show lines of maximum intensity 3 in the solar disk and are clearly but weakly present in Procyon.

CN.—In the case of *CN*, the $\Delta v = 0$ (λλ 3883, 3871, ...) and $\Delta v = -1$ (λλ 4216, 4197, ...) sequences of the violet system ($^2\Sigma \leftarrow {}^2\Sigma$) have been identified. The maximum line intensity reached is about 1. The *CN* red system lies, for the most part, outside the limits of Table 1; moreover, it is much weaker than the violet system and would not be observable in Procyon. Uhler and Patterson's[19] extensive wave-length table was used in making the identifications in the $\Delta v = 0$ sequence, in which only the λ 3883 (0, 0) and λ 3871 (1, 1) bands are recognizable. It is apparent that the maximum for the *CN* lines in the R branch of the (0, 0) band lies somewhere between R(25) and R(35). However, owing to the numerous cases of blending and masking, it would be imprudent to attempt even a rough determination of the rotational temperature for this molecule, using either the R or the P branches of the (0, 0) band or any of the other observed bands. Probably this observation of the approximate location of a *CN* maximum, which is in accord with what is to be expected in stellar atmospheres, best serves only to strengthen the identification of *CN*. T. Heurlinger's[20] monograph on band spectra provided the necessary laboratory material for the $\Delta v = -1$ sequence. In addition to the λ 4216 (0, 1) and λ 4197 (1, 2) bands, a number of the stronger members of the λ 4181 (2, 3) band have been recorded.

On high-dispersion laboratory spectra the higher members of both the P and the R branches are resolved into doublets usually having a separation less than 0.10 A. Owing to the character of the stellar lines and the dispersion of our plates, only the simple averages of the laboratory wave lengths of the doublet components have been entered in Table 1. Immediately preceding the wave length is given the following abbreviated description of the location of the line in the violet *CN* band system: the branch and particular band, indicated by its v', v'' numbers without the customary parentheses.

CH.—The *CH* wave lengths and molecular notations have been taken from the recent work of E. Fagerholm.[21] The λ 4300 ($^2\Delta \leftarrow {}^2\Pi$) system, together with the (0, 0) and (1, 1) bands of the λ 3900 ($^2\Sigma^- \leftarrow {}^2\Pi$) system, fall within the limits of Table 1. In the strongest *CH* system, that of the λ 4300 region, the (0, 0) band has definitely been identified; the maximum line intensity reached is about 2, the P branch appearing somewhat weaker than the Q and R branches. In Table 1 the *CH* identifications have been entered in the same way as for *CN*, except that the four rotational component lines corresponding to each value of K'' have not been averaged in wave length. In the Q and R branches the points of maximum intensity fall in the neighborhood of the lines Q(12) and R(12); but, although blending and masking effects are much less troublesome than for the *CN* bands, the obscuration is enough to make it well-nigh impossible to ascertain the true intensity distribution. The (1, 1) band of the above system is almost superposed on the (0, 0) band. A few faint lines corresponding to the expected strongest members of the Q and R branches have been entered, since they presumably form the "peaks" of these branches protruding just above the limit of visibility of the plates. The λ 3900 system is not recog-

[19] *Ap. J.*, **42**, 434, 1915.

[20] *Untersuchungen über die Struktur der Bandenspektra* (Lund, 1918).

[21] *Arkiv för mathematik, astronomi och fysik*, A, Vol. **27**, No. 19, 1941.

nizable, but this may arise from the crowding of atomic lines in the region of the (0, 0) band, over 90 per cent of the members of this band being masked.

Fagerholm's[22] analysis of the *CD* bands provides ample material on which to base a search for evidence of this molecule; however, no trace of *CD* is revealed in our table of measures.

CH and CN in F-type stars.—For the F-type stars mentioned in the introduction we offer the following comments concerning *CH* and *CN*. Albrecht's[23] table does not cover the region of the strongest *CN* bands; but in the region of the *CH* 4300 system, although he does not actually identify any features with *CH*, the stronger lines appear to be present in his table. Swings and Struve[24] found both *CH* and *CN* to be "perhaps very faint" in Procyon, but no trace of these molecules was found in α Persei by them or by Dunham.[25] Since these stars are, respectively, a dwarf and a supergiant of the same spectral type, it is natural to seek an explanation of the absence of molecular lines in α Persei in terms of an absolute magnitude effect. Unless we choose to suppose that *C* and *N* are more abundant in Procyon than in α Persei, the theory offers no explanation; for, according to the present approximations of H. N. Russell,[26] Y. Cambresier and L. Rosenfeld,[27] and M. Nicolet,[28] there should be little difference or even a slight enhancement in the abundance of *CH* and *CN* in passing from an early dwarf to a giant of the same spectral type. But, when comparing observations of very faint features, we must bear in mind that much depends on the contrast, quality, and dispersion of the spectrograms compared, on the observers' ability to see faint features, and on the method of recording—that is, the microphotometer tracing versus the measuring microscope.

Of course, the absence of *CH* and *CN* in α Carinae[29] and ε Aurigae[30] is not surprising, since both of these stars are of earlier spectral type than Procyon. In γ Cygni, Swings and Struve[31] found *CH* and *CN* to be extremely faint, whereas Roach[32] does not list any molecular lines whatsoever. Both *CH* and *CN* have been observed by Swings[33] on microphotometer tracings of δ Cephei, whereas Krieger[34] was unable to record any individual lines; but, as he remarks, a weakening in the continuous background is more easily observed on tracings than in the eyepiece of the measuring microscope.

Other molecules.—An F-type spectrum is not a promising one in which to conduct a search for any but the molecules most prominently present in the sun. For this reason no attention has been given to molecules which have not already been considered in astronomical sources. Following *CN* and *CH*, the next strongest band systems which have been identified in the spectrum of the solar disk are those of *OH* and *NH*. The bands of *OH* (the λ 3064 system) and the strongest bands of *NH* (the λ 3360 system) unfortunately lie outside the region covered in the present study. The (0, 0) band of the λ 4502 system ($^1\Pi \leftarrow {}^1\Sigma$) of *NH*, the only known band of this molecule falling within the limits of the observed spectrum, arises from an electronically excited state and is, according to H. D. Babcock,[35] faintly recognizable in the sun; as might be expected, it was found to be absent in Procyon. In view of the comparative weakness of the *CH* and *CN* lines in our spectra and the high ratio of intensity of the solar *CH*, *CN*, *OH*, and *NH* lines compared to those of the other solar molecules, one should not expect any evidence in Procyon of the remaining molecules present in the sun. A check on this expectation was made by

[22] *Ibid.*

[23] *Loc. cit.*

[24] *Phys. Rev.*, **39**, 142, 1932.

[25] *Loc. cit.*

[26] *Ap. J.*, **79**, 317, 1934.

[27] *M.N.*, **93**, 710, 1933; L. Rosenfeld, *M.N.*, **93**, 724, 1933.

[28] *Mém. Soc. R. Sc., Liège*, **2**, 89, 1937.

[29] Greenstein, *op. cit.*

[30] Frost, Struve, and Elvey, *op. cit.;* Morgan, *op. cit.;* and Swings and Struve, *Ap. J.*, **94**, 307, 1941.

[31] *Phys. Rev.*, **39**, 142, 1932.

[32] *Ap. J.*, **96**, 272, 1942.

[33] *M.N.*, **92**, 140, 1931.

[34] *Loc. cit.*

[35] *Ap. J.*, **102**, 154, 1945.

searching for the strongest lines of a number of the molecules which have been considered in the sun; no evidence was found for the molecules C_2, MgH, and SiH or for AlH, H_2, and SiF, which were once considered, at least tentatively, as showing in the solar spectrum but are really absent, according to Babcock's[36] recent study of the chemical compounds present in the sun.

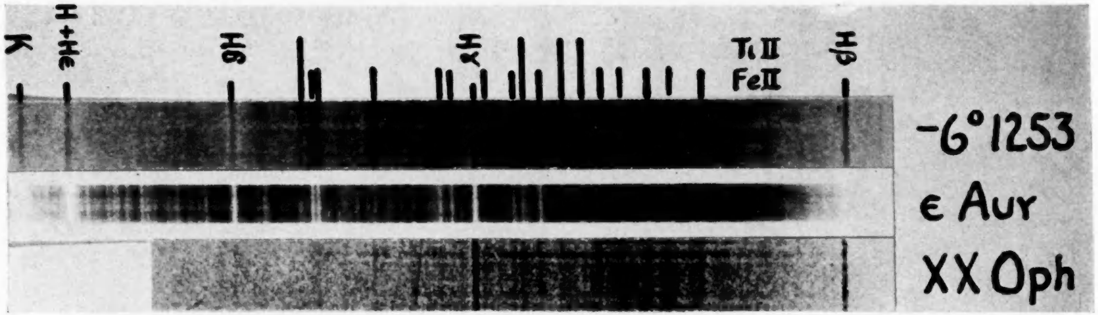
Our spectrograms were taken close to the time of Procyon's culmination; consequently, the telluric lines should be weak. Indeed, the only such atmospheric lines which appear (listed as "Atm." in Table 1) are those of intensity greater than about 3 in the *Revised Rowland*.[37] Those unblended telluric lines recorded on more than one plate show no real shift in their wave lengths uncorrected for radial velocity. In order that the stellar and telluric lines may form a consistent system in wave length, the uncorrected values have been entered in the first column of Table 1 for those lines considered to be, for the most part, of telluric origin. For the identifications of atmospheric lines in the fourth column the wave lengths and intensities from the *Revised Rowland* have been given.

The author is sincerely grateful to Dr. O. Struve for the table of measures, to Dr. P. Swings for his constant encouragement and counsel, and to Mrs. B. W. Sitterly (née Charlotte E. Moore) for her kind co-operation in discussing a number of critical identifications and in lending most of her *Revised Multiplet Table* before its publication.

[36] *Ibid.*

[37] St. John, Moore, Ware, Adams, and Babcock, *Pub. Carnegie Inst. Washington*, No. 396, 1928.

PLATE VIII



The spectrum of BD $-6°1253$, with ϵ Aurigae and Merrill's "iron star," XX Ophiuchi, for comparison. The spectrogram of BD $-6°1253$ was obtained on March 4, 1945, and that of XX Ophiuchi on June 3, 1944. The spectrograms were obtained with the 40-inch refractor.

NOTES

AN INTERESTING EMISSION-LINE STAR NEAR THE ORION NEBULA

In the course of the classification of spectra of faint stars in the neighborhood of the Orion nebula an emission-line star was found which appears to be new. It was first noticed on an objective-prism plate which had been taken with the 24-inch Schmidt telescope of the Case School of Applied Science in Cleveland. The exact nature of the spectrum could not be ascertained, other than that it was made up of a combination of emission and absorption features. A slit spectrogram was obtained later with the 40-inch telescope; it is reproduced in Plate VIII.

The spectrum originates from the star BD $-6°1253$, which appears to be immersed in the extensive nebulosity which extends a considerable distance east and south from the Orion nebula proper. This nebulosity absorbs background starlight and normally would be accounted a dark nebula; it does, however, shine feebly, and there are occasional patches which are fairly conspicuous. That it is an extension of the Orion nebula there can be little doubt. The star BD $-6°1253$ is the nucleus of a small, very bright condensation, which is catalogued as NGC 1999. The star itself has a photographic magnitude in the neighborhood of 10.8; on long exposures with small instruments, however, the star and nebula produce an almost stellar image having a photographic magnitude of approximately 8.5.

The spectrum is composed of very bright emission lines of *H*, *Fe* II, *Ti* II, and *Ca* II, which are superposed on a continuous spectrum containing broad absorption lines at $H\gamma$ and $H\delta$. From the latter it is estimated that the spectrum of the star itself corresponds to a dwarf of Class A or late B. The remarkable feature is the great strength of the enhanced emission lines of iron and titanium; they appear to be much stronger relative to the bright hydrogen lines than in Merrill's "iron star," XX Ophiuchi, at the time when the Yerkes plate of the latter star was obtained. The spectrum of BD $-6°1253$ will be more fully described at a later time.

On the assumption that NGC 1999 is an extension of the Orion nebula, we can derive its parallax from an investigation now in progress at the Yerkes Observatory of the distance of the stars and nebulosities in the Orion aggregate. The true distance modulus is in the neighborhood of

$$m - M = 8.0$$

which corresponds to a parallax of

$$\pi = 0\overset{''}{.}0025 \, .$$

The apparent photographic absolute magnitude of BD $-6°1253$ is therefore about $+2.8$; this value will, however, almost certainly require an appreciable negative correction because of absorption effects in the nebulosity.

We are indebted to Dr. J. J. Nassau for the opportunity to obtain the objective-prism plates used.

W. W. Morgan
Stewart Sharpless

Yerkes Observatory
Williams Bay, Wisconsin
January 1946

THE SPECTRUM OF HD 151932

The star HD 151932[1] has been considered by Beals[2] as the prototype of the WN7 spectral class. Its spectrum was previously described by Mrs. Payne-Gaposchkin[3] and by Swings,[4] of whom the latter gave a list of wave lengths in the range λλ 3888–6561. Last year Struve[5] studied the star for radial velocity, and the spectrograms secured suggested the presence of more emission features than the ones contained in Swings's list. Consequently, in June, 1945, the star was reobserved with the Cassegrain quartz spectrograph

TABLE 1*

λ	*I*	Identification
3889....	2	*He* I 3888.646 (10); *H*8 3889.051; *He* II 3887.44
3921....	1	*He* II 3923.48
3930....	1	
3937†...	1	*N* III 3938.52 (4); *N* III 3934.41 (3)
3942....	1	*N* III 3942.78 (1)
3960....	0	*He* I 3964.727 (4)
3971....	4	*H*ε 3970.074; *He* II 3968.43
4026....	5	*He* I 4026.189 (5); *He* II 4025.60
4057....	20	*N* IV 4057.80 (2)
4074....	0	....
4080....	1	*Ca* III 4081.74 (5)?
4088....	3	*Si* IV 4088.863 (10)
4098....	10	*N* III 4097.31 (10)
4103....	25	*N* III 4103.37 (9); *H*δ 4101.737; *He* II 4100.04
4110....	2	....
4116....	10	*Si* IV 4116.104 (8)
4122....	1	*He* I 4120.812 (3)
4200....	6	*He* II 4199.83; *N* III 4200.02 (6); *N* III 4195.70 (5)
4322....	1	*N* III 4323.93 (2)
4328....	1	*Si* IV 4328.22 (4)?; *N* III 4328.15 (3)
4335....	0	*N* III 4335.53 (4)
4340....	20	*H*γ 4340.468; *He* II 4338.67; *N* III 4339.52 (3)
4347....	0	*N* III 4348.36 (5)
4378....	6	*N* III 4379.09 (10)
4388‡...	....	*He* I 4387.928 (3)
4453....	0	....
4470....	8	*He* I 4471.477 (6)
4481....	1	*N* IV 4479 (P); *Mg* II 4481.226 (100)
4486....	1	*B* III 4487.46 (5)
4496....	1	*N* IV 4495 (P); *B* III 4497.58 (10)
4511....	10	*N* III 4514.89 (7); *N* III 4510.92 (6)
4520....	2	*N* III 4523.60 (4); *N* III 4518.18 (3)
4533....	1	*N* III 4534.60 (4); *N* IV 4528 (P)
4542....	15	*He* II 4541.49
4605†...	4	*N* V 4603.2 (P)
4615....	1	*N* V 4619.4 (P)
4636....	25	*N* III 4640.64 (10); *N* III 4634.16 (9); *N* III 4641.90 (7)
4685....	50	*He* II 4685.682
4735†...	....	*N* IV 4733 (P)
4862....	15	*H*β 4861.332; *He* II 4859.323
5207†...	0	....
5412....	20	*He* II 5411.524
5799....	2	*N* IV 5794 (P); *N* IV 5812 (P); *N* IV 5828 (P); *N* IV 5846 (P)
5819....	2	
5841....	2	
5877†...	15	*He* I 5875.634 (10)
6170....	0	*He* II 6170.6
6217....	4	....
6313....	2	*He* II 6310.8
6335....	0	....
6389....	1	*N* IV 6383 (P)
6411....	2	*He* II 6406.3
6564....	40	*H*α 6562.817; *He* II 6560.099; *He* II 6570.0
6592....	0	....
6680....	5	*He* I 6678.149 (6); *He* II 6683.2

* There is a weak absorption at λλ 4460 and 6283. Estimated intensity 0 indicates that the presence of the line is uncertain. The laboratory intensities are quoted from Moore's revised *Multiplet Table* (*Contr. Princeton U. Obs.*, No. 20, 1945).

† Indicates that there is a violet absorption.

‡ According to Swings. The line is not visible on the spectrograms taken by the writer.

of the 82-inch reflecting telescope of the McDonald Observatory. Two exposures of the star were made, one on Eastman 103*a*-F emulsion and the other on Eastman Process emulsion; the dispersion used was 55 A/mm at $H\gamma$. Table 1 gives the list of wave lengths, the estimated intensities, and the identifications.

JORGE SAHADE

YERKES AND MCDONALD OBSERVATORIES
December 28, 1945

[1] CD−41°10972. $\alpha = 16^{h}48^{m}5$; $\delta = -41°45'6$ (1945.0). The star is located in the cluster NGC 6231, in Scorpius; its photographic magnitude is 6.4.

[2] *Trans. I.A.U.*, **6**, 248, 1938; *J.R.A.S. Canada*, Vol. **34**, Pl. VIII, 1940.

[3] *Harvard Bull.*, No. 843, p. 7, 1927.

[4] *Ap.J.*, **95**, 112, 1942; *McDonald Contr.*, No. 40.

[5] *Ap.J.*, **100**, 189, 1944; *McDonald Contr.*, No. 96.

NOTE ON THE PERIOD OF U CEPHEI

Observations made on the nights of January 28 and February 2, 1946, of the light-curve of U Cephei at principal minimum have shown that the difference (O − C) between the observed time of central minimum and that computed on the basis of Wendell's elements,

$$\text{Mid-eclipse} = 2407890.301 + 2.4928840E\ ,$$

is equal to

$$(O - C)_{\text{Wendell}} = +0.198 \pm 0.005 \text{ days} \qquad (E = 9613)\ .$$

When this value of (O − C) is compared with the value computed on the basis of the equation given by Tchudowitchev,[1]

$$O - C = -0.0050 + 0.0000165E + 0.063 \cos\,(0.028^{\circ}E + 235^{\circ})\ ,$$

the agreement is satisfactory, as can be seen in Figure 1.

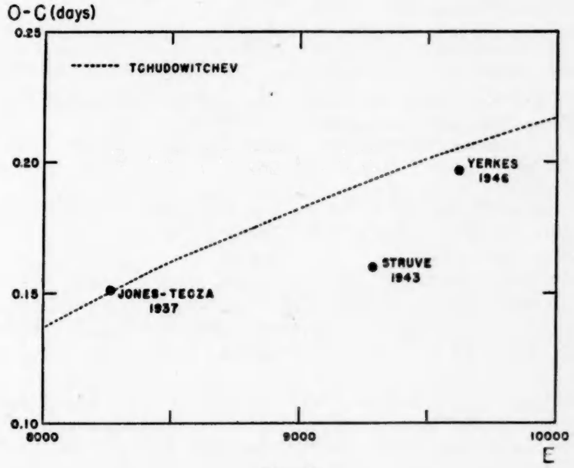


FIG. 1

This result is of interest, since Struve,[2] on November 8 and 13, 1943, found that Wendell's elements gave

$$(O - C)_{\text{Wendell}} = +0.160 \text{ days} \qquad (E = 9285)\ ,$$

which falls considerably below Tchudowitchev's curve, whereas observations made in 1937 by Jones[3] and Tecza[4] fell almost exactly on this curve.

Struve[2] suggested that the departure which he observed was due either to an error in Tchudowitchev's period for the apsidal motion or to residual fluctuations in the period. The present observations seem to confirm the latter suggestion.

JOHN G. PHILLIPS
ARNE SLETTEBAK

WILLIAMS BAY, WISCONSIN
February 27, 1946

[1] *Bull. Engelhardt Obs., Kasan U.*, No. 17, 1939.

[2] *Ap. J.*, **99**, 226, 1944.

[3] *Harvard Bull.*, No. 905, 1937.

[4] *Acta Astr.*, C, **3**, 131, 1937.

REVIEWS

Atlas der Restlinien, Vol. II: *Spektren der seltenen Erden*. By A. GATTERER and J. JUNKES, with the collaboration of V. FRODL. Vatican City: Vatican Press, 1945. Pp. 350+18 tables +45 photographic prints. $60.

Seven years after the appearance of the first volume of the *Atlas* of "ultimate lines" (*raies ultimes, Restlinien*), containing the spectra of 30 elements, which was reviewed in this *Journal*[1] the same authors have published this magnificent work for the spectroscopists.

This spectroscopic atlas fills a great need in the existing literature and furnishes a precise tool for the physicist and chemist. The second volume is divided into two parts: the first contains the text and the tables of wave lengths, the second the photographic reproductions of the spectra. It is dedicated to the Pope with these words:

> Pio XII Pont. Max./ optimarium artium fautori insigni dum omnia bello flagrant/ angelo pacis/ opus hoc/ ad scrutandam naturam intimam/ elementorum rariorum/ imaginibus luce expressis elaboratum/ grato pioque animo/ auctores dedicant.

The authors are to be commended for being able to finish their task in spite of the difficulties caused by the war. F. Gatterer gives an account in the Preface of the increasing difficulties:

> The quiet so necessary for scientific work was almost continuously disturbed by the roar of the guns, by the rattling of the machine guns, and the bursting of the bombs very close to the Specola. The danger was so great for the persons and instruments as to render it necessary to move away from Castel Gandolfo until the end of hostilities.

Notwithstanding these circumstances, the work is excellent and represents one of the first attempts to give the arc and spark spectra of all the rare earths in known and homogeneous excitation conditions, with large dispersion.

It is known that the greatest difficulty in studying spectra which are so rich in lines as are those of the rare earths is that of the impurities, because of the analogy of their chemical properties. We can, therefore, appreciate the ability and care exercised by the authors in the use of materials from various sources in correctly identifying the few lines of the foreign elements which are present in the final plates.

The photographs of the spectra were made with a direct-current arc, and sometimes at high voltage with a condensed spark, in order to make a preliminary classification of the lines; the conditions of exposure were accurately controlled with a photoelectric device.

In all, the *Atlas* contains the spectra of 18 elements: 14 of the rare earths and 4 of the homologues of lanthanum: scandium, yttrium, zirconium, and thorium, which have properties like those of the rare earths. For each element the existing literature is given, with a brief description of the lines and the band spectra, a table of the ultimate lines (*Analysenlinien*), an account of the impurities present in the spectrum, and the wave lengths measured by the authors, together with the intensities estimated in the arc and in the spark, the stages of ionization, and the bibliographical references.

The second part contains the photographs of the spectra on 45 plates printed on Agfa Lupex paper in size 30×40 cm. Each photograph gives in negative print (dark lines on white background) a spectral region for six different elements. The arc spectra are given on 10 plates for the region from 2265 A to 7600 A, while the spark spectra are on 5 plates, limited to the spectral region from 2265 A to 4530 A. For each spectral region and for each element the photographs show 5 spectra: 3 of the element with various exposures and 2 of the comparison spectrum of iron. In the spaces between the spectra, the wave lengths of the principal lines are written in, to the hundredth of an angstrom. The photographs are accompanied by a series of tables, which give the ultimate lines of each element, and by a general index of the tables with the values of the mean dispersion in the various spectral regions. The *Atlas* contains 41,605 wave lengths, of which about one-fourth have been measured by the authors.

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[1] *Ap. J.*, **91**, 479, 1940.

One Hundred Years of the Pulkovo Observatory. (In Russian.) Papers contributed by the MEMBERS OF THE OBSERVATORY STAFF. Moscow and Leningrad: Academy of Sciences of the U.S.S.R., 1945. Pp. 271.

The destruction of the Pulkovo Observatory during the siege of Leningrad is doubtless mourned most bitterly by those of us who were at one time connected with it. However, it cannot fail to depress every astronomer who is at all aware of the glorious tradition of this institution. Dr. Gould is said to have called it the astronomical capital of the world;[1] and Newcomb, writing of it in 1903,[1] said: "From its foundation it has taken the lead in exact measurements relating to the motion of the earth and the positions of the principal stars." Pulkovo's great heritage is well described in this centennial history, which was completed before the siege of Leningrad; but the introduction, which was written later, leaves little doubt as to the extent of the calamity. Very little has been salvaged, aside from a few of the larger objectives and a small part of the library, which was one of the richest astronomical libraries in the world.

The book is a collection of twenty-three articles written in 1940 and 1941 by sixteen Pulkovo astronomers to mark the observatory's hundredth anniversary in 1939. The first chapter gives a brief outline of the general course of events. Subsequent chapters deal with the histories of the programs of individual instruments, accounts of the various astronomical and geodetic expeditions, theoretical studies, and plans for the future. The description of the astrometric work opens with a survey of the entire astrometric program at Pulkovo. Similarly, there are two general chapters dealing with the development of astrophysics preceding the more detailed articles. Finally, there are chapters on the teaching of postgraduates in astronomy and geodesy, and on the time service, the library, the museum, and the archives.

It is widely recognized that the greatness of Pulkovo Observatory had its origin in the masterplan of its founder, Wilhelm Struve. It was his good fortune to be given a free hand in designing and constructing a first-class observatory for purely scientific research; probably it was the first government observatory which was not built as a naval or a geodetic institution, although it is true that the training of geodesists in practical astronomy was, from the start, made a small part of the observatory's activities. Struve's program was sidereal astronomy in the old sense. There were three aspects of it: (1) the precise determination of absolute right ascensions and declinations of a limited number of stars and the extension of this absolute system to a larger number of fainter stars; (2) new determinations of the constants of precession, nutation, aberration, and refraction; and (3) the observation of double stars. Much of this remains live research material to this day. That the program has yielded extremely valuable results which underlie our modern studies in stellar dynamics and statistics need hardly be pointed out. But it is well to realize that Struve himself clearly visualized the significance of his plan. In his "*Etudes d'astronomie stellaire*" he deals with the same problems of stellar astronomy which we now associate with the names of Seeliger, Schwarzschild, and Kapteyn.

At the outset there were five principal instruments: a transit instrument, a vertical circle, a meridian circle, a transit in the prime vertical, and a 15-inch refractor. The design of the instruments and the execution of the programs were notably successful, except in the case of the transit in the prime vertical. The chapters dealing with the details of these make very good reading for anyone interested in fundamental astronomy; but it seems to this reviewer that the one concerning the vertical circle by B. A. Orlov merits special mention. The determination of absolute declinations is perhaps the knottiest problem of fundamental astronomy, and Orlov has contributed an absorbing chapter on it, describing the difficulties and how they were surmounted, to a great extent, in the course of nearly one hundred years of uninterrupted work. According to Newcomb: "In the hands of C. A. F. Peters one observation with this instrument [the vertical circle] was worth as much as twenty, thirty or even forty made by routine observers with the meridian circle."[2]

The original 15-inch refractor and the 30-inch, erected in 1885, were primarily designated for double-star work; but, curiously enough, very little is said about it in this volume, in spite of nearly half a century of very valuable observations by Otto Struve. The 30-inch refractor was also used for observations of the satellites of planets, and as early as in 1892 it was being used for the determination of radial velocities. Incidentally, one of the first tasks for the 15-inch was a survey of all stars down to the seventh magnitude in order to obtain a working list for the meridian instruments, since there was at that time no *Bonner Durchmusterung*.

[1] Simon Newcomb, *Reminiscences of an Astronomer*, p. 309. Houghton, Mifflin & Co., 1903.

[2] Simon Newcomb, *Compendium of Spherical Astronomy*, p. 344. New York: Macmillan Co., 1906.

In 1893 an astrographic telescope of the standard type inaugurated the work in photographic astrometry. One of the most important contributions made by means of this instrument is the volume of excellent proper motions in the Selected Areas which form such a valuable independent check on those derived at Radcliffe and have, in addition, the advantage of being reduced to absolute values. It was with this instrument, I believe, that Kostinsky discovered his famous photographic effect, later known as the "Eberhard effect." The "zone astrograph," erected in 1929, was designed for the determination of the positions and proper motions of the AG stars. Pulkovo agreed to observe the zone from the North Pole to +70°. The work was almost completed as the account was being written, and the instrument had been transferred to the "southern" station at Simeis (latitude +44°!). This station has been in operation since 1908.

In the early development of astrophysics Pulkovo took relatively little part, although as early as in 1867 two Zöllner photometers were installed and used for various observations, and in 1883 Hasselberg was appointed as astrophysicist, with the task of investigating the spectra of molecular gases in connection with the constitution of comets. After Bredikhin assumed the directorship in 1890, much more attention was devoted to astrophysics, and Belopolsky and Kostinsky were brought from Moscow.

Belopolsky is perhaps best known for his work on radial velocities and variations in line intensities. His name will always be associated with such stars as δ Cep, η Aql, ζ Gem, β Per, αCVn, and Polaris. He established the long-period variation in the orbital elements of Polaris, and his constant interest in solar physics culminated in the design of a large solar spectrograph with a dispersion of 0.76 A/mm; this he used for studying the rotation of the sun. A number of astrophysicists trained by him (Shajn, Albitsky, Melnikov, and others) continue in spectroscopic research, and many new lines of research have been opened up at Pulkovo and also at Simeis with the 40-inch reflector (installed in 1925). We may mention a catalogue of radial velocities of early-type stars, an investigation of the *TiO* bands in the spectra of long-period variables, studies of blue supergiants, and research concerning the *CN* absorption bands as criteria of absolute magnitude. Trials were also in progress for the determination of radial velocities with an objective-prism camera.

Bredikhin was the first to work in theoretical astrophysics, and excellent work has been done in this field in recent years. It is understandable, but nonetheless greatly to be regretted, that the name of Gerasimovič does not appear on the pages of the book. His brilliant career as astronomer and director was suddenly cut short in 1937. This tragedy is not referred to in the book, so that the history of the last decade is somewhat inadequate.

The work of Tikhov and his collaborators on photometry and colorimetry is described in a chapter by Tikhov himself. He began to use the 6-inch "Bredikhin" short-focus camera almost as soon as it was installed in 1905. He introduced the method of color filters and applied it to the observation of eclipsing variables. It was then that he discovered the much-disputed "Tikhov-Nordman effect," and this controversial issue is presented impartially and with restraint. Tikhov devised the method of estimating stellar colors from a single photograph, which he applied in the determination of the colors of faint stars in the Selected Areas. This method was later developed in this country at the Lowell Observatory. At the same time Beliavsky in Simeis, using one of the two 5-inch cameras, obtained photographic magnitudes of a large number of stars, including the Coma cluster, 2777 stars in the north polar AG zone, and 9400 AG stars in the zone from +40° to +45°.

Pulkovo's contributions to stellar statistics date from Wilhelm Struve, and in recent years many of the younger astronomers have devoted much time to the subject; of the older men, the names of Shajn, Eigenson, and Ogrodnikov are well known in this country. The problems of interstellar absorption and diffuse nebulae have occupied much of their attention. In some instances new observational techniques have been introduced, as with the "zone-astrograph" in connection with magnitudes and star counts.

The famous astronomical library at Pulkovo, which Newcomb[1] referred to as "perhaps the most complete in existence," had its origin in a collection made by Struve in 1838. It grew immensely in quantity and value when the libraries of Olbers and Naumann were purchased in 1841 and 1854, respectively. Frequent additions continued to be made, and by 1939 there were more than ninety thousand titles in the catalogue. Among the most precious acquisitions were the twenty-two volumes of Kepler's manuscripts presented to the Russian Academy by Catherine the Great and later transferred to the Pulkovo Observatory; these were kept in a special safe, and it is to be hoped that they were removed before the destruction of the building. However, we have no assurance of this, and the very existence of the manuscripts is barely mentioned.

Possibly this may be attributed to the point of view of the author of this section, as illustrated by the following quotation from Lenin with which the chapter concludes: "The fame and value of a library do not depend on the rarities it contains, such as 16th century editions or 10th century manuscripts, but rather they depend upon the extent of the circulation of its books among the common people. . . . ."

The book, as a whole, is a very readable account of the development of many phases of astronomy during the last hundred years. The love of astronomy and of Pulkovo Observatory, in particular, is evident in all the authors, young and old; and painstaking care has been taken to have the outward appearance of the volume reflect this attitude. It is printed on excellent paper, and the typography is very pleasing, with appropriate initial blocks at the beginning of each chapter and with attractive vignettes scattered throughout the book. There are many illustrations of astronomers and instruments.

A tradition of long standing at Pulkovo when I was there was its liberal spirit of free research. To some extent this was an outcome of the fact that the six senior astronomers were elected to their positions, in those days, by the Russian Academy of Sciences. Two occurrences during my rather brief association with the observatory may serve to illustrate this point. In the first case I was absent from Pulkovo as a junior member of an eclipse expedition when a new comet made its appearance. Before Dr. Backlund, the director, permitted Numerov to use the 15-inch refractor to observe it, he telegraphed me for my approval, since I was one of the two regular observers with that instrument! The second instance, chosen from a number that I recall, occurred when there was a scientific controversy between the director and one of the astronomers; this matter was referred for discussion to the general meeting of astronomers, while the director abstained from participating. In spite of the fact that an overwhelming majority took the director's point of view, the latter permitted the astronomer to publish his discussion, although not as an observatory publication. From conversations with older Pulkovo astronomers at that time, I learned that this liberal spirit was not simply characteristic of Backlund but that it had existed there since the time of the first director, who was regarded as *primus inter pares*. After reading the present volume I infer that something of the same atmosphere continued until the observatory's destruction.

A. N. VYSSOTSKY

Leander McCormick Observatory

Roemer and the First Determination of the Velocity of Light. By I. BERNARD COHEN. New York: Burndy Library, Inc., 1944. Pp. 63.

Astronomers, physicists, and all who are interested either in the history of science or in the varied activities of noted scientists will welcome this booklet, which contains far more than the title seems to suggest.

It contains not only an account of Roemer's determination of the velocity of light but also (1) a review of the opinions of his predecessors and contemporaries regarding the finiteness of that velocity, (2) remarks of his contemporaries regarding his determination, and (3) much biographical material. There is a bibliography of thirty-one entries concerning Roemer and his work, and in the one hundred and nineteen footnotes about eighty additional sources of information are listed.

It is to be regretted that Section 32, relating to thermometers, contains serious errors, not the least being the conclusion expressed in its final sentence: "Thus it is quite clear that it was Roemer who invented the modern thermometer and that the Fahrenheit thermometer should, in all fairness, be called the Roemer thermometer"! That claim, first advanced in a milder form by Kirstine Meyer (*Nature*, **82**; 296–98, 1910; **139**; 585–86, 1937), is not justified by the known facts and seems to rest on misinterpretations of memoranda and correspondence not intended for publication. The reviewer hopes to consider this section in more detail at another time. Suffice it now to say that he believes that J. Newton Friend was entirely correct in stating (*Nature*, **139**; 586, 1937) that what Fahrenheit learned from Roemer was not the thermometric scale but a handy procedure for calibrating thermometers so that they will accord with a certain antecedently defined scale. In spite of this unfortunate section, the booklet is of great value and of much interest.

The author explains that the main text of this booklet—which was copyrighted in 1942 but which appeared "on the 300th anniversary of the birth of Olaus Roemer, who was born on

September 25, 1644"—was originally printed in *Isis*, **31**; 327–79, 1940. But as only a few copies of the issue (No. 84) containing it reached the United States, it was decided to issue this American edition, and advantage was taken of the opportunity so offered to add certain "Addenda et Corrigenda" and a new index of names.

The booklet contains a portrait of Roemer, three facsimiles taken from Horrebow's *Basis astronomiciae* (title-page, and illustrations of the Round Tower and of Roemer's transit instrument), and facsimiles (1) of Roemer's report to the Paris Academy of Sciences—"Demonstration touchant le mouvement de la lumière" (*Jour. des Scavans*, Dec. 7, 1676)—(2) of the English translation of the preceding (*Phil. Trans.*, **12**; 893–94, 1677), and (3) of a manuscript folio, discovered in 1913, containing a list of eclipses of Jupiter's first satellite, in Roemer's handwriting.

Mr. Cohen has placed the historian of science in his debt by bringing together within a small compass so much information regarding Roemer and his work and by naming the original sources from which it was obtained.

N. Ernest Dorsey

Washington, D.C.

A Textbook of Elementary Astronomy. By Ernest Agar Beet. Cambridge: At the University Press, 1945. Pp. x+110. $2.00.

The author, who is connected with the Nautical College at Pangbourne, has written this book for use in schools. It "more than covers the syllabus suggested by the Science Masters' Association." The approach to the subject is experimental and historical, rather than mathematical.